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MODEL-BASED AND DATA-DRIVEN DIGITAL TWINS FOR RAILWAY VEHICLE-TRACK INTERACTION MONITORING

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Summary. The concept of digital twins is a promising alternative approach for condition monitoring in the railway sector. In this work a digital twin for the vehicle-track interaction is presented, which is based on a physics-informed encoder-decoder convolutional neural network. The digital twin can be used bi-directionally. On the one hand, it can estimate the longitudinal rail profile from axle-box accelerations and, on the other hand, axle-box accelerations can be estimated from the rail profile. In this way, the model can be used to get actual information on the condition of the rail from onboard data and additionally to predict vehicle reactions to a specific rail longitudinal profile. The model is trained and tested on real data acquired with a shunter locomotive.

1 INTRODUCTION

Digital twins, as digital representations of physical assets or processes, have become popular tools for a variety of asset management and monitoring applications. They were primarily used in the manufacturing industry, but are also increasingly being used in other fields such as civil engineering and structural health monitoring.

Specifically, a digital twin is a digital, indistinguishable copy of a real asset that allows to provide information on the past and current condition of its counterpart. It simplifies the analysis and the condition monitoring, leading to effective forecasting of the status of the physical asset and its predictive maintenance.

As a part of the critical infrastructures, rail transportation systems require comprehensive and frequent monitoring. For this reason, digital twins are particularly interesting for the railway sector and offer the potential to improve the reliability and safety of rail transport.

Specifically, the vehicle-track interaction monitoring is important for maintaining ride quality, reducing wheel-rail impacts, rolling noise and improving safety and reliability. Indeed, the major source for vibration and noise lies in the wheel-rail interaction. Its surveillance, therefore, could lead to high improvements but the current inspection processes are expensive. Traditional methods of acquiring measurement data involve manual devices or dedicated vehicles, both of which are time-consuming and impractical during normal operations.
The use of in-service trains for data acquisition is a cost-effective alternative to conventional data acquisition. In this context, axle-box accelerometers, which measure vibrations related to the dynamic vehicle-track interaction, have proven to provide valuable information on the wheel [1] and track condition [5]. These monitoring techniques could enable digital twins of the infrastructure allowing to implement predictive maintenance strategies and reducing maintenance costs. However, besides onboard sensors, the development of suitable algorithms is required to link the the dynamic vehicle response, namely the axle-box accelerations (ABA) to rail surface irregularities. In this paper, this very challenge is tackled by introducing a digital twin for the vehicle-track interaction.

The remainder of this paper is structured as follows. In section 2 different physics based and data driven modelling methods for digital twins of dynamic systems are introduced and how they can be combined is explained. In section 3 the data, its acquisition and processing and the modelling method used in this study are detailed. In section 4 the results are presented and discussed. Finally, in section 5 a conclusion is given.

2 MODELLING APPROACHES

The decision on how to create a digital twin model depends on the system knowledge, data availability and application need. Particularly, two different approaches can be distinguished: physics-based modelling and data-driven modelling. Both Methods are based on different prerequisites with corresponding advantages and disadvantages, which are explained in more detail below.

2.1 Physics Based Modelling

The general idea underneath physics-based modelling is the ability to represent phenomena through well defined dynamics and mathematics. By understanding the role of forces, multi-bodies interactions and kinematic chains it is possible to describe the targeted system via differential equations to model input-output relations. However, two main obstacles have to be overcome: highly non-linear systems are difficult to model, especially when high-order differential equations are involved. Approximations have therefore to be introduced, by describing the dynamics with lower-order equations or via linearisation methods. Additionally, model parameters such as masses or equivalent spring-damper coefficients have to be estimated. This might be a lengthy and costly process according to the optimisation problem, and dedicated algorithms and solvers must be adopted. Based on the problem complexity, multiple solutions can be found, of which some of them might be nonphysical. It is therefore important to correctly evaluate their physical meaning and choose the correct one.

According to the selected model and its level of approximation, high prediction accuracy can be obtained. Nevertheless, it is important to find the best trade-off between model computational times and fidelity. High fidelity models are indeed characterised by complex equations that might take non-negligible time to be solved, becoming not suitable for real-time applications. Instead, the introduction of approximated or linearised versions leads to shorter computing times at the expense of some accuracy.

Besides the method relies on the selected model and its accuracy level, it is almost data-independent, meaning that high quality data is needed only for the parameter optimisation process and no large datasets are required, making it suitable for scenarios in which acquisition of reference data is an expensive process.

Another substantial advantage of such approach is the capability of getting insights about the dynamics of the system, such as internal forces, kinematics or energy dissipation, since the respective components in the differential equations can be isolated and studied. This directly leads to a higher understanding of the system and its possible failure.

Regarding the specific case, modelling a shunting locomotive is a difficult task that would require the introduction of several non-linear components. Since the goal of this digital twin is to represent the generic behaviour of such a system, approximations can be introduced to fasten the modelling process.

The first approach is to completely neglect the system structure and exploit the input and output nature. Since the considered input is acceleration data collected at the axle-box level, and the estimated output is the vertical displacement along the track, a simple double integration of the signal, adequately filtered to avoid drift effects, can provide a good output estimation [5].

Another approach consists in modelling and optimising a simple yet widespread linear time-invariant (LTI) system called quarter car model (Fig. 1). It allows to describe a suspension system by taking into account spring-damper contributions, from both wheels and suspensions. The model is characterised by the following differential equations

$$\begin{aligned} m_s\ddot{x}_s &= -k_s(x_s - x_{us}) - c(\dot{x}_s - \dot{x}_{us}) - (F_c + F_k) \\ m_{us}\ddot{x}_{us} &= k_s(x_s - x_{us}) + c(\dot{x}_s - \dot{x}_{us}) - k_{us}(x_{us} - u) + (F_c + F_k) \end{aligned} \tag{1}$$

that can be written in the classical state-space form

$$\begin{cases} \dot{x} = \mathrm{A}x + \mathrm{B}u \\ y = \mathrm{C}x + \mathrm{D}u \end{cases} \tag{2}$$

by introducing the following state matrices

$$A = \begin{bmatrix} -\frac{c}{m_s} & \frac{c}{m_s} & -\frac{k_s}{m_s} & \frac{k_s}{m_s} \\ \frac{c}{m_{us}} & -\frac{c}{m_{us}} & \frac{k_s}{m_{us}} & -\frac{k_s+k_{us}}{m_{us}} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{3}$$

$$B = \begin{bmatrix} 0 \\ \frac{k_{us}}{m_{us}} \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

$$C = \begin{bmatrix} \frac{c}{m_{us}} & -\frac{c}{m_{us}} & \frac{k_s}{m_{us}} & -\frac{k_s+k_{us}}{m_{us}} \end{bmatrix} \tag{5}$$

$$D = \begin{bmatrix} \frac{k_{us}}{m_{us}} \end{bmatrix} \tag{6}$$

Particularly, the lumped mass m_{us} models the wheel and k_{us} its stiffness. m_s is instead used to model the axle frame and k_s and c the primary suspensions. By correctly choosing their values, the system results to be stable. Since the input for this case is acceleration y and the desired output is the vertical displacement $u(t)$, the model has to be inverted to be further used. Specifically, this can be achieved via the method illustrated in [2]

$$\begin{aligned} &\mathrm{D}^* = \mathrm{D}^{-1} \\ &\mathrm{C}^* = -\mathrm{D}^{-1}\mathrm{C} = -\mathrm{D}^*\mathrm{C} \\ &\mathrm{B}^* = \mathrm{B}\mathrm{D}^{-1} = \mathrm{B}\mathrm{D}^* \\ &\mathrm{A}^* = \mathrm{A} - \mathrm{B}\mathrm{D}^{-1}\mathrm{C} = \mathrm{A} - \mathrm{B}^*\mathrm{C} = \mathrm{A} + \mathrm{B}\mathrm{C}^* \end{aligned} \tag{7}$$

With such manipulation, the inverted system in state-space representation turns to be

$$\begin{cases} \dot{x} = \mathrm{A}^*x + \mathrm{B}^*y \\ u = \mathrm{C}^*x + \mathrm{D}^*y \end{cases} \tag{8}$$

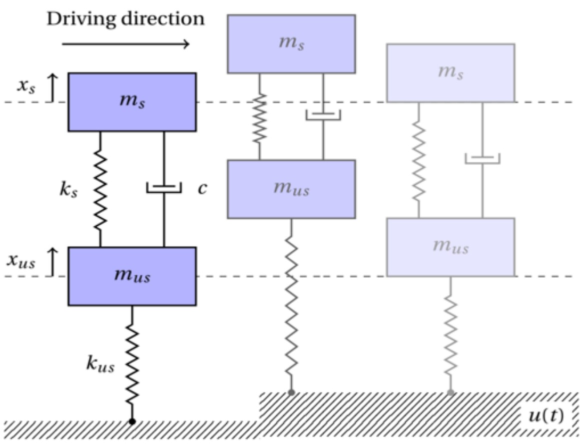

Figure 1: Representation of the quarter car (LTI) model as mechanical suspension system describing the dynamic vehicle-track interaction [11].

2.2 Data-Driven Modelling

Opposite to the physics-based, data-driven modelling relies on the high availability of data to discover patterns and underlying physical laws of the targeted system. It is often exploited when there is a limited knowledge about its dynamics and data collection is not costly. Indeed, by means of statistical tools and machine learning methods, it is possible to identify the correlation between the system inputs and its output.

According to the complexity of the system, simple statistical tools such as linear regression can perform well. For more complex or non-linear systems, instead, more powerful techniques must be adopted, including machine and deep learning solutions. In both cases the system is seen as a black box, whose parameters are automatically learned during the training process. However, there is no guarantee that such values have a physical meaning, thus it is impossible to get additional insights such as forces or energies apart from the system output.

For the data-driven modelling a large amount of data is needed and ground truth values are crucial for the model to be properly trained. Particularly, this approach is known to depend on the quality of the training data to guarantee high accuracy. Pre-processing techniques can be used to filter out noise and outliers from datasets, but this might not be enough to guarantee the desired quality level.

Among several techniques, machine learning methods have become the main tool to implement data-driven modelling, thanks to their possibility of handling both linear and non-linear systems. Additionally, with the black-box approach and the automatic learned parameters, high abstraction manipulation can be achieved, outperforming classical statistics methods.

Particularly, in sequence-to-sequence (Seq2Seq) modelling, Convolutional Neural Network (CNN), Recurrent Neural Network (RNN) and Transformers are usually adopted, according to the type of application. For time series and sequences, methods as RNN especially Long-Short Term Memory (LSTM) are preferred, as they are able to keep track of the input evolution over time thanks to internal state variables, but other solutions from the natural language and audio processing are also becoming increasingly widespread. CNNs, on the contrary, are much quicker to train as they can be fully parallelised during training [4]. The convolutional properties of CNNs enables them to model linear and non-linear time-invariant systems. In this context, Del Álamo, for example, developed a type of CNN (U-net) for ill-posed inverse problems where the forward operator is unknown [3].

2.3 Data and Physics Combined

In scenarios in which labelled datasets are available and some information regarding the physical system is known, a combination of physics-based and data-driven modelling can be developed to ultimately achieve better performance than either individual approach on its own. Indeed, this hybrid approach leverages the strengths of both methods. Physics-based models provide interpretability and adherence to known physical laws, while data-driven models, particularly neural networks, excel at capturing complex patterns from large datasets. This hybrid approach allows for improved accuracy, robustness, and generalisation, especially in scenarios where purely data-driven models might overfit or fail to extrapolate accurately.

Generally, in this hybrid implementations the model knowledge is combined with one or multiple neural networks, by introducing physics-based constraints in the loss function. Alternative implementations consist in using the network as ordinary differential equations solvers to estimate the state of the system for further computations, as well as injecting physics constraints in the network by adequately choosing its structure, e.g. applying linear layers to enforce linearity or using convolution operations to mimic transfer functions properties.

For this reason, this kind of hybrid model is known as Physics-Informed Neural Network

(PINN) and it opens the doors for tackling a wide range of problems in computational science. It can be used for substituting numerical solvers for partial differential equations (PDEs), as well as a data-driven approach for model inversion and systems identification [9].

3 MATERIAL AND METHODS

In this section the data set and its acquisition is introduced, the pre-processing steps are detailed and the proposed modelling approach for the digital twin is explained.

3.1 Data acquisition

The data used in this study was acquired with an onboard multi-sensor system installed on a shunter locomotive (Fig. 3) [5] operating on the railway network of the inland harbour of the city of Braunschweig (Germany, Fig. 2). The sensor system comprises two onboard accelerometers mounted at the left and right axle box of the shunter locomotive's front axle, which measure the dynamic vehicle-track interaction at a sampling rate of 20.625 kHz. In addition, an inertial measurement unit (IMU) in the vehicle cabin and a Global Navigation Satellite System (GNSS) are used for vehicle positioning and georeferencing of the sensor data [10]. A front-facing and rear-facing camera can provide environment perception [8] but are not relevant for this study (Fig. 3b).

The German Aerospace Center (DLR) have been carrying out measurements with this kind of onboard sensor systems during day-to-day shunting operations at the Braunschweig harbour since 2015. However, for this study dedicated measurement runs were carried out on the track segment highlighted in Figure 2. A total of 18 journeys were recorded, nine in each direction, three of which were carried out at speeds of around 25 km/h, 17 km/h and 12 km/h respectively.

The longitudinal rail profile was measured with a hand-pushed trolley (Fig. 3a). The measured height values are sampled every 2 mm. In order to obtain highly accurate and track-selective positions of the rail profile data, the positions of the measurement start and end points were recorded with a GNSS receiver and projected onto the associated tracks [5]. In this way, track positions were assigned to each measuring point.

Figure 2: Railway network of the Braunschweig. Red line indicates the test area for this study. Aerial image: Digital Orthophoto, 2014, Stadt Braunschweig (Abteilung Geoinformation)

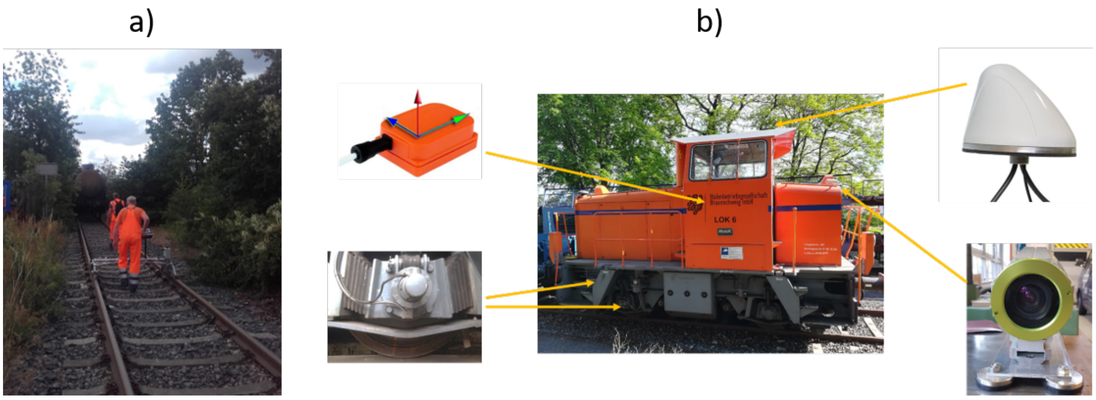

Figure 3: a) Conventional measurement trolley, b) Shunter locomotive with multi-sensor system, sensors from top-left clockwise: IMU in the vehicle cabin, GNSS antenna on the cabin roof, camera (driver's perspective view), ABA. Image source: DLR.

3.2 Pre-processing

The challenge of the modelling task in this study is that both the input data as well as the output data are sensor data, which are inherently noisy and limited by the specific sensor characteristics (e.g. sampling rate, bandwidth etc.). Furthermore, the data are sampled in different domains. The on-board data are gathered in the time domain at a sampling frequency of 20,625 Hz, while the reference data are sampled in the spatial domain every 2 mm. Both data sets are georeferenced using a digital map of the railway network. The resulting coordinates are track-ID and distance on the track. Since the onboard data and reference data are georeferenced individually, they are not perfectly aligned after georeferencing. To overcome these challenges, suitable pre-processing is required, which is described below.

In a first step, the ABA data is band-pass filtered forwards and backwards using a second-order Butterworth filter with cut-off frequencies of 1 Hz and 1,000 Hz and resampled at 2,667 Hz. The filtering reduces the noise while still providing a sufficiently large bandwidth to detect track irregularities with a wavelength between approximately 0.01 m and 3 m given the speed of the shunter locomotive.

In order to compensate for any remaining spatial shifts between the two data sets, the following steps are performed: The ABA data is double integrated to obtain displacements, which are then interpolated to the the discrete distance points of the reference data. In this domain the spatial shift between the ABA and reference data can be calculated by means of cross-correlation [6]. The reference data are corrected for this shift and mapped to the discrete time steps of the ABA data. For the forward and back transformation between time and spatial domain the vehicle speed from the georeferencing is used. The transformation to the time domain is important, since the physical relationship between the rail profile and the ABA is described with respect to time (see section 2.1). The reference data in the time domain is then band-pass filtered using the same filter previously used to filter the ABA data.

Finally, the data was spilt into training, validation and testing data sets. The testing data set comprises ten journeys in total measured at different speeds and directions. Before training the input as well as the target data sets were normalised by subtracting the mean and dividing by the standard deviation of the training data sets in order to make the training faster and more stable.

3.3 Proposed Modelling Approach

The aim of the model used in this study is to approximate the dynamic vehicle-track interaction related to imperfections of the rail longitudinal profile. Pure physics based modelling is limited for our approach since the model parameters describing the mechanical spring-damper system are unknown. Additionally, the broad frequency content of the signals to be modelled entails a high level of computational complexity. Therefore, the goal is to develop a primarily data-driven surrogate model, which incorporates some physical knowledge or constraints.

As detailed in section 2, the vehicle track interaction can be described as an LTI system. Nevertheless, it is well known that the wheel-rail interaction is nonlinear, specifically the contact stresses are usually a nonlinear function of the deformation. In contact mechanics, the wheel-rail contact is often described as Hertzian contact problem [7]. Therefore, it is important to use a model that can represent this non-linearity. In addition, the inverse of an LTI system is only causal and stable for LTI systems with minimum phase, which cannot be guaranteed for the dynamic wheel-rail interaction. These requirements leads us to the use of CNNs as they are powerful tools to model nonlinear time-invariant systems and therefore build the backbone of the model used in this study. In order to comply with the non-linearity, the convolutional layers in the model use nonlinear activation functions.

The model consist of two sub-models following an encoder-decoder architecture (Fig. 4), where the encoder represents a surrogate model for the inverse problem, namely predicting the longitudinal rail profile from the ABA and the decoder a surrogate model for the forward problem reconstructing the ABA form the rail profile. Both, encoder and decoder are trained together but can be used separately for inference.

To provide physically meaningful results, a physical constraint is imposed on the model. The simplest physics model that does not depend on any mechanical systems' parameters is the assumption that the displacement of the axle equals the rail profile or in mathematical terms the ABA is equal to the second derivative of the rail profile with respect to time. This physical assumption is incorporated in the model by adding an additional term to the loss function. The loss function then reads:

$$L = \|\hat{u}(\Phi_{enc}) - u\|_2^2 + \lambda\beta\|\hat{a}_d(\Phi_{enc}, \Phi_{dec}) - a\|_2^2 + \lambda(1-\beta)\|\hat{a}_{ph}(\Phi_{enc}) - a\|_2^2 \ , \tag{9}$$

where $\|\cdot\|_2^2$ denotes the squared L^2-norm. The first term represents the misfit between the predicted state $\hat{u}$, which is the longitudinal profile recovered from the encoder-CNN and the measured ("true") rail profile u. The second term is a measure of the misfit between the output from the decoder CNN, the reconstructed acceleration $\hat{a}_d$, and the "true" acceleration a measured with the ABA sensor and the third term incorporates the physical constraint, where $\hat{a}_{ph}$ is the second derivative of the predicted state $\hat{u}$ with respect to time.

The training is performed by minimising the loss function with respect to the model parameters, namely the weights of the CNNs of the encoder (Φ_{enc}) and decoder (Φ_{dec}). The different contributions to the loss are weighted by the loss term weights λ and β, which were found empirically.

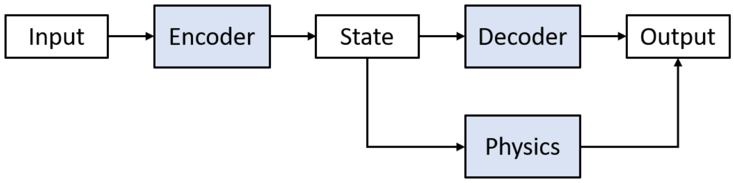

Figure 4: The physics informed encoder–decoder architecture: The input is the ABA data. The encoder consist of convolutional layers and predicts the state, namely the longitudinal rail profile. The output contains two reconstructions of the ABA data. The first is reconstructed from the state via the decoder, which, like the encoder, consists of convolutional layers, and the second via the physics model (second derivative of the state with respect to time.)

4 RESULTS AND DISCUSSION

In this section the results from model training and testing are presented. Only the data from the right-hand rail was used for training and validation, while testing was carried out with the data from the left-hand rail of a journey that was not considered during training.

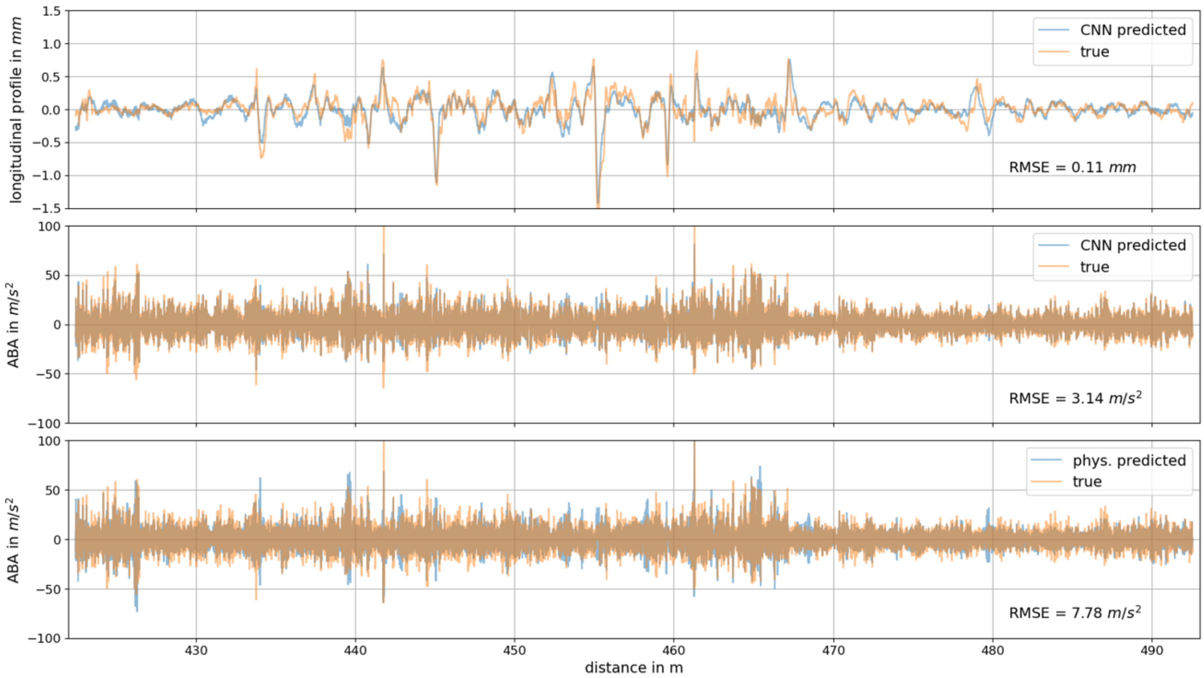

Figure 5: Training results. Top: predicted (blue line) versus measured (true) rail profile, middle: predicted (blue line) versus measured (true) ABA, bottom: second derivative of predicted rail profile (blue line) versus measured (true) ABA.

The comparison of the training results (Fig. 5) with the test results (Fig. 6) shows that the rail longitudinal profile is predicted with sufficient accuracy using the encoder CNN for both the training data and the test data. The root mean square errors (RMSE) are around 0.1 mm. It can be noticed, that the predicted rail profile is less spiky than the measured "true" one. This behaviour is reasonable considering the different measurement systems. The ABA system is mounted on a heavy locomotive able to push dirt and other small obstacles of the track while the hand-pushed system might not.

The difference between the predicted ABA from the decoder CNN using the training data and test data is more distinct. However, in both cases the CNN-based predictions are better than those using the simple physical model. Furthermore, the reconstruction losses of the CNN decoder output and the physical model provides measures of the trustworthiness of the model in the absence of reference data.

In order to evaluate the effect of the physics constraint, the model was trained without this constraint, which is equivalent to setting $\beta = 0$ in equation 4. The testing results of this model are shown in Fig. 7. The RMSE of the predicted longitudinal rail profile is similar to the physics-constrained model. However, the reconstruction of the ABA data is much worse, especially that of the physical model. This is due to the fact, that small changes in the rail profile can lead to large changes in ABA, which makes the regularisation provided by the physics constraint essential.

Additionally, the physics constraint improves stability during training and the generalisation capabilities of the network.

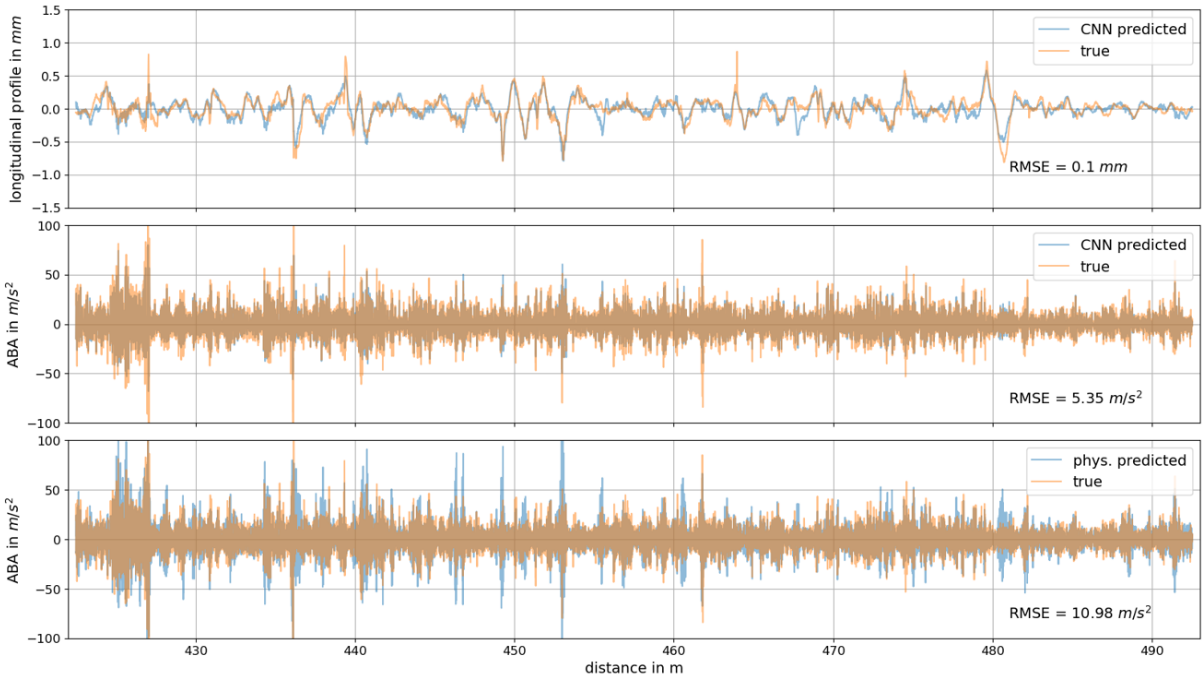

Figure 6: Test results. Top: predicted (blue line) versus measured (true) rail profile, middle: predicted (blue line) versus measured (true) ABA, bottom: second derivative of predicted rail profile (blue line) versus measured (true) ABA.

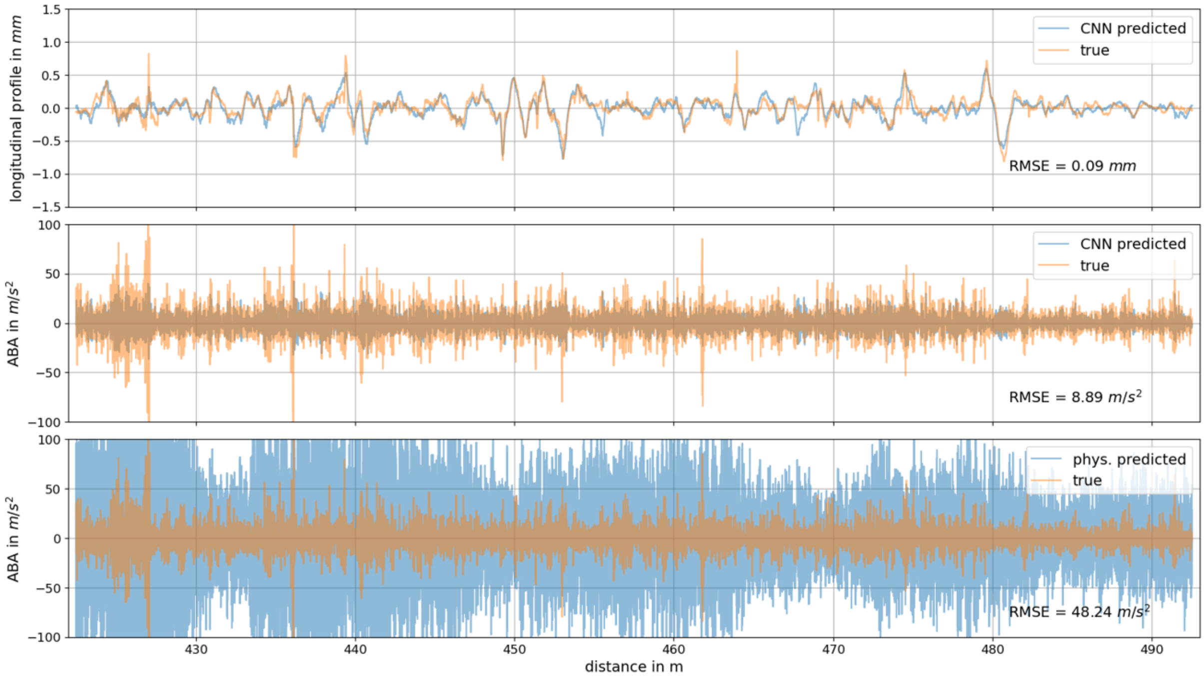

Figure 7: Test results without physics constraint. Top: predicted (blue line) versus measured (true) rail profile, middle: predicted (blue line) versus measured (true) ABA, bottom: second derivative of predicted rail profile (blue line) versus measured (true) ABA.

5 CONCLUSIONS

Condition monitoring with in-service trains provide data continuously, which can be used as input to a digital twin of railway networks containing past and present information on the rail condition. Specifically, in this paper it was shown that ABA can be used to reconstruct the longitudinal rail profile employing a digital twin model of the dynamic vehicle-track interaction. It was demonstrated, that in the absence of physical model parameters, data-driven models provide a promising alternative, especially if combined with physical knowledge. Incorporating physical constraints into the data-driven model enhances stability and ensures physically meaningful results.

The autoencoder-like structure of the proposed model provides a surrogate model for the forward and inverse problems and can therefore be used to predict the longitudinal rail profile from the dynamic vehicle response and vice versa to predict vehicle response from the rail profile.

The presented approach can readily be applied to other forward or inverse problems of dynamic systems.

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