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SEMI-ANALYTICAL FAILURE PREDICTION OF ADHESIVE JOINTS BY FINITE FRACTURE MECHANICS

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Abstract. The current contribution suggests a semi-analytical structural model for an adhesive joint with a closed-form higher-order description of the adhesive layer and the potential occurrence of a debonding crack. This enables a highly efficient failure prediction by the concept of Finite Fracture Mechanics, employing a coupled failure criterion that consists of a stress and an energy subcriterion. The comparison with accompanying finite element calculations and experimental findings demonstrates the high predictive quality of this approach.

1 INTRODUCTION

In countless mechanical structures, the use of adhesive bonding is a very advantageous way to join load-carrying components, in particular when these components are relatively thin. A common application area is lightweight construction, where typically two or more relatively thin sheet-like adherends are joined through an intermediate adhesive layer. This joining technique has the advantage that no screws or rivets are necessary and that the load transfer is distributed over a larger overlapping area. The actual load transfer behaviour, however, is somewhat demanding. Due to the given overlap geometry and the generally dissimilar stiffness properties of the adherends and the adhesive, local stress concentrations occur at the edges of the joints, which may trigger the onset of debonding cracks and subsequent failure, thus determining the effective strength of the given adhesive joints.

The particular advantages of adhesive joints are discussed in detail for instance in [1]. A good overview of available numerical evaluation procedures for adhesive joints can be found in [2]. Purely analytical approximate analyses of adhesive joints go back to the early works of Volkersen in 1938 [3] and Goland and Reissner in 1944 [4] for single-lap joints under tensile loading, where the adherends were modeled as simple plates connected by a thin adhesive layer that only transfers shear stresses or shear and peeling stresses, respectively. A decisive generalization to arbitrary configurations and loadings was made by Bigwood and Crocombe in 1989 [5] by a sandwich-type model in which only the overlap region is considered and submitted to appropriate cross-sectional forces and moments. For thicker adhesive layers, Ojalvo and Eidinoff [6] in 1978 suggested a linear displacement approach for the adhesive layer. Other analytical approaches can be found in the review of da Silva et al. [7] from 2009.

Although all the mentioned works in some way capture the stress concentrations at the edges of the adhesive layer, predicting actual overloading of an adhesive joint, the occurrence of debonding cracks and the subsequent failure are more advanced challenges. Since stress-based strength criteria do not apply well to stress concentrations and the common fracture mechanical concepts do not describe crack initiation, in 2002 Leguillon [8] introduced a coupled criterion in form of a combination of a strength and an energy criterion, postulating the instantaneous formation of a crack in the framework of Finite Fracture Mechanics as it has been suggested by Hashin in 1996 [9]. This kind of concept has proven its usefulness in various kinds of situations [10], [11] and will also be used in the following. The current contribution in particular focusses on the case of relatively thick adhesive layers where a higher-order displacement approach within the adhesive layer is appropriate for a realistic stress analysis and failure prediction [12].

2 HIGHER-ORDER MODELING APPROACH

As initially suggested by Bigwood and Crocombe [5], this paper also considers a sandwich-type model for the overlap region where loads are applied as cross-sectional forces and moments as indicated in Fig. 1. The adherends are indexed with (1) and (2) for the upper and lower adherend with the corresponding layer thicknesses h_1 and h_2. The adhesive layer in between is denoted by (a) with thickness t. Since there is no load in the depth direction the model merely is two-dimensional assuming a plane strain condition.

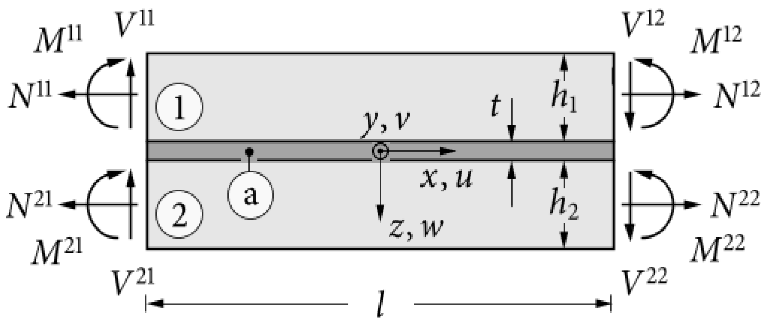

Figure 1: General sandwich-type model with geometric dimensions and applied loading

The key concept of the current modeling is to describe the displacements of the adherends by Mindlin kinematics (which means first-order shear deformation theory), the displacements of the adhesive layer, however, by a third-order deformation approach. This leads to the following representations for the horizontal and vertical displacements of the adherends (with $i=1$ for the upper adherend and $i=2$ for the lower adherend)

$$u^{(i)}(x,z_i)=u_0^{(i)}(x)+z_i\,\psi^{(i)}(x), \quad w^{(i)}(x,z_i)=w_0^{(i)}(x) \tag{1}$$

where the local coordinates z_i are centered in the respective midplanes.

For the adhesive layer beyond this additional warping deformation functions $\phi(x)$ and $\chi(x)$ are introduced so that

$$u^{(a)}(x,z) = \frac{1}{2}(\hat{u}^{(1)}+\hat{u}^{(2)}) + \frac{z}{t}(\hat{u}^{(2)} - \hat{u}^{(1)}) + \phi_u(1-4\frac{z^2}{t^2}) + \chi_u(z-4\frac{z^3}{t^2}), \tag{2}$$

$$w^{(a)}(x,z) = \frac{1}{2}(\hat{w}^{(1)}+\hat{w}^{(2)}) + \frac{z}{t}(\hat{w}^{(2)} - \hat{w}^{(1)}) + \phi_w(1-4\frac{z^2}{t^2}) + \chi_w(z-4\frac{z^3}{t^2}). \tag{3}$$

Herein the quantities $\hat{u}^{(i)}$ and $\hat{w}^{(i)}$ denote the displacements of the adherends evaluated at their interfaces to the adhesive layer.

From these displacement representations corresponding strains and through Hooke's law also respective stresses can be derived which are not given here but can be found in more detail in [12].

In total, the equations above represent the behavior of the adhesive joint by 10 unknown deformation functions and their derivatives with respect to the horizontal coordinate x for which the notation $(.)'$ is used. These deformation functions can be given in a summarized vector notation by

$$\boldsymbol{\Phi} = [u_0^{(1)}, u_0^{(1)\prime}, w_0^{(1)}, w_0^{(1)\prime}, \psi^{(1)}, \psi^{(1)\prime}, u_0^{(2)}, u_0^{(2)\prime}, w_0^{(2)}, w_0^{(2)\prime}, \psi^{(2)}, \psi^{(2)\prime}, \phi_u, \phi_u', \chi_u, \chi_u', \phi_w, \phi_w', \chi_w, \chi_w'] . \tag{4}$$

In order to determine the solution of the deformation functions in $\boldsymbol{\Phi}$ and to adapt it to the given geometry and the underlying load case, use is made of the principle of minimum total potential energy

$$\Pi = \Pi_{int} + \Pi_{ext} , \tag{5}$$

i.e. the sum of the stored strain energy and the external potential related to the given external loading. Accordingly, the variation of the total potential must vanish so that

$$\delta\Pi = 0 \tag{6}$$

applies. If the variation process is actually performed, it eventually leads to a set of 10 linear differential equations of second order, or equivalently, to a set of 20 differential equations of first order which by use of respective matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ can formally be described as

$$\boldsymbol{A\Phi}' + \boldsymbol{B\Phi} = \boldsymbol{0} . \tag{7}$$

This system of equations can be solved by a standard exponential solution approach leading to an eigenvalue problem to be solved. Finally, from the obtained deformation functions all deformations of the adhesive joint are available at any point and through standard kinematics and Hooke's law also the according stress distributions. The described approach has been implemented in MATLAB and yields all required result quantities in about 1 second.

For validation a numerical comparative modeling has been performed by means of the finite element program ABAQUS for the single-lap configuration shown in Fig. 2. In doing so the overlap length has been chosen as l=25mm, the adhesive thickness as t=0.5mm and the adherends' thicknesses as h_1=h_2=2mm. The finite element model consists of a very fine mesh of eight-node biquadratic plane strain continuum elements with approximately 1300000 degrees of freedom and an average computation time of 130 seconds. For more details, see [12]. Fig. 3 shows the resulting predictions for the shear stress distribution along the centerline of the adhesive and along the upper and lower interfaces with a good agreement. For the obtained peel stresses the corresponding results are given in Fig. 4 again showing a pretty good agreement.

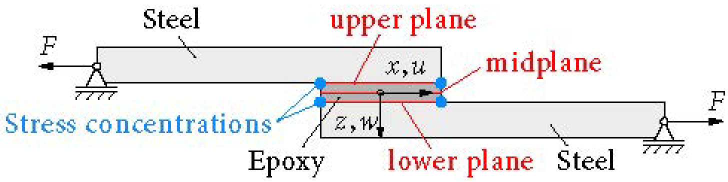

Figure 2: Considered single-lap joint under tensile loading

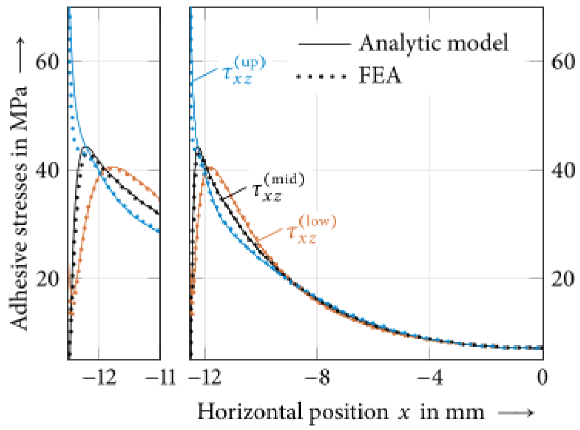

Figure 3: Shear stress distributions calculated along the centerline and the upper and lower interfaces

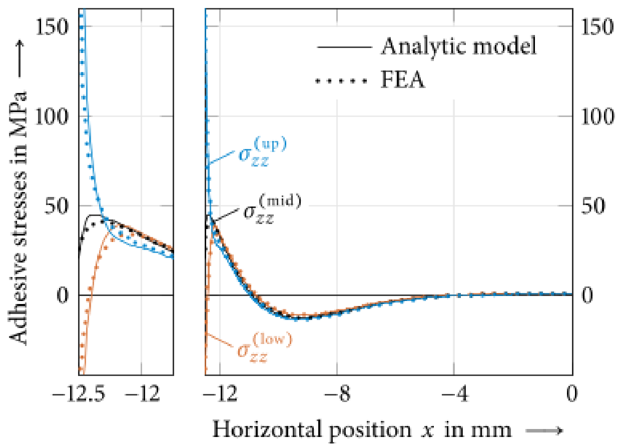

Figure 4: Peel stress distributions along the centerline and the upper and lower interfaces

2 ASSESSMENT OF EFFECTIVE STRENGTH

When the adhesive joint is overloaded or when the applied load reaches the effective strength of the joint, typically a debonding crack is generated starting at the edges between the adhesive layer and the adherends where the high stress concentrations occur. Following Leguillon [8] such a crack of finite length Δa and surface area $\Delta A = d\Delta a$ (where d is the depth of the considered joint) will form instantaneously when, according to a coupled criterion, a stress and an energy criterion, are satisfied simultaneously. For the energy criterion the so-called incremental energy release rate $\bar{G}$ is needed which is defined as

$$\bar{G} = -\frac{\Delta\Pi}{\Delta A} = \frac{1}{\Delta A}\int_0^{\Delta A} G(A)\,dA. \tag{8}$$

To enable crack initiation, this incremental energy release rate must exceed the fracture toughness of the adhesive layer or of the interface to the adherends which means

$$\bar{G} \geq G_c\,. \tag{9}$$

The energy release rate can be calculated through the so-called virtual crack closure integral which is composed of a mode I and a mode II part:

$$\bar{G} = \bar{G}_I + \bar{G}_{II} \qquad \text{with} \tag{10}$$

$$\bar{G}_I = \frac{1}{2\Delta a}\int_0^{\Delta a} \sigma_{zz}\left(w^{(up)} - w^{(low)}\right)ds, \tag{11}$$

$$\bar{G}_{II} = \frac{1}{2\Delta a}\int_0^{\Delta a} \tau_{xz}\left(u^{(up)} - u^{(low)}\right)ds. \tag{12}$$

For the stress-based strength criterion of the coupled criterion here the following quadratic stress criterion is taken:

$$(\sigma_{zz}/\sigma_c)^2 + (\tau_{xz}/\tau_c)^2 \geq 1\,. \tag{13}$$

Thus in total three material parameters are used in the assessment, namely the tensile strength σ_c, the shear strength τ_c and the fracture toughness G_c. Depending on the case of application and the involved materials of course other stress and energy criteria can also be used if they are considered as more appropriate.

In order to determine the effective strength or failure critical force F_c for the adhesive single-lap joint in the case of an applied tensile force F the lowest force fulfilling the coupled criterion has to be determined. This can be formulated as the following optimization problem:

$$F_c = \min\{F \text{ subject to } f(\sigma) \geq 1 \text{ along } \Delta a \text{ and } \bar{G} \geq G_c\}. \tag{14}$$

Herein in addition to the critical load magnitude also the finite length Δa of the crack is unknown but is obtained at the end of the optimization process.

For the quantitative determination of the incremental energy release rate the respective crack opening is needed and therefore the finite crack of length Δa has to be included into the structural model of section 2. This can be done by a subdivision of the adhesive joint into a cracked and an uncracked part as it is sketched in Fig. 5. In doing so in the cracked part an additional deformation function has to be included representing the crack opening. Beyond this at the horizontal position of the crack tip respective continuity conditions of the deformation functions and the related dynamic cross-sectional quantities have to be taken into account. With this modification and extension in essence the structural analysis works in the same manner as described in section 2.

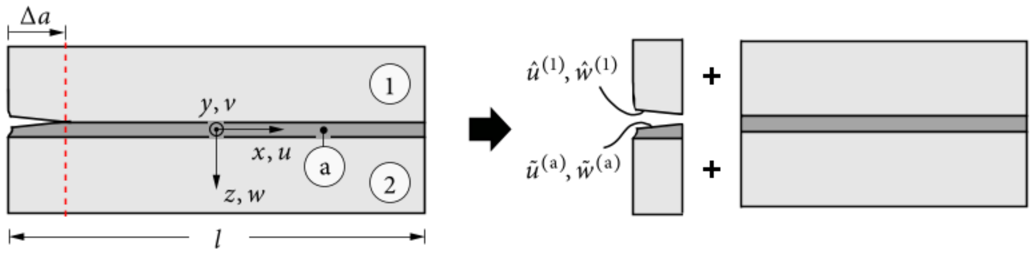

Figure 5: Subdivision of the adhesive joint into cracked and uncracked parts

With a correspondingly extended MATLAB implementation the incremental strain energy release rate can be calculated as a function of the crack length Δa as it is displayed for an example in Fig. 6, also showing the two portions of mode I and mode II. For comparison the same has been done by detailed finite element analyses, as can be seen with a pretty good agreement. This is remarkable insofar, as the structural behavior around the crack tip is very complex and can only be taken into account in an approximate manner by the described closed-form analytical approach.

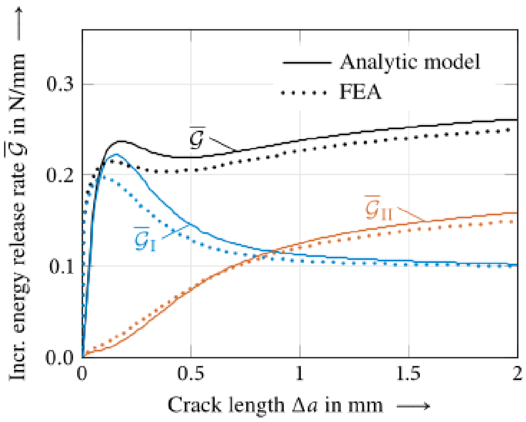

Figure 6: Energy release rate of the analytical and numerical model for steel adherends connected with adhesive layer of AV138 under a tensile load of 8kN

As can also be seen from Fig. 6 the mode II portion of the energy release rate and the total energy release rate show a clearly non-monotonic behavior. As a consequence, the optimization procedure for the effective strength is a bit more demanding, since it may happen that the coupled criterion is met for relatively short crack lengths on the left side of the local maximum in Fig. 6 or for considerably longer cracks more on the right side where the strain energy release rate exceeds the amount of the local maximum. With a sufficiently general optimization algorithm (also implemented in MATLAB) in each case the effective strength and the corresponding crack length can be identified. For an example situation with available actual test results Fig. 7 shows results for the effective strength as a function of the adhesive thickness in comparison to a comparative finite element implementation and in comparison to experimental findings with very satisfying agreement. In addition, the calculated crack lengths are given. It can be seen that smaller adhesive thicknesses result in larger crack sizes and vice versa. This is due to the non-monotonic behavior of the energy release rate.

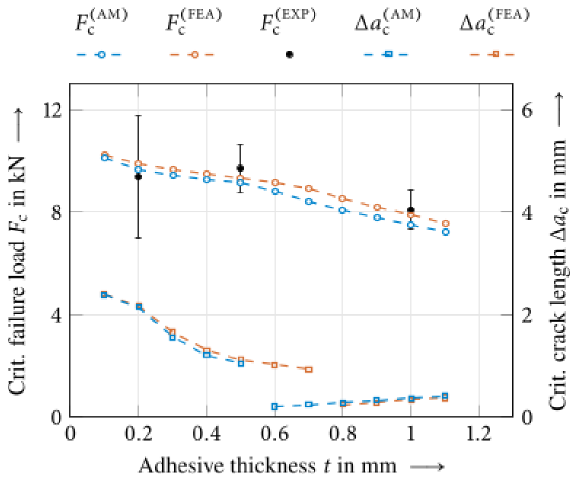

Figure 7: Mechanical failure loads and crack lengths for different adhesive thicknesses *t* according to analytical model (AM) in comparison with finite element analyses (FEA) and experimental findings (EXP) of [13]

3 CONCLUSIONS

In the framework of finite fracture mechanics, a semi-analytical failure prediction has been performed using the coupled stress and energy criterion of Leguillon. The analytical structural model is based on an advanced third-order approach for the adhesive layer and allows to take into account larger thicknesses of the adhesive layer. By a respective optimization procedure, the effective strength of the adhesive joint can be determined. With an own MATLAB implementation a very satisfying agreement with accompanying finite element calculations and with experimental findings has been obtained. The main advantages of the presented analysis procedure are its efficiency and the possibility to perform parameter studies in a very short time. Due to the sandwich-type model concept this kind of analysis can be easily done for different joint configurations and varying load cases with relatively low computational effort.

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