# FINITE ELEMENT-BASED SIMULATION OF LARGE WIND TURBINES WAKE USING THE ACTUATOR LINE METHOD

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Abstract. The numerical simulation of wind turbines and wind farms aerodynamics represents an open challenge in computational mechanics. It involves multi-physics and multi-scale phenomena, turbulent flows at very large Reynolds numbers, atmospheric boundary layer features, and rotor machinery flow features and dynamics. The geometrically resolved Computational Fluid Dynamics (CFD) is recognized as the highest-fidelity approach for wind turbine simulations but it has still a too high computational cost if employed for wind farm flow analysis. For this application, several reduced-order models have been formulated to obtain reliable results at a sustainable computational effort. Among the others, Large Eddy Simulations (LES) with Actuator Line Model (ALM) represents a valid middle-fidelity alternative for accurately simulating the wind turbine wakes dynamics and its interaction with the atmospheric boundary layer turbulence. Most implementations of the ALM are derived for volume-based CFD solvers. In this work we present the implementation of this model in a Finite Element Method (FEM) framework, which allows the use of a Residual Based Variational Multiscale (RBVMS) method to model the turbulent flow field, instead of the standard LES formulation. The ALM-VMS formulation is applied to study a 5MW and a 15MW wind turbine rotors, comparing the results with data available in literature in terms of aerodynamic variables of main interest, such as rotor loads and aerodynamics and near and far wake features.

## 1 INTRODUCTION

In recent decades, wind technology has emerged as a key component in the global strategy for transitioning to green energy [1, 2]. The constantly increasing dimensions of the wind turbines (WT) has made clear the need of highly reliable simulation tools to compensate the technological and economical impossibility of testing machines of radius higher than 120m [3] and wind farms of several kilometres of extension. Therefore, numerical simulations play a crucial role in validating new technological solutions and accurately assessing loads and aerodynamic losses (such as wake losses). This capability is essential for optimizing the design and operation and maintenance (O&M) of power plants, ultimately reducing associated costs.

One of the primary challenges in this field is the aerodynamic simulation of WT and wind farm flows. Geometrically resolved CFD [4, 5] currently offers the highest fidelity in simulations but comes with significant computational costs. Instead, the simulation tools commonly adopted in industrial applications are based on the Blade Element Momentum (BEM) theory [6], a low-fidelity approach that, through enhancement and correction modelling [7, 8], is used in the implementation of very fast tools for aero-servo-elastic simulations of the wind turbines. While this method provides quick estimates of WT dynamics, it lacks accuracy in off-design conditions and does not address wake and WTs interactions. Higher-order models, such as the Vortex Lattice Model (VLM) [9], Vortex Particle Method (VPM) [10], or panel methods [11], can resolve wake interactions but cannot model neither the entire wind farm flow in complex domains nor the Atmospheric Boundary Layer (ABL).

It is then clear how the optimal method would be a compromise between the high-fidelity fully resolved CFD and the high-efficiency BEM. For this reason, mid-fidelity methods are rapidly gaining attention, being able to simulate the wind flow through turbulent CFD and solve all the features of the ABL [12, 13, 14]. These methods significantly reduce the computational costs associated with simulating the wind turbine rotor. Notable among these are the Actuator Disk Method (ADM) [15] and the Actuator Line Method (ALM) [16, 17].

Most implementations of the ALM are derived for finite volume CFD solver (Sørensen, 1995; Michelsen, 1992), with the most used opensource library of ALM for finite volume CFD (SOWFA [18] and turbinesFoam [19]) based on LES. Korobenko et al. [20] demonstrated the potential of using the ALM in the framework of finite element and Isogeometric Analysis (IGA) [21], obtaining highly accurate description of the atmospheric turbulence for different class of ABL stability with the RBVMS method, which have been validated and applied successufully on a large range of fluid mechanics applications [22, 23, 24, 25].

This work presents the results of implementing the Actuator Line Model in the FEM-based framework developed by the authors [26, 27, 28, 29, 30], with the novel application to largescale wind turbines. The potential of using the VMS model for solving the turbulent flow field generated by the rotor and wake is verified by comparing the performance of two different wind turbines with the available literature. A first case focuses on the IEA Wind 15MW reference wind turbine [3], comparing the results with those of another authors' work [31] on the same turbine. A second case focuses on the NREL 5MW reference wind turbine [32], comparing the results with those of Troldborg et al. [33] and focusing in particular on the capability of the solver of properly deriving the wake dynamics both in the near and far wake. Both cases will have as reference results from geometry-resolved CFD simulations, for a correct assessment of the performance of this novel ALM-VMS implementation.

A general overview of the methodology used to conduct the analysis will be provided in Section 2. Section 3 will then describe the two different test cases used in this work and the results of these analysis will be discussed in Section 4.

## 2 METHODOLOGY

## 2.1 COMPUTATIONAL FLUID DYNAMICS MODEL

Let  $\Omega_1 \subset \mathbb{R}^{n_{sd}}$  be the spatial domain of the fluid, with boundary defined by  $\Gamma_1$ . The Navier– Stokes equations of incompressible flows can be written on  $\Omega_1$  and  $\forall t \in (0, T)$  as

$$
\rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} - \mathbf{f} \right) - \nabla \cdot \boldsymbol{\sigma}_1 = 0, \tag{1}
$$

$$
\nabla \cdot \mathbf{u} = 0, \tag{2}
$$

where  $\rho$  is the density, u the velocity, f the body force, and the stress tensor is defined as  $\sigma_1(p, u) = -pI + \mu \varepsilon(u)$ . Here, p and  $\mu$  are the pressure and the viscosity, I is the identity tensor, and  $\varepsilon(\mathbf{u}) = \frac{1}{2} ((\mathbf{\nabla}\mathbf{u}) + (\mathbf{\nabla}\mathbf{u})^T)$  is the strain-rate tensor.

We assume then finite-dimensional trial solution and test function spaces for the coarse scale fluid velocity and pressure, respectively defined as  $\mathbf{u}^h$ ,  $p^h$ , and  $\mathbf{w}_1^h$ ,  $q_1^h$ , where the superscript "h" indicates that the finite-dimensionality of the function.

The RBVMS formulation of the Navier-Stokes equations in  $\Omega_1$  used in the present case study is derived according to [22, 23, 34, 35, 28] as follows:

$$
\int_{\Omega_{1}} \mathbf{w}_{1}^{h} \cdot \rho \left( \frac{\partial \mathbf{u}^{h}}{\partial t} + \mathbf{u}^{h} \cdot \nabla \mathbf{u}^{h} - \mathbf{f} \right) d\Omega + \int_{\Omega_{1}} \varepsilon(\mathbf{w}_{1}^{h}) : \sigma_{1}^{h} d\Omega \n+ \int_{\Omega_{1}} q_{1}^{h} \nabla \cdot \mathbf{u}^{h} d\Omega \n+ \sum_{e=1}^{n_{el}} \int_{\Omega_{1}^{e}} \frac{\tau_{\text{SUBS}}}{\rho} \left( \rho \mathbf{u}^{h} \cdot \nabla \mathbf{w}_{1}^{h} + \nabla q_{1}^{h} \right) \cdot \mathbf{r}_{\text{M}} d\Omega \n+ \sum_{e=1}^{n_{el}} \int_{\Omega_{1}^{e}} \nu_{\text{LSIC}} \nabla \cdot \mathbf{w}_{1}^{h} \rho \mathbf{r}_{\text{C}} d\Omega \n- \sum_{e=1}^{n_{el}} \int_{\Omega_{1}^{e}} \tau_{\text{SUBS}} \mathbf{w}_{1}^{h} \cdot (\mathbf{r}_{\text{M}} \cdot \nabla \mathbf{u}^{h}) d\Omega \n- \sum_{e=1}^{n_{el}} \int_{\Omega_{1}^{e}} \frac{\tau_{\text{SUBS}}^{2} \mathbf{r}_{\text{M}}}{\rho} \cdot (\nabla \mathbf{w}_{1}^{h}) \cdot \mathbf{r}_{\text{M}} d\Omega = 0,
$$

where

$$
\mathbf{r}_{\mathbf{M}} = \rho \left( \frac{\partial \mathbf{u}^h}{\partial t} + \mathbf{u}^h \cdot \nabla \mathbf{u}^h - \mathbf{f} \right) - \nabla \cdot \boldsymbol{\sigma}_1^h, \tag{3}
$$

$$
\mathbf{r}_{\mathbf{C}} = \nabla \cdot \mathbf{u}^{h} \tag{4}
$$

are the residuals of the momentum and continuity equations. The superscript "e" is the element counter,  $n_{el}$  is the number of elements, and  $\tau_{\text{SUPS}}$  and  $\nu_{\text{LSIC}}$  are the stabilization parameters, for which we use the expression given in [36].

#### 2.2 ACTUATOR LINE MODEL

The aerodynamic model of the turbine rotor blades follows that of a standard ALM [16], using the convention of [37] to define a generic blade section. In this aerodynamic model, the blade is discretized into actuator line elements, and its effect on the flow-field is represented by substituting the 3D blade geometry with a momentum source field added to the body force term f within the Navier-Stokes equations. Mean cross-sectional geometric and aerodynamic properties of the blade are defined for each element. A research algorithm is used to locate the center of each element within a fluid volume element, where the velocity is interpolated using the FEM shape functions. This velocity, along with tabulated aerodynamic coefficients, is then used to calculate the total aerodynamic force at the centroid. The force are then distributed onto the fluid domain using a three-dimensional kernel function, expressed as

$$
\mathbf{f}_{\varepsilon} = \mathbf{f} \otimes \eta_{\varepsilon} \,, \tag{5}
$$

where

$$
\eta_{\varepsilon}(r) = \frac{1}{\varepsilon^3 \pi^{3/2}} exp\left[-\left(\frac{r}{\varepsilon}\right)^2\right]. \tag{6}
$$

Here r is the distance between the center of the actuator element under consideration, origin of the force source, and the measuring point, and  $\varepsilon$  is a constant that defines the strength of the regularization function. In this work the chosen strategy for computing  $\varepsilon$  is the one proposed by Troldborg et al [38], which indicates that a good approximation of  $\varepsilon$  would be at least twice the local grid cell length.

Also, Martinez et al. reports in [39] that one of the main requirements of the ALM is that  $\Delta b/\Delta x$  < 0.75, where  $\Delta b$  is the average blade element length and  $\Delta x$  is the most refined cell dimension in the rotor zone of the computational domain.

A loss correction based on the lifting line theory is applied to the lift coefficient used in the element force computation, to account for end effects of the finite blade at both root and tip. It's implementation follows the one reported in [40].

## 3 CASE STUDY

#### 3.1 Wind turbines

The two turbines chosen for this study are the IEA Wind 15MW [3] and the NREL 5MW [32] reference wind turbines.

The 15MW horizontal axis wind turbine (HAWT) features 117 m long blades, constrained to the rotor hub at a distance of 3.97 m from the rotor axis, for a resulting rotor diameter of 242 m. For this turbine, a coning angle of 4 deg is used, which, considering the blade tip prebend of 4 m, leads to an upstream tip deflection of 12 m. Since no tower is considered in this case study, the rotor is centered in the computational domain.

The aerodynamic properties of the turbine blades are defined for 50 uniformly-spaced airfoil sections along the blade span, giving actuator elements of 2.387 m. With a rotor cell dimension of 5 m for the adopted mesh resolution, this discretization respects the requirement presented in 2.2 with a final ratio of  $\Delta b/\Delta x = 0.477$ .

The 5MW HAWT features 61.5 m long blades, constrained to the rotor hub at a distance of 1.5 m from the rotor axis. With no coning angle for this case study, the resulting rotor diameter is 126 m. The rotor is mounted at an height of 90 m from the ground, to reproduce the reference literature configuration. No tower is considered in this case study.

The aerodynamic properties of the turbine blades are defined for 62 uniformly-spaced airfoil sections along the blade span, giving actuator elements of 1.008 m. Given a rotor cell dimension of 3.34 m for the adopted mesh resolution, the final ratio between actuator elements and local cell dimension is  $\Delta b / \Delta x = 0.3$ .

#### 3.2 Computational domain and numerical setup

The computational domain for the 15MW case study consists of a cubic box of side length 10 D, with the rotor centered in the domain. The mesh, shown in Figure 1, is a hexahedral mesh containing 1.15 M 8-nodes hexahedron finite elements. A refinement is applied in the rotor zone, extending 1.2 D in both Y and Z directions, and extending 40 m upstream and 1 D downstream of the rotor, to obtain a minimum element dimension of 5 m.



Figure 1: Computational domain for the 15MW case study, a) general view and b) detail of refinement.

The computational domain for the 5MW case study is the same as defined in [33]. The domain extends 8 D upstream and 12 D downstream of the turbine, which is located 90 m above the level of the ground, and has the shape of a half-cylinder of radius 8 D. The mesh, shown in Figure 2, is a hexahedral mesh containing 2.7 M cells. A refinement is applied in the rotor zone, extending 1.5 D both in Y and Z direction, to obtain a 3.34 m fluid element dimension. This refinement is then extended 1 D upstream and 7 D downstream of the turbine, with the elements gradually increasing in dimension towards the far field.



Figure 2: Computational domain for the 5MW case study, a) general view and b) detail of refinement.

Both the study cases have uniform inflow conditions at the inlet patch, with an inflow velocity of  $7.5 \,\mathrm{m/s}$  for the 15MW rotor and  $8 \,\mathrm{m/s}$  for the 5MW rotor, while a slip condition is applied to all the other boundary patches. They both have run for 500 s of physical time, with the timeaveraged flow field quantities computed from 350 s for the 15MW case and from 250 s until the end for the 5MW case. For the 15MW case the timestep size is 0.05 s, while it is 0.025 s for the 5MW case, with the coupled non-linear system of equations solved using three iterations of Newton-Rhapson for each timestep.

#### 4 RESULTS

The analysis of the results is presented for the two separate cases, focusing on the turbine performance in the 15MW case and on the wake dynamics in the 5MW case.

#### 4.1 15MW case study results

The analysis of the performance of the 15MW turbine rotor are presented in terms of aerodynamic loads and angle of attack along the blade span. These results, shown in Figures 3 and 4, are compared to those of [31], focusing in particular on the comparison with results obtained using the ALM implemented in OpenFOAM [19] and run within a Detached Eddy Simulation (DES), a geometry resolved Reynolds Averaged Navier Stokes (RANS) simulation of a singleblade periodic sector, and a BEMT-based solver (*Aerodyn v.15* [41]).

Figure 3 shows a very close match of the normal force distribution with the other ALM code, with the same loss of aerodynamic load towards the tip. The tangential force distribution shows a slight under prediction of the load in the central part of the blade with respect to the geometry resolved data, while behaving very similarly to the BEMT model.

Figure 4 shows the geometric angle of attack distribution along the blade span. ALM and BEMT codes show an under prediction of the values in the first third of the blade span with respect to the geometry resolved case, while the results are very similar for the most loaded



Figure 3: Normal (top) and tangential (bottom) aerodynamic force distribution over the 15MW blade, averaged over the last rotor revolution [31].



Figure 4: Geometric angle of attack distribution over the 15MW blade, averaged over the last rotor revolution [31].

portion of the blade.

## 4.2 5MW case study results

This section will focus on a comparison of the wake statistics with the results obtained by Troldborg et al. [33]. Figure 5 offers a plot of the vorticity magnitude in a meridian plane of the domain, passing through the rotor center. The wake structure keeps ordered until 5D after which the turbulent wake starts to mix with the undisturbed flow field. A Q-criterion contour is shown in Figure 6 showing the development of stable tip vortex structures in the near wake.

The velocity deficit and the turbulent kinetic energy in the wake are plotted in Figures 7 and 8, respectively. Figure 7 shows how the current ALM implementation gives a velocity deficit in the turbine wake that closely resembles that of the ALM-ADM of the reference. In particular, in the middle part of the blade the deficit appears slightly overestimated, while towards root and tip two different phenomena are worth mentioning. First, as the distance from the rotor increases, the tip velocity deficit become lower than the reference. Second, the velocity profile



Figure 5: Vorticity magnitude in the wake of the 5MW rotor (top view).



Figure 6: Q-criterion contour ( $Q = 0.035$ ).



Figure 7: Stream-wise velocity deficit in the wake of the 5MW turbine rotor [33].



Figure 8: Turbulent kinetic energy in the wake of the 5MW turbine rotor [33].

in the middle of the rotor suggests that either a better end effect for the blade root or the presence of the hub should be considered to better match the reference data.

The turbulent kinetic energy profiles in Figure 8 gives similar results to the ALM-ADM implementations of the reference, confirming a very low turbulence production in the wake behind a rotor modelled using these techniques.

## 5 CONCLUSIONS

This study presented an investigation on the use of an Actuator Line Model in a FEM-based tool for wind turbine performance assessment. The theory underlying the problem has been described in the second section and a brief focus on the turbine modelling process has been done in the third section. Two different test cases were chosen for the study, to demonstrate the validity of the predictions of the tool with respect to reference literature.

The 15MW rotor test case demonstrated the capabilities of the tool of correctly predicting

the rotor performance. Although only few aerodynamic results have been taken into consideration, they showed errors within acceptable ranges, as demonstrated by the reference literature. They also suggested the use of a better end effect model for the ALM, to properly catch the aerodynamics towards the blade tip.

The 5MW rotor test case focused instead on demonstrating the capabilities of the tool of assessing the main aerodynamics characteristics in the wake dynamics. The comparison with the reference literature showed a dynamic development of the wake in line with the prediction of similar codes, but underlined the necessity of taking into consideration a better end effect for the blade root or the presence of the rotor hub, to reduce the error in the rotor center.

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