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The Pseudo-*N*-values: proposal and practice

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ABSTRACT

The *N*-value by the standard penetration test (SPT) is commonly used for site characterisation in geotechnical engineering. S-wave velocity, typically estimated by borehole measurement or seismic survey, is also indicative of the strength of the ground.

Many researchers attempted to find precise relationships between these parameters. However, *N*-values estimated from S-wave velocity using these formulae are subject to substantial errors, and the errors are inevitable due to the different nature of the parameters.

The formula for pseudo-N value was first proposed in 2011 as $N = (60/Vs)^{2.5}$ (Suto, 2011). This is a simplified formula derived from the formulae found by the previous authors. By using this simple formula as a common practice, with understanding of existence of error, the results can be compared from site to site.

This presentation first compares the *N*-value and S-wave velocity in their natures, methods, practice and cost. Then it examines the previously published formulae and proposed formula of pseudo-N value. Some examples of use of the pseudo-N values are also presented at the end.

Keywords: *N*-value; S-wave velocity; pseudo-*N*-value; soil strength

1. Introduction

The standard penetration test (SPT) is commonly used for site characterisation in geotechnical engineering. This geotechnical parameter obtained from SPT is called "*N*-value". This is a non-dimensional value of count of blows of a cone of a standardised size, shape and material to penetrate a standardised depth driven by a standardised force. These standards are rather arbitrarily decided and the specification is enshrined in ASTM D1586/D1586M-18e1. This value is indicative of the strength of ground material and various conditions under the condition at the time of the SPT operation.

The multi-channel analysis of seismic waves (MASW) method estimates S-wave velocity structure. The S-wave velocity is a physical property of the material also indicative of its strength. Therefore, there is some correlation between the S-wave velocity and the *N*-value.

As the seismic surveys are typically conducted along survey lines, the distribution of S-wave velocity is usually expressed as 2-dimensional sections. This contrasts with the SPT which measures *N*-values only along the depth axis at one drill location.

Seismic surveys are generally less expensive than SPT for the same data density. It is not invasive to the ground of the sites.

Many researchers attempted to find relationships between these parameters. While these formulae are valid for the environment of the investigations undertaken, they cannot be applied ubiquitously.

2. *N*-values and S-wave velocity

In engineering context, ASTM D1586/D1586M-18e1 defines the procedure of SPT and use of *N*-values.

These standard values and procedure are somewhat arbitrarily defined and it presents little scientific justification in terms of physics concerned. Nevertheless, the *N*-value is a long established standard in practice and verbal descriptions of the sites are based on these values.

On the other hand, S-wave velocity is a physical property with a dimension $[L][T]^{-1}$ or commonly expressed by m/s in the SI System.

It can be directly measured by an S-wave velocity logging and a surface-to-borehole measurement in boreholes, and a seismic cone penetration test (SCPT). These surveys provide S-wave velocity profiles along the depth axis at one point of the ground surface.

S-wave velocity is also estimated by seismic surveys. such as S-wave reflection survey, S-wave refraction survey and surface wave surveys. Recently the surface wave methods are more commonly used than reflection and refraction methods. This is, perhaps, because field data acquisition to achieve sufficient signal to noise ratio for the S-wave reflection or refraction analysis is difficult.

There are two modes of the surface wave seismic surveys: passive and active methods. Passive survey records ambient vibration of the ground from natural and cultural sources. As these signals are dominated by long wavelength components of the surface waves, the passive wave method is capable of probing deep S-wave velocity structure to hundreds of metres to a few kilometres. This is, therefore used to investigate basin structures and large-scale town planning. The active surface wave method uses artificial impact source to stimulate the ground surface; typically by sledgehammer and weight dropping. While the surface wave theory and its possibility of use for ground investigation was described in 1970s (Aki and Richards, 1975, for example), it gained popularity in the 1990s, when advance of computer technology made its data analysis feasible. The MASW method (Park, et al., 1999) uses a linear seismic array for data acquisition.

Unlike the borehole surveys and SCPT, the seismic survey works on the ground surface. As it surveys along a line, it can cover number of points along the line in relatively short time. With several lines, it can achieve area coverage, too.

Table 1 summarises differences between S-wave velocity and *N*-value.

Table 1. Nature of S-wave velocity and *N*-value

	S-wave velocity	***N*-Value**
Measurement	Indirect measurement by seismic survey or borehole logging	Direct measurement by the standard penetration test (SPT)
Physical significance	Physical property Dimension: $[L][T]^{-1}$ Analytical relationship with other physical parameters	Standard parameter dimensionless Relationship with other parameters by empirical correlations
Establishment	Practical use since 1990s or later. MASW by Park et al (1999)	Well established since early 20th Century.
Coverage	Typically 2D section along a line 3D volume is possible	1D along the depth axis only
Environmental impact	Non-invasive	Disturbes ground surface
Cost	Low per point	Higher cost

As seen in the comparison in the second row of Table 1, S-wave velocity is a physical property of the material with dimension $[L][T]^{-1}$. With some other physical properties of the same material, such as density and P-wave velocity, measured or assumed, S-wave velocity can in turn be used to derive various physical properties like rigidity, Young's modulus and Poisson's ratio. On the other hand, the *N*-value has little backing of physics. For this reason, some conscientious geophysicists consider S-wave velocity is superior to *N*-value in describing strength of the ground (Inazaki (2006), for example).

The descriptive terms of the nature of the soils are defined according to *N*-values. Tables 2 and 3 show some examples of site characterisation using the *N*-values. The terms of these examples are commonly used all over the world.

Table 2. Terms and *N*-values for essentially cohesive soils

Terms	SPT *N*-Value	
Very soft	0 - 2	Not suitable for civil structures, good for park.
Soft	2 - 4	
Medium	4 - 8	Good for very light structure using proper methods.
Stiff	8 - 15	Good for low load bearing structures.
Very Stiff	15 - 30	Good for moderate load bearing structures.
Hard	>30	Good for high load bearing structures.

from Haque, et al. (2013)

Table 3. Terms and *N*-values for essentially non-cohesive soils

Terms	SPT *N*-Value	
Very loose	0 - 4	Not suitable for civil structures, good for park.
Loose	4 - 10	Good for very light structure using proper methods.
Medium dense	10 - 30	Good for low load bearing structures.
Dense	30 - 50	Good for moderate load bearing structures.
Very dense	> 50	Good for high load bearing structures.

from Haque, et al. (2013)

3. Relating S-wave velocity with *N*-values

From 1970s, many researchers attempted to find relationships between *N*-values and S-wave velocity from comparison under various locations and conditions. Some of them are from collection of several locations and others are site-specific. Inevitably, these formulae do not *exactly* convert S-wave velocity to *N*-value: there is always some estimation error. This is because these are fundamentally different properties as mentioned in the previous section: S-wave velocity is only one aspect of the nature of the ground and the N-value represents overall character of the soil.

Bellana (2009) studied previous effort of relationship between N-values and S-wave velocity and compiled over twenty formulae (Table 4). In this table S-wave velocity is in m/s. More recently, Crice (2022) examined the formulae and supported OYO's recent formula: $Vs = 97\ N^{0.314}$ or $N = (Vs/97)^{3.18}$.

Table 4. *Vs*-*N* relationships by various authors

Author(s)	ID	Formula
hibata (1970)	A	-
Ohba and Torinuma(1970)	B	$Vs = 84\ N^{0.31}$
Imai and Yoshimura(1975)	C	$Vs = 76\ N^{0.33}$
Ohta *et al.* (1972)	D	-
Fujiwara (1972)	E	$Vs = 92.14\ N^{0.337}$
Ohsaki and Iwasaki (1973)	F	$Vs = 81.4\ N^{0.39}$
Imai *et al.* (1975)	G	$Vs = 89.9\ N^{0.341}$
Imai (1977)	H	$Vs = 91\ N^{0.337}$
Ohta and Goto (1978)	I	$Vs = 61.4N^{0.5}$
Seed and Idriss (1981)	J	$Vs = 84\ N^{0.31}$
Imai and Tonouchi (1982)	K	$Vs = 96.9\ N^{0.314}$
Sykora and Stokoe (1983)	L	-
Jinan (1987)	M	$Vs = 116.1\ (N + 0.3185)^{0.31}$
Okamoto *et al.* (1989)	N	-
Lee (1990)	O	-
Athanasopoulos (1995)	P	$Vs = 107.6\ N^{0.36}$
Sisman (1995)	Q	$Vs = 32.8\ N^{0.51}$
Iyisan (1996)	R	$Vs = 51.5\ N^{0.516}$
Kamai (1966)	S	$Vs = 19\ N^{0.6}$
Jafari *et al.* (1997)	T	$Vs = 22\ N^{0.85}$
Kaku *et al.* (2001)	U	$Vs = 68.3\ N^{0.292}$
Jafari *et al.* (2002)	V	-
Hasanocebi and Ulusay (2006)	W	$Vs = 904\ N^{0.309}$
Ulugergerli and Uyank (2007)	X	$Vsu = 23.291\ \mathrm{Ln}(N)+405.61$
Ulugergerli and Uyank (2007)	Y	$V_{SL} = 52.9\ e^{0.011N}$
Dikmen (2009)	Z	$Vs = 58\ N^{0.39}$
Pitlakis *et al.* (1999)	AA	$Vs = 84\ N^{0.31}$
Hasanocebi and Ulusay (2006)	AB	$Vs = 104\ (N_{60})^{0.26}$

From Bellana (2009)

4. Critical examination of the conversion formulae

These formulae were made by fitting a line on the cross-plot of S-wave velocities and *N*-values. Many of these formulae have a common form of:

$$Vs = aN^k \quad \text{or} \quad N = \left(\frac{Vs}{a}\right)^{\frac{1}{k}} \tag{1}$$

with ranges:

$$19 < a < 107 \tag{2}$$

and:

$$0.2 < k < 0.8 \tag{3}$$

These ranges are considered very large. This suggests there is not a global equation which possibly relates the two parameters consistently. Whichever the equation is used, there is always an error in “conversion”. This is unavoidable in the fitting a line on a cross-plots. In fact, *N*-value is a result of combined effect of many elements of the ground in which S-wave velocity is a major contributor. There are other contributors, such as P-wave velocity, density, rigidity, Young’s modulus, Poisson’s ratio, porosity, compaction, grain size, soil type, degree of disturbance, burial history, temperature and moisture of the ground at the time of measurement, and so on whatever the conditions, quantifiable or not:

$$N = f(Vs, Vp, \rho, G, E, \nu, \varphi, \dots, a, b, c, \dots x, y, z \dots.) \tag{4}$$

The attempt to represent *N*-value by S-wave velocity alone is similar to projecting multi-dimensional space onto a fewer dimensional space. Therefore, presence of discrepancy similar to “rounding error” is inevitable.

Paying the due respect to all the effort of collecting and analysing the data to derive these formulae of previous works, the present author wonders the justification of such a precision with so many decimal places. In practice, as seen in Tables 1 and 2, the *N*-values are used in the context of loose ranges. When the requirement of *N*-value is between 8 and 15, for example, argument whether actual value is 9.345 or 12.987 is irrelevant.

5. Pseudo-*N*-value

From the above consideration, Suto (2011) proposed "pseudo-*N*-value ($\tilde{N}$)" with a formula between S-wave velocity and *N*-value using simple parameter set:

$$Vs = 60\tilde{N}^{0.4} \quad \text{or} \quad \tilde{N} = \left(\frac{Vs}{60}\right)^{2.5}. \qquad (5)$$

These simple parameters are arbitrary chosen within the range of (2) and (3), yet reasonably fit to the data originally compiled by Bellana (2009). Figure 1 shows the comparison of these formulas in the *N*-*Vs* plane.

As the pseudo-*N*-values are calculated from seismic survey result, it can display the structure beyond the depths SPT can feasibly reach or the strength SPT can only describe as "refusal".

Table 5 is estimated S-wave velocity ranges for the descriptive terms of soil classification

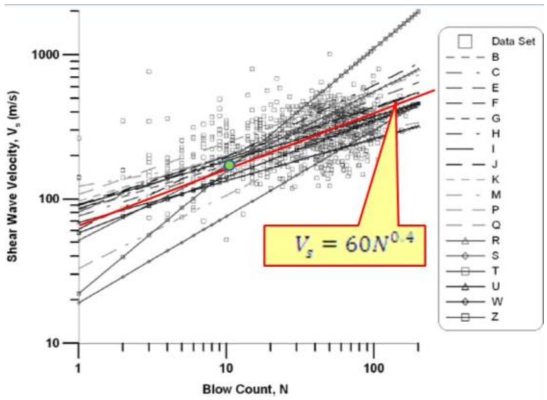

Figure 1. *N*-*Vs* plot by Bellana (2009) superimposed by the line of the proposed equation.

corresponding to Tables 1 and 2.

Table 5. Soil Classification with estimated S-wave velocity.

Essentially Cohesive Soils			Essentially Non-Cohesive Soils		
Terms	**SPT *N*-Value**	**S-wave Velocity (m/s) ***	**Terms**	**SPT *N*-Value**	**S-wave Velocity (m/s) ***
Very soft	0 - 2	0 – 79	Very loose	0 - 4	0 -104
Soft	2 - 4	79 - 104	Loose	4 - 10	104 – 151
Firm	4 - 8	104 - 138	Medium dense	10 - 30	151 – 234
Stiff	8 - 15	138 - 177	Dense	30 - 50	234 – 287
Very stiff	15 - 30	177 - 234	Very dense	> 50	> 287
Hard	> 30	> 234			

* calculated by $V_s = 60\tilde{N}^{0.4}$

6. Use of pseudo-*N*-value

As seismic surveys are carried out on a line, S-wave velocity structure is typically presented as a section along the line. Pseudo-*N*-values structure is estimated by converting S-wave velocity using Equation (5), and expressed in a section. Achieving an equivalent data density by SPT is, on the other hand, very costly.

Having a common formula to estimate *N*-value from S-wave velocity, comparison between sites where MASW surveys are carried out, hence experience in one site may be practiced in another site.

Figures 2 and 3 are examples of pseudo-*N*-value sections in essentially cohesive and non-cohesive environments.

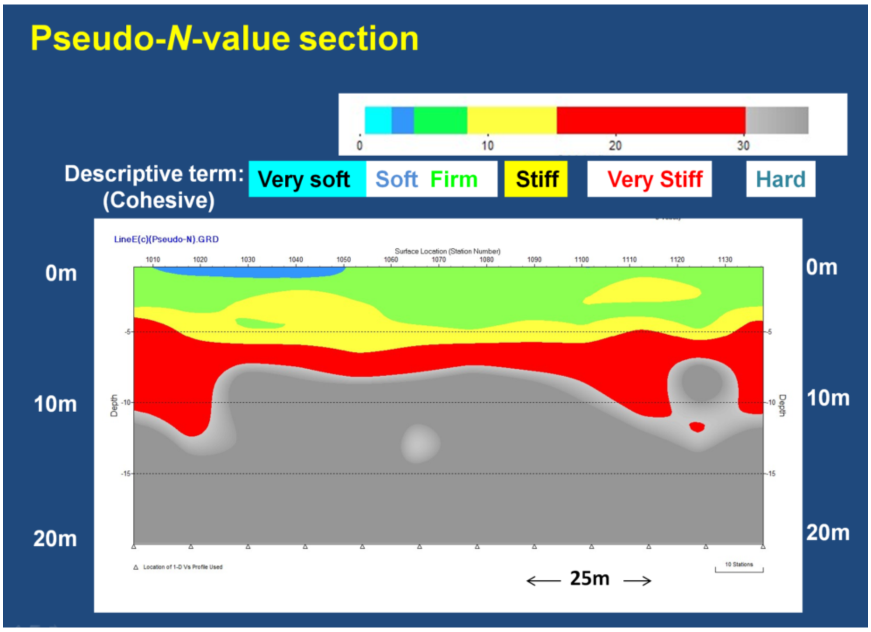

Figure 2. An example of pseudo-*N*-value section in cohesive environment

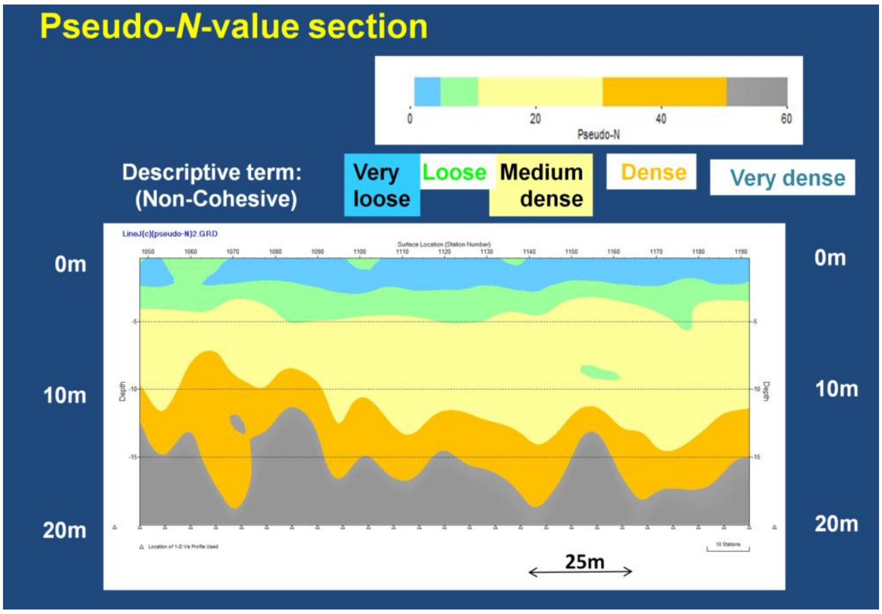

Figure 3. An example of pseudo-*N*-value section in non-cohesive environment

7. Discussion and conclusions

SPT *N*-value is commonly used by geotechnical engineers and it is well established as a descriptor of strength of the ground. S-wave velocity is a physical property. Both are related to the strength of the ground. S-wave velocity is estimated by seismic surveys, and can cover a large number of points quickly and inexpensively.

There is some correlation between *N*-value and S-wave velocity estimated by MASW seismic surveys. Many formulae to relate them have been proposed with an unreasonable precision. Conversion using these formulae inevitably incurs some errors. This is because the essential difference in the aspects these surveys measure:

S-wave velocity is only one physical property while *N*-value is a result of combined effects of many features of the ground. As all the formulae do not *exactly* relate these parameters, a simple formula is proposed as "pseudo-*N*-value". This formula is simple and provides as good an approximation of *N*-value as the other formulae with great precisions.

The pseudo-*N*-values are displayed along the survey line, like a velocity section, showing structure in terms of *N*-value.

Using the common formula, character of geotechnical sites are compared globally. The simple formula of equation (5), $\tilde{N} = \left(\frac{Vs}{60}\right)^{2.5}$, is proposed for the common use.

When geotechnical engineers see the seismic survey result expressed in pseudo-*N*-value, they must be aware that it is not exactly the same as *N*-value measured by SPT survey. Pseudo-*N*-value is an estimate of SPT *N*-value from S-wave velocity and the conversion inevitably causes some discrepancy.

However, using pseudo-*N*-value enables estimating spatial distribution or regional variation of *N*-values. By using the same formula ubiquitously comparison between different sites will be possible.

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