LAMINATE-LIKE FUNCTIONALLY GRADED COMPOSITES

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Summary. Despite their advantageous properties such as high stiffness-to-weight ratio, fiberreinforced laminates may suffer from characteristic failure modes such as fiber-matrix debonding or delamination. This paper introduces laminate-like functionally graded composites, which offer remedies against mentioned structural deficiencies thanks to their unitary structure. Besides their mechanical advantages, these materials can be additively manufactured, circumventing the lengthy fabrication processes of classical laminates. The proposed material concept is demonstrated on a simple flat plate, whose free vibration modes are computed via finite element analysis. The results show that natural frequencies computed for the graded composite plate converge to the ones obtained for the equivalent homogenous structure as the number of grading waves is increased. The presented results also form guidelines about the necessary mesh resolution to accurately model plates with different spatial grading frequencies.

1 INTRODUCTION

Fiber-reinforced laminates are extensively used in aerospace, automotive, and wind energy industries since efficient technologies in these sectors require sustainable, strong, and lightweight materials [1]. Besides their high stiffness-to-weight ratio, composite laminates can be stiffness-tailored to yield optimal mechanical responses [2–5]. However, fiber-reinforced laminates are prone to certain drawbacks despite the mentioned advantages. Due to its nonuniform structure, the material can suffer from additional failure types such as fiber-matrix debonding or delamination [6]. In addition, laminate manufacturing typically involves many steps such as fiber production, lamination, resin infusion, and curing. This series of operations lengthens the production process and increases expenses [7]. There are also certain design problems, where the ideal laminate consists of infinitely many layers (e.g., optimal design for the maximum fundamental frequency [8,9]), which cannot be achieved in practice.

This study aims to introduce novel laminate-like functionally graded composites to overcome the mentioned deficiencies of fiber-reinforced composites. This new type of composite architecture can be produced using functionally graded materials with specific patterns. Similar to the principle employed in a previous study [10], the stiffer and heavier material provides the primary load-carrying function, while the lighter and softer material transfers the loads and reduces the weight. The resulting composite has similarities with conventional laminates but its unitary structure offers significant advantages including improved machinability, suitability for additive manufacturing, and superior damage resistance.

To demonstrate the feasibility of the concept, finite element analyses are performed to perform the free vibration analyses for the flat plates made of the proposed graded composite and equivalent homogeneous material. The comparison of the results indicates that the shown periodical grading approach is promising for achieving unitary composites with favorable properties. The case studies also serve as guidelines about the necessary spatial grading frequencies to achieve sufficient structural uniformity as well as the required mesh resolution to achieve solution convergence.

2 PARAMETRIZATION AND MODELING

This section covers the details regarding the graded plate structure, material properties, finite element analysis, and material homogenization.

2.1 Functionally graded plate

The proposed material concept is demonstrated on a fully clamped square plate (Fig. 1). This combination of the plate geometry and boundary conditions is a popular laminate optimization case that leads to notable differences between constant and variable stiffness designs [11,12], which have straight and curved fibers, respectively. Therefore, it has been deliberately chosen to pave the way for future studies, which may address the application of the presented grading approach to variable stiffness plates with curved stiffening paths. The length (*l*), width (*w*), and thickness (*t*) of the plate have been selected as 400, 400, and 4 mm, respectively. The material is sinusoidally graded in the x direction, and the total number of grading waves is denoted by λ . Hence, the average volumetric ratio of each material is 0.5.

Figure 1: Schematics of the functionally graded plate model. The number of grading waves in the x direction is $\lambda = 2$ in this example.

2.2 Material properties

Aramid and polyamide 12 (PA12) are used as the stiffer and softer FGM constituents, respectively (see Table 1 for the relevant physical properties). The reason behind this selection is that both materials are polymers that can be used together in the additive manufacturing of composites [13]. The ease of manufacturing will facilitate future work regarding experimental validation of the proposed laminate-like material.

	Aramid [14]	PA ₁₂ [15]
Elastic modulus (GPa)	112.0	$2.0*$
Poisson's ratio	0.36	$0.37*$
Density (g/cm^3)	1.44	1.02

Table 1: Physical properties of Aramid and PA12.

* Average values across different printing directions and tensile test speeds.

2.3 Finite element analysis

The FEA is performed using the commercial software COMSOL. Fully clamped boundary conditions are applied by restraining the translational and rotational degrees of freedom for all edges. The plate is discretized using shell elements with quadratic displacement field. The required number of elements is investigated through convergence studies (see Sec. 3.2 for the details). Natural frequencies and mode shapes are computed as the responses of interest.

2.4 Material homogenization

In addition to FGM plate, an equivalent structure with the homogenized material properties is also analyzed for comparisons. For this purpose, COMSOL is used to create a repeating unit cell and determine its homogenized elasticity tensor. Figure 2 depicts the cubic unit cell that is discretized using 4 096 hexahedral elements (16 elements per axis). The obtained elasticity tensor is used as the input for the anisotropic material properties of the homogeneous plate.

Figure 2: Finite element model of the FGM unit cell used to obtain the homogenized material properties.

3 RESULTS AND DISCUSSION

The section covers the free vibration analysis results for the FGM plate. First, a convergence analysis is presented, where the natural frequencies are computed for different numbers of grading waves and finite elements. Then, the mode shapes obtained for different spatial grading frequencies are compared to the one computed using equivalent homogenized material.

3.1 Convergence analysis for grading frequency and mesh resolution

In this subsection, the spatial frequencies that make the FGM structure sufficiently uniform are determined. To this end, the first three natural frequencies of the FGM and equivalent homogenized plates are compared with each other. The number of elements required to accurately represent the patterned material also depends on the grading frequency. Therefore, both mesh resolution and grading frequency are treated as parameters in the investigations.

Figure 3 shows the results of the convergence analyses. The preliminary inspections revealed that the number of elements should be at least eight times the number of grading waves to capture the material distribution accurately. Hence, the results are computed until this limiting proportion for each mesh. As seen in the graphs, the natural frequencies of the graded plates converge to those of the homogeneous counterparts. For the mesh consisting of 256 elements along each of the x and y axes, the maximum difference between the natural frequencies computed for 16 and 32 grading waves is less than 2% across all modes. Therefore, this mesh comprising a total of 65 536 elements is chosen for the subsequent analyses.

Figure 3: Convergence of the first three natural frequencies of the FGM plates for different FE meshes and the increasing number of spatial grading waves.

3.2 Mode shapes

Following the convergence study, the mode shapes are analyzed for the selected mesh comprising 256 × 256 elements. Figure 4 shows the mode shapes for the plates, for which the

grading (in the x direction) starts and ends with the softer material (PA12). The mode shapes exhibit drastic changes when the parameter λ is altered between low values. This effect is very apparent when comparing the first or third modes calculated for the *λ* values of 1 and 2. As *λ* is increased, the mode shapes converge to those of the homogeneous panel shown in the last row. Close examination of the modes shapes also reveals that kinks form at the regions where the concentration of the softer material is high. The number of kinks is equal to λ - 1 for this grading configuration.

Figure 4: First three natural modes of the clamped square FGM plates, for which the grading (in the x direction) starts and ends with the softer material (PA12). The modes are computed for different numbers of spatial grading waves (*λ*) as well as for the equivalent homogenized material.

Next, the panels graded using the stiffer material (Aramid) at the two ends (in the x direction) are analyzed (Fig. 5). The significant changes in the mode shapes across low *λ* values can also be observed in this case. Even switching occurs between the second and third modes when *λ* is increased from 1 to 2. Kinks also appear for this grading arrangement, where their total number is equal to λ itself. For $\lambda < 8$, the altered number and location of the kinks causes the shapes to be notably different than the ones presented in Fig. 4. However, these differences diminish with increasing *λ*.

Figure 5: First three natural modes of the clamped square FGM plates, for which the grading (in the x direction) starts and ends with the stiffer material (Aramid). The modes are computed for different numbers of spatial grading waves (*λ*) as well as for the equivalent homogenized material.

4 CONCLUSIONS

This paper presents a new composite architecture, which relies on using periodically graded stiff and soft materials to attain laminate-like properties. The mechanical properties of the proposed composite were investigated by performing free vibration analyses on plate structures. The responses were also computed for the equivalent homogenous plates with the identical volumetric ratio of the constituent materials.

The mesh convergence analyses demonstrated that the number of elements should be at least eight times the grading waves to sufficiently capture the material gradient. As the number of grading waves is increased, the natural frequencies and mode shapes of the graded plates approach those of the homogenous ones. Close frequency values were achieved for 32 grading waves, where the differences became less than 1% for the second mode and around 3% for the first and third modes. The accurate representation of this configuration required using 256 elements along each in-plane axis, resulting in 65 536 elements in total. When a higher number of grading waves or more complex geometries are used, the necessary number of elements may get excessively large and pose a significant challenge for simulation-driven optimization tasks. Therefore, the development of meshless analysis techniques (e.g., Rayleigh-Ritz [16] or spectral Chebyshev [17]) is crucial for the computationally efficient analysis of graded composites.

The periodical grading concept proposed in this study paves the way for the development of composites with more complex patterns. The presented results are exclusive for a clamped square plate with longitudinal stiffening. Extensive future work is required to explore the effect of diverse variables including panel geometry, boundary conditions, material properties, and volumetric ratio. The grading approach can also be extended to design variable stiffness panels, which are subject to strict limitations on fiber curvature when made of classical laminates [18].

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