IX International Conference on Computational Methods for Coupled Problems in Science and Engineering
COUPLED PROBLEMS 2021
E. Oñate, M. Papadrakakis and B. Schrefler (Eds)

HIERARCHICALLY DECOMPOSED FINITE ELEMENT METHOD FOR THE COUPLED FOUR FIELDS OF THE FLUID-STRUCTURE-PIEZOELECTRIC-CIRCUIT INTERACTION

Prakasha Chigahalli Ramegowda*, **Daisuke Ishihara**†

*†Department of Intelligent and Control Systems,
Kyushu Institute of Technology, 680-4 Kawazu, Iizuka, Fukuoka, 820-8502, Japan
e-mail: prakasha@solid.mse.kyutech.ac.jp

Key words: Fluid-structure-piezoelectric-circuit interaction, hierarchal decomposition, Partitioned iterative method, Finite element method

Abstract. A partitioned iterative method based on hierarchical decomposition is proposed for providing numerical modeling and analysis of the piezoelectric energy harvester which is involved in coupled fluid-structure interaction, coupled electro-mechanical, and a controlling electrical circuit for piezoelectric structural applications in energy harvesting. This circuit-integrated piezoelectric structural application in energy harvesting surrounded by fluid media takes the form of natural four-way coupling of fluid flow, the structure, the electromechanical effect of the piezoelectric material, and the electrical circuit. This can be formulated exactly as a fluid-structure-piezoelectric-circuit interaction. These coupled four fields are hierarchically decomposed into the fluid-structure interaction, structure-piezoelectric interaction, and piezoelectric-circuit interaction interactions. Then these subsystems are decomposed into each field. The proposed finite element method enables to reuse of existing techniques because of its modularity. Furthermore, scalability to multiphysics and multisystem couplings is expected. There are some numerical approaches in particular monolithic coupling methods are studied which are computationally expensive and leads to an ill-conditioned coefficient matrix. Nevertheless, accurate modeling for predicting the characteristics of this four-way coupling using partitioned methods has not yet been developed. This method enables an investigation of piezoelectric structures in fluid with complex geometry, material composition, and attached electrical circuits to the harvester. A flexible piezoelectric bimorph harvester in the converging channel is analyzed to demonstrate the efficiency of the proposed method. The results indicate that the method captures the coupled effect accurately.

1 Introduction

The circuit-integrated piezoelectric devices surrounded by fluid media take the form of natural four-way coupling of fluid flow, the structure, the electromechanical effect of the piezo-

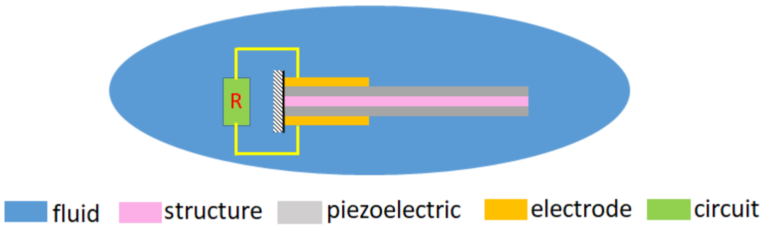


Figure 1: Subdomains of the multiphysics coupled problem

electric material, and the electrical circuit. This type of multiphysics coupling is a common phenomenon that appears in flow-induced piezoelectric energy harvesting [1, 2]. As shown in Fig. 1, this piezoelectric energy harvesting technology simultaneously involves the coupled interaction of a composite piezoelectric structure and a surrounding fluid, the electric charge stored in the piezoelectric material, and a controlling electrical circuit attached to the harvester. The large deformation of a thin piezoelectric device by the fluid flow causes a strong interaction with the electric field and the surrounding fluid, and inversely, these two fields significantly affect the structural behavior. Also, the electrical charge in the circuit affects the electromechanical behavior of the piezoelectric oscillator because of the coupled direct-piezoelectric and inverse-piezoelectric effects. At the same time, the piezoelectric device behaves as a kind of capacitance for the circuit, which affects the electrical charge in the circuit. This can be formulated exactly as a fluid-structure-piezoelectric-circuit interaction. To predict the functional properties of such smart future devices and to increase their performances, a mathematical and numerical model of the complex physical system is required to allow a systematic computational analysis of the complex phenomena and multiphysics coupled properties.

On the mathematical modeling front, Amini et al. [3] recently proposed a numerical model for modeling piezoelectric energy harvester from fluid-structure interactions. Their numerical model was a combination of FEM for structure-piezoelectric-circuit interaction in which single degree of freedom (SDOF) is used for electric circuit and open FOAM solver for fluid. In their finite element models considered a linear variation of electrical potential through-the-thickness of the piezoelectric continuum. Several research works [4–12] clearly pointed out that the electromechanical coupling is partial in case of a linear approximation. Yang [13], Klinkel and Wagner [14] was demonstrated that linear approximation was incorrect. Further, Yang [13] investigated and shown that a quadratic approximation of electric potential through-the-thickness of the piezoelectric continuum is necessary in bending-related problems. Akaydin et al [15]. analyzed energy harvesting from unsteady turbulent flow by placing a piezoelectric beam in the wake of a circular cylinder at high Reynolds number. They used commercial FLUENT software to analyze the coupled interactions of the aerodynamics of turbulent flow, the piezoelectric structure under vibration, the electrical response of the piezoelectric material and the harvester electronic circuit. They used SDOF model for the piezoelectric beam. Both the piezoelectric structure and electric circuit governing equations are reduced to a second-order- ordinary differential equations. Erturk et al [16] investigated that SDOF model may yield highly inaccurate

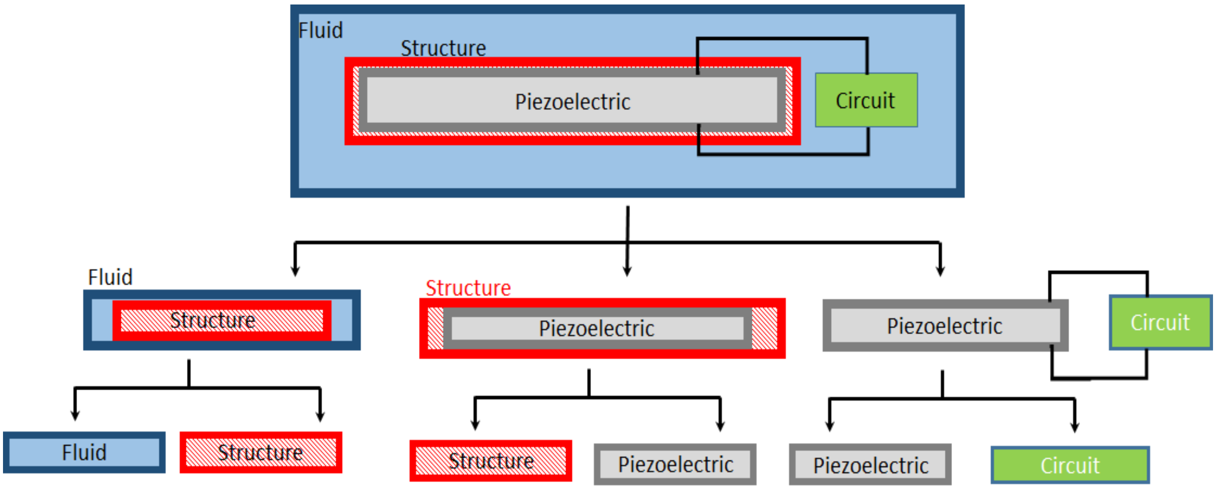


Figure 2: Hierarchical decomposition of the fluid-structure-piezoelectric-circuit interaction .

results for transverse vibrations of piezoelectric beams and can not accurately predict the power outputs of the harvester. Very recently, Ravi et al. [17] had proposed simultaneous solution method also called as monolithic method wherein the fluid, the piezoelectric structure with electrode, and the electrical circuit is formulated as a single monolithic equation and solved simultaneously using FEM. Monolithic method solves the coupling term directly, that is, it is strongly coupled, and as a consequence, it is numerically stable and accurate in general [18–20]. Nevertheless, this formulation leads to an ill-conditioned coefficient matrix [21–23]. Also, software modularity is difficult with monolithic methods, that is, it is difficult to reuse the existing technologies developed in different domains, and ad hoc software development is required [10, 24]. Therefore, development of a numerical simulation for fluid-structure-piezoelectric-circuit interaction using FEM considering software modularity, scalability to multiphysics and multisystem coupling, strong coupling, and solution accuracy is very important. This can be expected using a partitioned iterative coupled algorithms [25–29].

In the proposed method, the fluid-structure-piezoelectric-circuit interaction system is decomposed into subsystems hierarchically. This system is decomposed into the fluid-structure interaction (FSI), structure-piezoelectric interaction, and piezoelectric-circuit interaction using a iterative algorithm is similar to the partitioning of structure-piezoelectric-circuit interaction [30, 31], structure-piezoelectric interaction [11, 12], fluid-structure-piezoelectric interaction [25], and fluid-structure-electrostatic interaction[26]. This type of framework can be characterized as hierarchical decomposition. Here, the FSI subsystem is further split into the fluid-structure velocity field and the pressure field using the projection method [32]. Then, the structure-fluid velocity field is further partitioned into the structure velocity field and the fluid velocity field using the explicit time integration for the fluid. The structure-piezoelectric interaction is decomposed into the structure field (inverse-piezoelectric field) and the direct-piezoelectric field. Similarly, the piezoelectric-circuit interaction system is decomposed into the direct-piezoelectric field and the circuit. In essence, the fluid-structure-piezoelectric-circuit interaction system is decomposed into each field of four distinct fluid, structure, direct piezoelectric, and circuit solvers, as shown in Fig. 2. A thin flexible piezoelectric bimorph harvester in a converging channel is solved using the proposed method. Results demonstrated that the proposed method is accurate analyzes coupling phenomena.

2 Strong-form governing equations

The complete set of mathematical model equations of the strong-form governing equations of the coupled problem of the fluid, structure, piezoelectric material connected to the electrodes, and the electrical circuit is given here. Eqs. (1)–(4) represents the Navier-Stokes equations for an incompressible fluid. The arbitrary Laqrangian-Eulerian formulation is employed.

$$\rho^f \dot{v}_i^f + \rho^f (v_j^f - \hat{v}_j^f) v_{i,j}^f = \sigma_{ji,j}^f + \rho^f g_i^f \text{ in } \Omega_f \quad (1)$$

$$v_{i,i}^f = 0 \text{ in } \Omega_f \quad (2)$$

$$v_i^f = \bar{v}_i^{\mathrm{f}} \text{ in } \Gamma_{\mathrm{f}}^v \quad (3)$$

$$\sigma_{ij}^f . n_j^f = \bar{g}_i^{\mathrm{f}} \text{ in } \Gamma_{\mathrm{f}}^\sigma, \quad (4)$$

Eqs. (5)–(9) represents the inverse piezoelectric effect of the piezoelectric material. Thesed equations also represents elastic structure if piezoelectric constant tensor e_{ijk} is zero. In the domain occupied by the piezoelectric material, the present electric field representing the direct-piezoelectric effect is described by Eqs. (10)–(14). For the piezoelectric material, constitutive relations (6) and (11) represent the electromechanical coupling (inverse-piezoelectric and direct-piezoelectric interaction) through the piezoelectric constant tensor e_{ijk}

$$\rho \dot{v}_i = \sigma_{ji,j} + g_i, \quad \text{in } \Omega_P \quad (5)$$

$$\sigma_{ij} = C_{ijkl}^E S_{kl} - e_{kij} E_k \text{ in } \Omega_P \quad (6)$$

$$S_{ij} = 1/2(u_{i,j} + u_{j,i}) \quad \text{in } \Omega^P \quad (7)$$

$$\sigma_{ij} n_j = \bar{g}_i \quad \text{in } \Gamma_P^\sigma \quad (8)$$

$$u_i = \bar{u}_i \quad \text{in } \Gamma_P^u \quad (9)$$

$$D_{i,i} = q \quad \text{in } \Omega^P \quad (10)$$

$$D_i = e_{ikl} S_{kl} + \varepsilon_{ik}^S E_k \quad \text{in } \Omega_P, \quad (11)$$

$$E_i = -\phi_{,i} \quad \text{in } \Omega_P \quad (12)$$

$$D_i n_i = \bar{q}, \quad \text{in } \Gamma_P^{\mathrm{q}} \quad (13)$$

$$\phi_i = \bar{\phi}_i \quad \text{in } \Gamma_{\mathrm{p}}^\phi \quad (14)$$

Eqs. (15)–(19) describes electrical circuit attached to the electrodes covering the piezoelectric material. Eq.(15) represents the electrical circuit described using Kirchhoff's lawwith load resistance R. Eq.(19) represents the equi-potential condition on the electrode making use of the surface boundary charge $\bar{q}$ in (13). Eqs.(19) and (30) are the potential continuity and charge continuity, respectively, representing a native coupling between circuit and the electrode.

$$R\dot{Q} + V_p = V_c \quad \text{in } \Gamma_e \quad (15)$$

$$\bar{\phi}_+ - \phi = 0 \quad \text{in } \Gamma_e^+ \quad (16)$$

$$\bar{\phi}_- - \phi = 0 \quad \text{in } \Gamma_e^- \quad (17)$$

$$V_p = \bar{\phi}_+ - \bar{\phi}_-. \quad \text{in } \Gamma_P^e \quad (18)$$

$$Q = \int_{\Gamma_e^+} q_+ \, \mathrm{d}\Gamma_{\mathrm{e}} = -\int_{\Gamma_e^+} q_+ \, \mathrm{d}\Gamma_{\mathrm{e}} \quad (19)$$

The interaction conditions on the interface between the fluid and the piezoelectric structure are imposed using the following geometric compatibility (20) and equilibrium conditions (21):

$$v_i^{\mathrm{f}} = v_i \equiv v_i^{\mathrm{fs}}, \quad \text{on } \Gamma_{\mathrm{fs}}, \qquad (20)$$

$$\sigma_{ij}^{\mathrm{f}}.n_j^{\mathrm{f}} + \sigma_{ij}.n_j = \bar{g}_i^{\mathrm{f}s}, \quad \text{on } \Gamma_{\mathrm{fs}}, \qquad (21)$$

3 Finite element equations and coupling strategy

The weak form involving the unknown variables of fluid velocity $\mathbf{v^f}$, fluid pressure $\mathbf{P^f}$, piezoelectric structure velocity $\mathbf{v}$, the electrical potential with in the piezoelectric material ϕ, and the total circuit charge Q is obtained using the weighted residual method for the above set of strong form governing equations. Employing the standard finite element discretization procedure leads to an algebraic set of equations and using heirarchical decomposition and partitioned iterative solver for system of algebraic equations one obtain a solution method for the multiphysics problem. Eqs. (22)–(25) represents the finite element spatial discretization to the incompressible Navier-Stokes Eqs. (1)–(4). Eqs.(26) and (27) represents the FE coupled inverse-piezoelectric and direct-piezoelectric effect of a piezoelectric material. The pure structural part of this coupled system is obtained using Eq.(26) with $\mathbf{K_{u\phi}}\phi = \mathbf{0}$ shown in Eq.(28). Similarly the electrodes are analyzed using pseudo-piezoelectric method leading to Eqs.(28) and (29) [12, 33, 34],

$$_{\mathrm{L}}\mathbf{M}^{\mathrm{f}}\mathbf{a}^{\mathrm{f}} + \mathbf{N}^{\mathrm{f}} + \mathbf{C}^{\mathrm{f}}\mathbf{v}^{\mathrm{f}} - \mathbf{G}^{\mathrm{f}}\mathbf{p}^{\mathrm{f}} = \mathbf{g}^{\mathrm{f}}, \qquad (22)$$

$$_{\mathrm{T}}\mathbf{G}^{\mathrm{f}}\mathbf{v}^{\mathrm{f}} = \mathbf{0}, \qquad (23)$$

$$\mathbf{v}_{\mathrm{c}}^{\mathrm{f}s} \equiv \mathbf{v}_{\mathrm{c}}^{\mathrm{f}} = \mathbf{v}_{\mathrm{c}} \qquad (24)$$

$$\mathbf{Q}_{\mathrm{c}}^{\mathrm{f}} + \mathbf{Q}_{\mathrm{c}} = \mathbf{g}_{\mathrm{c}}^{\mathrm{f}s}, \qquad (25)$$

$$\mathbf{M_{uu}}\ddot{\mathbf{u}} + \mathbf{K_{uu}}\mathbf{u} + \mathbf{K_{u\phi}}\phi = \mathbf{g}, \qquad (26)$$

$$\mathbf{K_{\phi u}}\mathbf{u} + \mathbf{K_{\phi\phi}}\phi = \mathbf{q_{ext}} + \mathbf{q_c} \qquad (27)$$

$$\mathbf{M_{uu}}\ddot{\mathbf{u}} + \mathbf{K_{uu}}\mathbf{u} = \mathbf{g}, \qquad (28)$$

$$\mathbf{K_{\phi\phi}}\phi = \mathbf{q_{ext}} + \mathbf{q_c}, \qquad (29)$$

Eqs.(30) and (31) represents the continuities of the electric potential and the electric charge at the interface between the piezoelectric continuum connected to the electrodes and the electric circuit,

$$\mathbf{V_p} = \bar{\phi}_+ - \bar{\phi}_- \qquad (30)$$

$$\mathbf{q_c} = \int_{\mathrm{S^c}} \mathbf{N}_\phi \left(\mathrm{Q}/\mathrm{S^c}\right) dS - \int_{\mathrm{S^c_-}} \mathbf{N}_\phi \left(\mathrm{Q}/\mathrm{S^c_-}\right) dS, \qquad (31)$$

where $\mathbf{M}^{\mathrm{f}}$ is the mass matrix of the fluid, $\mathbf{N}^{\mathrm{f}}$ is the convective term vector of the fluid, $\mathbf{C}^{\mathrm{f}}$ is the diffusion matrix of the fluid, $\mathbf{G}^{\mathrm{f}}$ is the divergence operator matrix of the fluid, $\mathbf{a}^{\mathrm{f}}$ is the acceleration vector of the fluid, $\mathbf{v}^{\mathrm{f}}$ is the velocity vector of the fluid, $\mathbf{p}^{\mathrm{f}}$ is the pressure vector of the fluid, $\mathbf{g}^{\mathrm{f}}$ is the external force vector acting on the fluid, $\mathbf{Q}^{\mathrm{f}}$ is the equivalent internal force vector including all effects of the fluid, $\mathbf{Q}$ is the equivalent internal force vector including all effects of the piezoelectric structure, the subscript L stands for lumping of the matrix, and the subscript T stands for transpose of the matrix. In the ALE formulation, the fluid convective term $\mathbf{N}^{\mathrm{f}}$ is expressed as $\mathbf{N}^{\mathrm{f}}(\mathbf{v}^{\mathrm{f}} - \hat{\mathbf{v}}^{\mathrm{f}})\mathbf{v}^{\mathrm{f}}$, where $\mathbf{M}_u u$ is the mass matrix of the piezoelectric structure, $\mathbf{K}$ is the mechanical stiffness matrix of the piezoelectric material, $\mathbf{K}_{\phi\phi}$ is the dielectric stiffness matrix of the piezoelectric material, $\mathbf{K}_{\mathrm{u}\phi}$ is the piezoelectric coupling coefficient matrix of the piezoelectric material, $\mathbf{u}$ is the vector of the mechanical displacements, ϕ is the vector of electric potentials, $\mathbf{g}$ is the vector of the external mechanical forces, $\mathbf{q_{ext}}$ represents

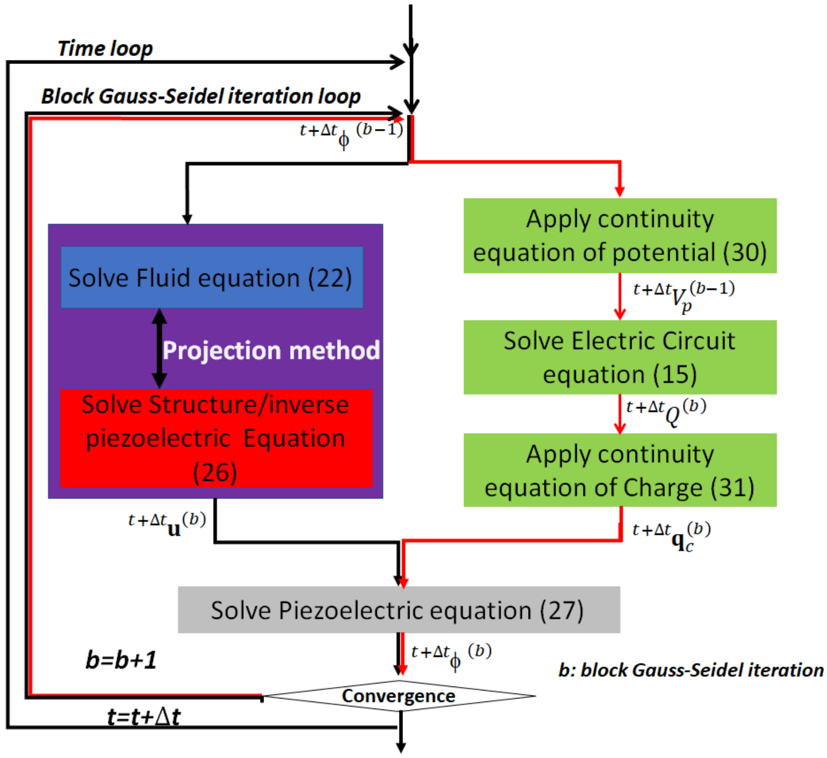


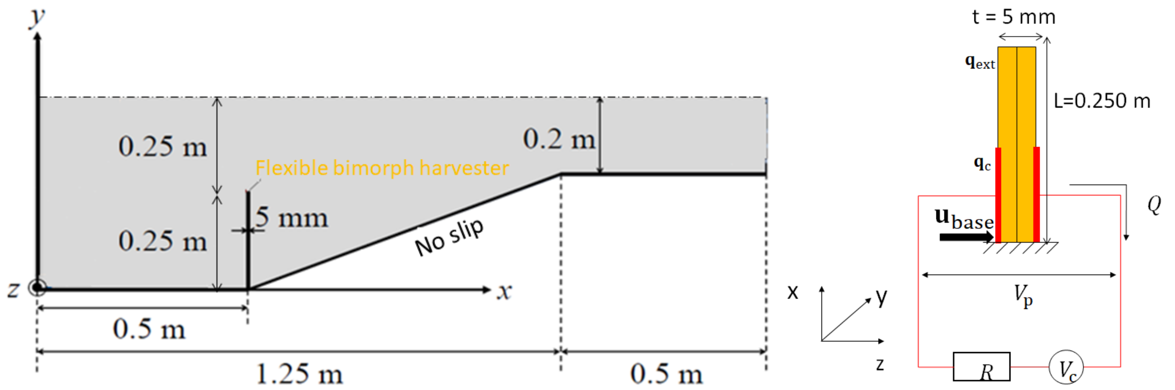
Figure 3: Analysis flow of the proposed hierarchically decomposed partitioned iterative algorithm for the fluid-structure-piezoelectric-circuit interaction

the vector of the external surface density charges on the piezoelectric material and $\mathbf{q_c}$ represents the vector of the external charge supplied by the circuit. Fig. 3 shows the analysis flow chart of the proposed method for the fluid-structure-piezoelectric-circuit interaction using the partitioned iterative algorithm. The projection method [32] using the algebraic splitting is used to solve the fluid-structure interaction. And finally, all the fields are coupled using loop union through Block Gauss-Seidel iterative procedure, as shown in Fig. 3.

4 Numerical example: A thin flexible piezoelectric bimorph harvester in a converging channel

4.1 Problem setup

A flexible piezoelectric bimorph harvester in the converging channel is analyzed in order to demonstrate the coupling between fluid, piezoelectric structure, and the circuit. The fulud domain is modeled using stabilized linear equal-order-interpolation velocity-pressure elements [35] (2,982 nodes and 8,400 elements), the structural part of the piezoelectric structure is modeled using mixed interpolation of the tensoral components shell elements [36] (22 nodes and 10 elements), while the electrical part of the piezoelectric bimorph is modeled using 20 node hexahedron solid elements (503 nodes and 60 elements). The fluid is air and the bimorph is made of



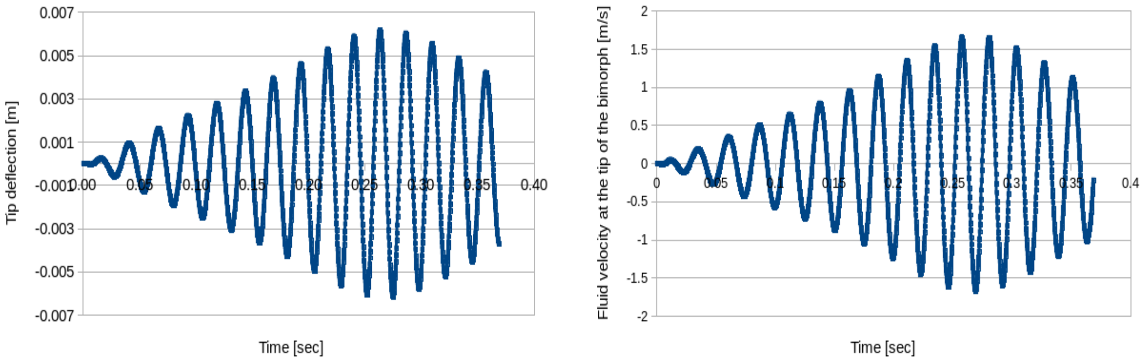
(a) Flexible piezoelectric bimorph energy harvester in fluid channel (b) Close view of harvester in fluid.

Figure 4: A flexible piezoelectric bimorph energy harvester in a converging fluid channel subjected to mechanical base excitation $\mathbf{u_{base}}$ and attached to an circuit with load resistance R.

PZT-5A material. The base acceleration of level is set as $a_0 = 9.81\mathrm{m/s}^2$. The forced displacement at the fixed end of the bimorph x = 0 as a function of time t is given as $\mathrm{u_{base}} = \mathrm{u_0 sin}\omega t$, as shown in Fig. 4. The forced acceleration amplitude a_0 and the forced displacement amplitude are related through $u_0 = a_0\, \omega^2$, where ω is the excitation frequency in rad/sec. The value of electrical resistance is set as $R = 1.0M\Omega$ to study open circuit condition and $R = 1.0K\Omega$ to study short circuit condition. The time increment is set as 0.001 sec for open circuit, while 5.0×10^{-5} sec for short circuit condition, satisfying the critical time increment for electric circuit [30] and the Courant number condition for FSI. [32]. The bimorph dimensions are set as length $\mathrm{L} = 250$ mm, width $\mathrm{w} = 20$ mm, and $\mathrm{t} = 2\mathrm{t_p} = 5.0$ mm. The variables from the shell elements (inverse piezoelectric solver) to solid elements (direct piezoelectric solver) and vice verse are exchanged using a transformation method [11].

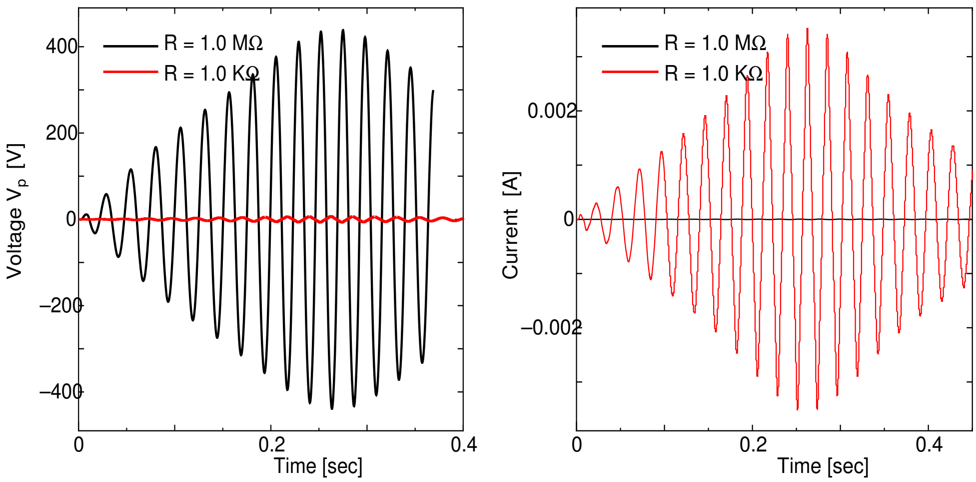
4.2 Results and discussions

Figures 5(a) and 5(b) shows the time histories of the tip deflection of the bimorph harvester when excited at $f = 40.0$Hz (near resonance) and the fluid velocities at the fluid node attached to the tip of the bimorph, respectively. As shown in these figures, we can see a beat phenomena due to the resonance. The method can capture the resonance characteristic accurately. Fig. 6(a) shows the time histories of the generated voltages due to the piezoelectric effect flowing through the attached circuit at load resistance R=1.0MΩ (black curve: open circuit condition) and R=1.0KΩ (red curve: short circuit condition). As shown in this figure, in the case of short circuit configuration the voltage is very very less (close to zero) than that of the open circuit condition because of the no piezoelectric effect, and therefore, no resistance is offered, hence, the current is maximum in the circuit, and vice-versa, that is, the generated voltage is maximum at open circuit condition, as shown in Fig. 6(b). The simulation results follow the standard electric circuit theory.



(a) (b)

Figure 5: Time histories of tip deflection of the bimorph (a) and the fluid velocity at the tip of the structure (b).



(a) (b)

Figure 6: Time histories of generated voltage (a) and the current flowing through the circuit (b).

5 Concluding remarks

In this study, a partitioned iterative method is proposed for the four-way coupling of fluid, the structure, the electromechanical effect of the piezoelectric material, and the electrical circuit based on hierarchical decomposition. These coupled four fields are hierarchically decomposed into the fluid-structure interaction, structure-piezoelectric interaction, and piezoelectric-circuit interaction interactions. Then these subsystems are decomposed into each field. Nevertheless, accurate modeling for predicting the characteristics of this four-way coupling using partitioned methods is first developed in this work. Using the proposed method, a flexible piezoelectric bimorph harvester in the converging channel is analyzed to demonstrate the coupling between fluid, piezoelectric structure, and the circuit. The results indicate that the method captures the coupled effect accurately.

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