Use of in-situ tests to predict stress increase component for foundation settlement

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Abstract

Because soil resists applied stresses in the x, y, and z directions, the ratio of horizontal to vertical stresses, k_o, determines the soil stress distribution. The dilatometer test predicts the unit weight of the soil and the horizontal stress ratio at rest, ko, providing the necessary input to determine the stress increase applied to the soil.

The Boussinesq stress distribution assumes the soil has a linear elastic stress-strain relationship. This distribution has no input from the soil's material properties. Harr (1977) proposed using the normal probability distribution with k_o input to more accurately compute stress distribution. The authors show the Harr stress distribution for different values of k_o and compares them with the Boussinesq stress and Westergaard stress distributions. The authors also present some case studies of stress distribution measurements and proposes modern instrumentation needed for additional research to determine the best prediction method.

Keywords: Dilatometer test, settlement, stress distribution, k_o step blade.

1. Introduction

Supporting structures on shallow spread footings costs less than either ground improvement or deep foundation solutions. Geotechnical design engineers should pursue and determine if a shallow foundation will satisfactorily support the proposed load from the structure. They should carefully evaluate the new imposed stresses on the existing soil to make accurate settlement predictions. The Boussinesq stress distribution appears in every geotechnical text book, and thus geotechnical engineers often use this distribution for their design. The Boussinesq stress distribution assumes the supporting soil behaves as a linear elastic, isotropic, "perfect" soil. Often the soil has aged and has over-consolidated and does not behave as a "perfect" soil.

Harr (1977) computed stress distributions based on probability theory and the coefficient of horizontal or lateral earth pressure, k_o, and they have rested peacefully in his text book for 47 years. Geotechnical engineers should intuitively and logically reason that predicting how the supporting soil will resist the new stresses should depend on the existing horizontal and vertical geostatic stresses. Using Harr's equations, stress bulbs and stress factors for different values of k_o are presented and compared to the Boussinesq and Westergaard stress distributions. Essentially, the Boussinesq stress distribution, only by chance, works out to an equivalent k_o of about 0.4 generally leading to a reasonable but often conservative predictions. For higher k_o values, the existing soil provides more support for the new stresses, resulting in smaller stress bulbs and stress factors. Marchetti (1998) illustrates the importance of lateral stress in Figure 1.

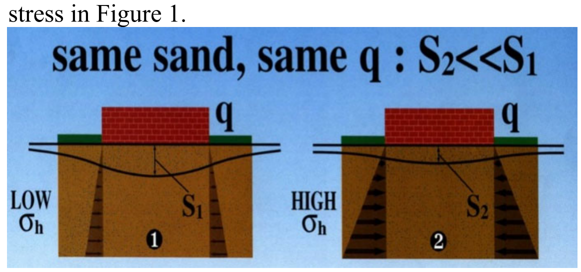


Figure 1: The importance of lateral stress for settlement

2. Boussinesq stress distribution

Boussinesq (1885) assumed the soil behaves as a homogeneous, isotropic, semi-infinite, elastic continuum having a constant value of modulus of elasticity. From those assumptions and not using any soil properties, he developed stress distributions in the underlying soil from applied loads/stresses placed on the soil.

To compute the stress under a square or rectangular footing, the engineer should use superposition, dividing the footing into four equal smaller footings; computing the applied stress beneath a corner of a smaller footing; and then multiplying the computed stress by 4. Boussinesq computes the stress under a corner of a square or rectangular footing using the following equation:

$$\Delta\sigma_v = \tan^{-1}\left\{\frac{a*b}{z*ABZ}\right\} + \frac{a*b*z}{ABZ} * \left\{\frac{\frac{1}{AZ}+\frac{1}{BZ}}{2\pi}\right\} * q \qquad (1)$$

where $\Delta\sigma_v$ = increase in vertical stress in soil at depth of z
q = applied stress at bottom of footing
a = footing width
b = footing length
z = depth of stress
AZ = $a^2 + z^2$
BZ = $b^2 + z^2$
ABZ = $\sqrt{AZ + BZ + z * z}$

For the applied stress beneath a circular footing with radius = r, Boussinesq computes the soil's stress with the following equation:

$$\Delta\sigma_v = \frac{1-z^3}{(r^2+z^2)^{1.5}} * q \qquad (2)$$

3. Westergaard stress distribution

Westergaard (1938) computed stress distribution in soil assuming the soil had thin sheets of infinite rigidity to better model sedimentary soil. His method uses Poisson's ratio, υ, to model lateral displacement. For applied stress on a square or rectangular footing, the stress transferred to the soil at the corner of the footing computes from the following equation:

$$\Delta\sigma_v = \frac{1}{2\pi}\left\{\cot^{-1}\sqrt{\eta^2\left(\frac{1}{m^2}+\frac{1}{n^2}\right)+\eta^4\left(\frac{1}{m^2n^2}\right)}\right\} * q \qquad (3)$$

where $\eta = \sqrt{\frac{1-2\nu}{2-2\nu}}$
ν = Poisson's ratio
m = a/z
n = b/z
a = footing width
b = footing length
z = depth
q = applied footing stress

For a circular footing, Westergaard uses the following equation for stress in the underlying soil:

$$\Delta\sigma_v = \left\{1 - \frac{\eta}{\sqrt{\eta^2 + \left(\frac{r}{z}\right)^2}}\right\} * q \qquad (4)$$

where r = radius of footing

For a Poisson's ratio of 0.3, the Westergaard stress distribution was almost identical to the Boussinesq stress distribution. For lower values of Poisson's ratio, the Westergaard stress bulb was smaller than the Boussinesq stress bulb; while for higher values of Poisson's ratio, the Westergaard stress bulb was larger than the Boussinsq stress bulb.

4. Harr k_o stress distribution

Harr modelled new stresses similar to a "leaky" water faucet. When the water droplet struck the ground, then it either deflected to the left or right, taking an unknown direction. When it struck the next underlying soil particle, the droplet again deflected to either the left or right and again in an unknown direction. Figure 2 depicts the likely outcome of the water droplet with increasing depth and direction choices. The droplet tends to end closer to the center than the outside and follows a normal or Gaussian probability distribution curve, having the traditional "bell" shaped curve with end limits of negative and positive infinity. The area under the entire curve equals exactly 1.000 because the probability of the event or stress occurring always equals one. For positive values, the area under the normal probability distribution curve as shown in Figure 3 has the following formula:

$$\varphi(z) = \frac{1}{\sqrt{2\pi}}\int_0^z \exp\left[-\frac{U^2}{2}\right] dU \qquad (5)$$

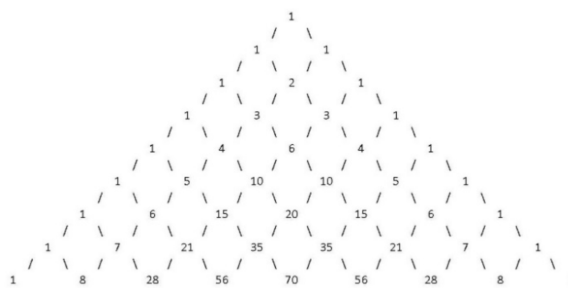


Figure 2: Outcome of Droplet from Leaky Faucet

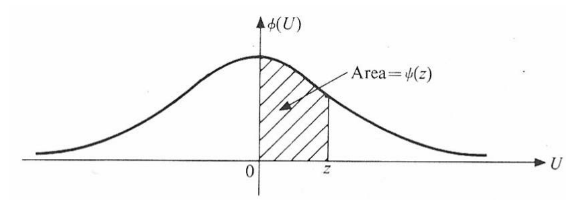


Figure 3: Area under the Normal Probability Distribution Curve

Excel computes this area using the "norm.s.dist" function – 0.5 (which removes the area to the left of zero). For a rectangular footing with sides having lengths of 2a and 2b, the engineer can use superposition multiplying the stress factor calculated at a corner of a footing with side lengths of a and b by 4. Harr found that the corner of that footing has the following equation for the expected value of stress:

$$\Delta\sigma_v = \varphi\left[\frac{a}{z\sqrt{K}}\right]\varphi\left[\frac{b}{z\sqrt{K}}\right] * q \qquad (6)$$

To compute the expected stress offset from the corner, Harr found the following more complicated equation:

$$\Delta\sigma_v = \left\{\varphi\left[\frac{x+a}{z\sqrt{K}}\right] - \varphi\left[\frac{x-a}{z\sqrt{K}}\right]\right\}\left\{\varphi\left[\frac{y+b}{z\sqrt{K}}\right] - \varphi\left[\frac{y-b}{z\sqrt{K}}\right]\right\} * q \qquad (7)$$

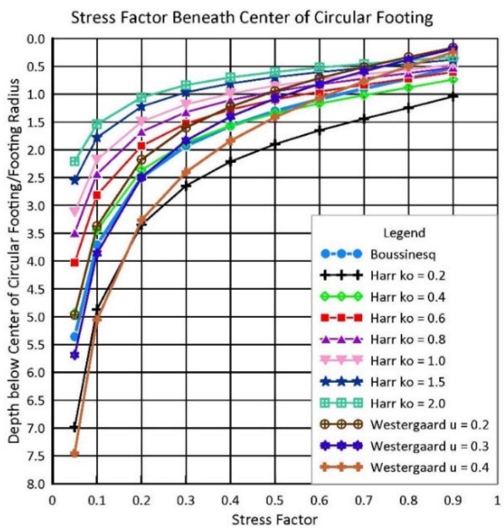
For a uniform load over a circular area with radius = r, Harr found the following expected stress at the center:

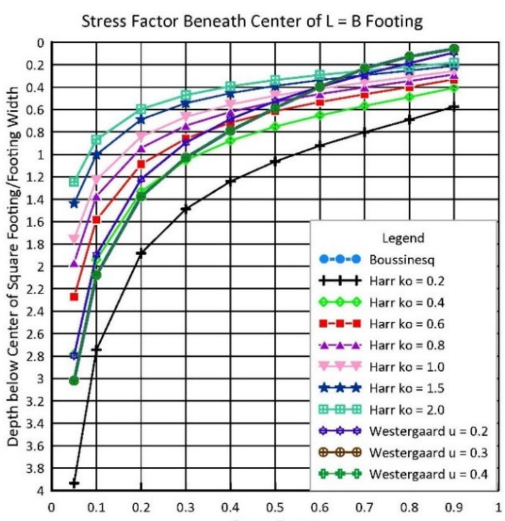
$$\Delta\sigma_v = 1 - exp\left(-\frac{r^2}{2Kz^2}\right) * q \qquad (8)$$

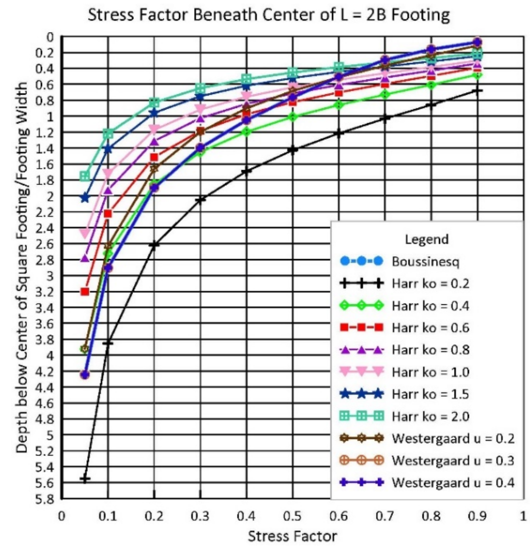
Researchers from the Corp of Engineers in Vicksburg, Mississippi installed stress cells into test sections filled with 1) sand and 2) clayey silt and applied different model footing stresses to them. Harr found that his stress distribution based on k_o accurately predicted the stress in the soil from their experiment.

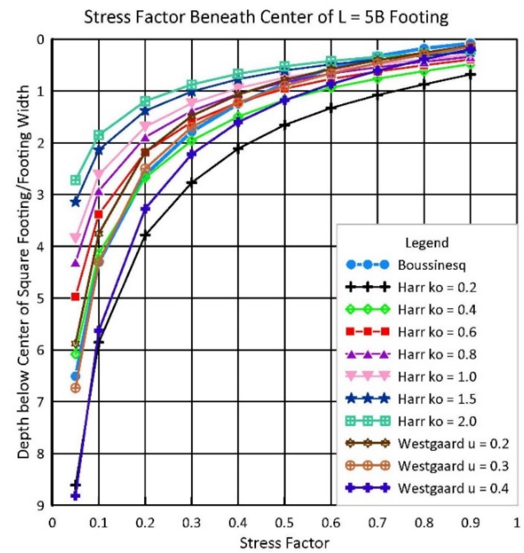
5. Comparison of Boussinesq, Westergaard and Harr ko methods

Using the above stress equations, the author computed the depths for stress factors from 0.05 to 0.9 for circular, square, L=2B, L=5B and L=10B rectangular footings for Boussinesq, Westergaard (Poisson's ratio of 0.2, 0.3, and 0.4) and Harr (k_o = 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.2, 1.5 and 2.0) methods. Figures 4a to 4e present the calculated stress at the center of circular, square, L=2B, L=5B and L=10B for those methods. Higher values of k_o have smaller stress bulbs. Interested readers can download the complete analyses and stress bulbs at www.insitusoil.com.









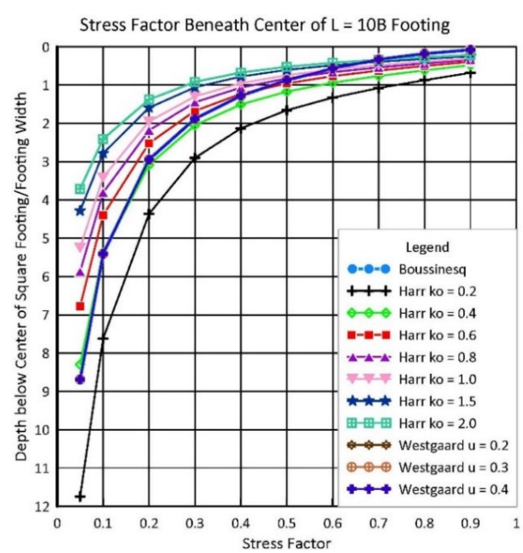


Figure 3a-e: Stress factors comparisons for Boussinesq, Westergaard and Harr k_o Methods for a) circular, b) square, c) L=2B, d) L=5B, and e) L=10B

6. Evaluating k_o

Schmertmann (1982) shows that k_o can range from 0.2 to 6.0. A k_o value of 0.2 represents a loose unaged soil, while high k_o values represent over-consolidated and aged soil. The k_0 step blade test, flat plate dilatometer test and soil pressuremeter test can evaluate k_o.

k_0 step blade test: Dr. Dick Handy (1982) invented the k_o step blade to measure this difficult to obtain soil parameter. The blade contains four steps starting from thin to thick from its bottom to top. At each step, the engineer increases pressure against its circular membrane moving it outward until it separates or lifts-off from the blade. The engineer measures this lift-off pressure, which equals the soil's horizontal stress. After pushing the blade to the next thicker step at that test depth, he/she measures the pressure that pushes the membrane to lift-off. Even the thinnest step causes disturbance to the horizontal stresses when the blade pushes into the soil. By plotting the blade thickness versus the log horizontal stress, engineer extrapolates the horizontal stress at a blade thickness of zero (Figure 5), computing the horizontal stress at rest. Handy (2008) documents ±10% accuracy for this method.

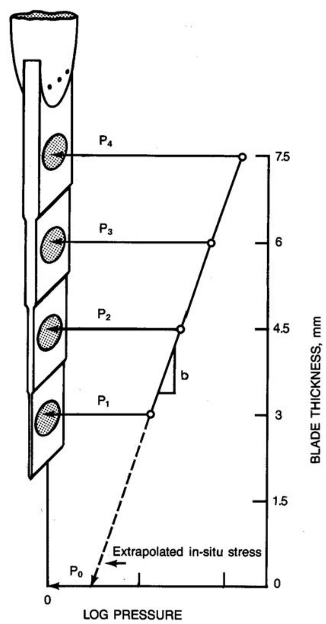


Figure 5: Determining the horizontal pressure at rest from k_0 step blade test

Flat blade dilatometer test: For clay and silt (I_D < 1.2), Marchetti (1980) correlation predicts k_o value reasonably well (Figure 6). His formula follows:

$$k_o = (K_D/1.5)^{0.47} - 0.6 \qquad (I_D < 1.2) \qquad (9)$$

Where K_D = the dilatometer horizontal stress index

From large chamber testing of sand, Schmertmann (1983) found that Marchetti (1980) formula did not predict k_o for sand and developed the below formula using both K_D and ϕ'_{ax} (Figure 7).

For $I_D > 1.2$ $k_o =$

$$\frac{40 + 23 * K_D - 86 * K_D * (1 - \sin\phi'_{ax}) + 152 * (1 - \sin\phi'_{ax}) - 717 * (1 - \sin\phi'_{ax})^2}{192 - 717 * (1 - \sin\phi'_{ax})}$$

Eq. (10)

where ϕ'_{ax} = triaxial angle of internal friction

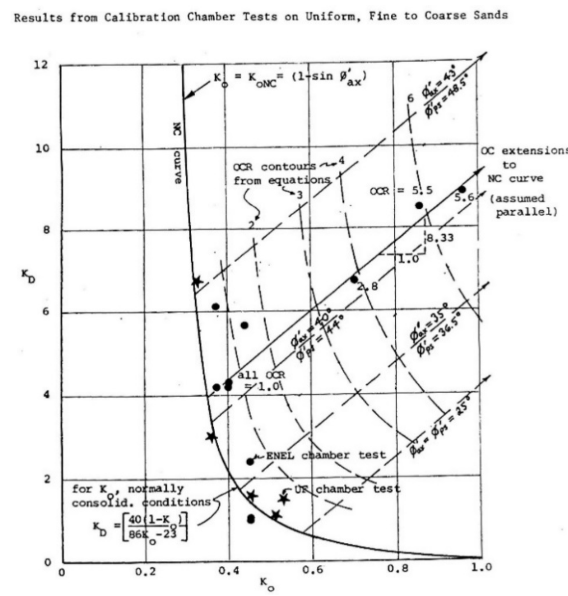


Figure 6: K_D correlation with k_o for I_D < 1.2

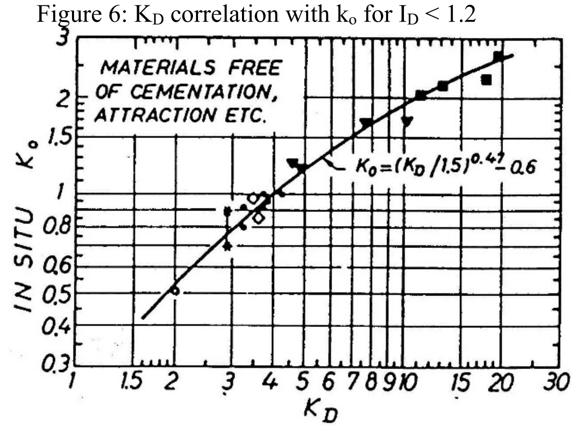


Fig. 7: K_D and ϕ'_{ax} correlation with k_o for I_D > 1.2

Soil pressuremeter test: Pressuremeter tests can estimate the horizontal pressure at rest by determining the pressure, P_{OH}, where the pressuremeter membrane contacts the soil borehole wall. Figure 8 shows this point where the elastic line intersects the initial line. The engineer can estimate the vertical stress knowing the test depth and approximate unit weight of the soil. k_o equals P_{OH} divided by the vertical stress.

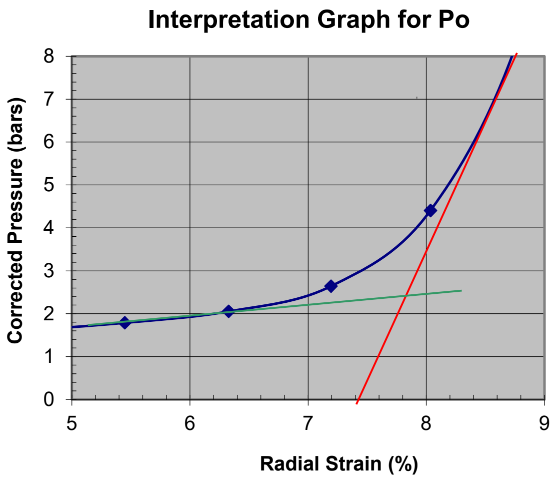


Figure 8: Determining P_{OH} from the pressuremeter test

7. Computing Settlement

Both the dilatometer and pressuremeter tests statically deform the soil measuring its stiffness, predict the soil's existing horizontal stress, and can make accurate settlement predictions. The dilatometer has the advantage of performing more deformation tests at closer intervals, while the pressuremeter has the advantage of performing deformation tests in stiffer soil that cannot be penetrated with direct push equipment and testing soil that contains some gravel and occasional cobbles with a slotted steel casing pressuremeter.

Dilatometer test: The dilatometer measures the constrained deformation modulus and horizontal stress (using the thrust measurement for cohesionless soil) at close depth intervals (10 or 20 centimeters). From each test, the geotechnical engineer can compute the applied stress and settlement as separate rows in a spreadsheet and sum the settlement column to get the total predicted settlement using Schmertmann's method (1986). Failmezger and Bullock (2004) show how well the dilatometer predicted constrained deformation moduli compares with laboratory consolidation calculated moduli in alluvial and residual soils as Figure 9.

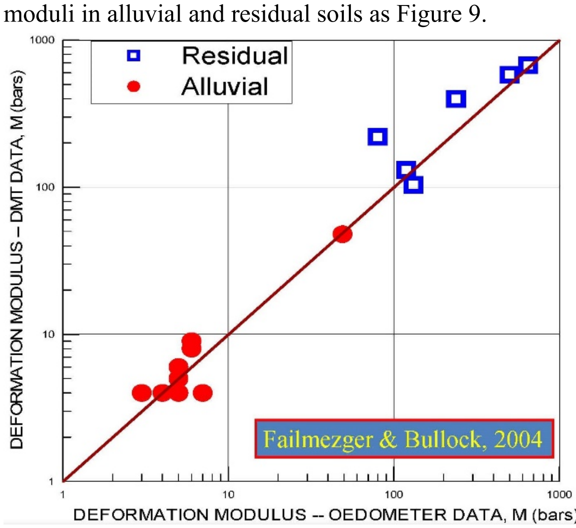


Figure 9: Comparisons of Lab and DMT Deformation Moduli

Soil pressuremeter test: The pressuremeter test measures the pressuremeter modulus and the horizontal pressure. The geotechnical engineer can predict settlement using either the French method, originally developed by Louis Menard (1958) or Briaud (2013) that uses the entire pressuremeter curve to represent a footing load test.

8. Comparisons with Footing Load Test Data

Briaud and Gibbens (1994), Schmertmann (2005), and Anderson, Townsend, and Rahelison (2007) performed footing load tests and predicted settlement prior to performing the load test (class "A" prediction). Briaud and Gibbens performed the load test using a 3 meter square footing and failed the sand in bearing capacity. Anderson, Townsend and Rahelison performed the load test using a 1.82 meter circular footing and used dead weights but did not fail the soil, and Schmertmann used a steel loading plate with a radius of 0.102 meters and measured the horizontal stresses at numerous points. Figure 10 shows the Briaud and Gibbens and Figure 11 shows the Anderson, Townsend, and Rahelison load test and settlement prediction for Boussinesq and Harr stress distributions from a nearby dilatometer test soundings. Figure 12 shows Schmertmann's load test results originally printed as Figure 5 from his paper. The Harr k_o method predicted the settlement well for the Briaud and Gibbens data for the soil at the lower stresses in the elastic deformation range, over-predicted the settlement for the Anderson, Townsend and Rahelison data, and underpredicted the vertical stresses for the Schmertmann data.

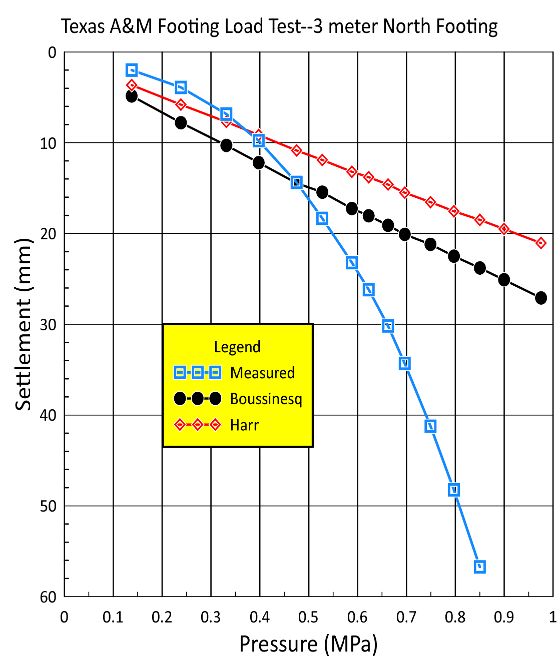


Figure 10: Briaud and Gibbens

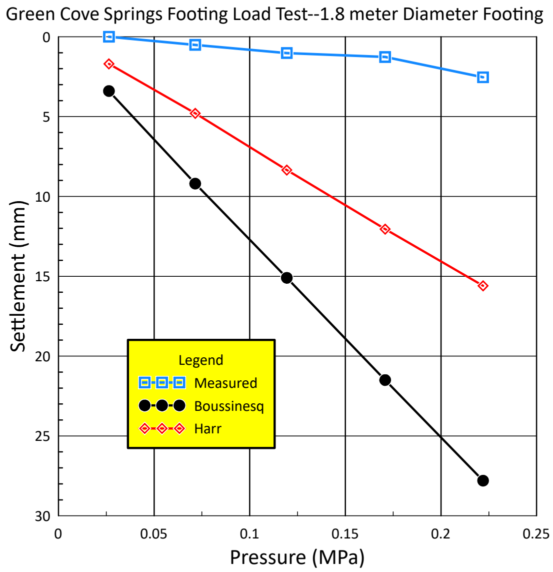


Figure 11: Anderson, Townsend and Rahelison

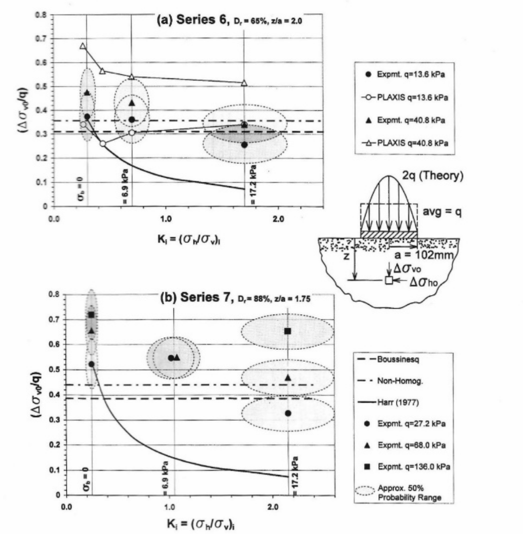


Figure 12: Schmertmann Experimental Load Test

Instrumentation of monitor field performance

To understand the stress distribution in the soil under footings, instrumentation can measure the new deformation and vertical and horizontal stresses from placing a footing on the supporting soil. This instrumentation should include borehole extensometers, rod anchors, piezometers, soil pressure cells, strain gages, and automated motorized total stations. The engineer should compare these new stresses and deformations with his/her predicted values from Boussinesq and Harr k_o stress distributions.

The following schematics (Figure 13 [profile view] and Figure 14 [plan view]) depict possible instrumentation layouts for a typical column footing.

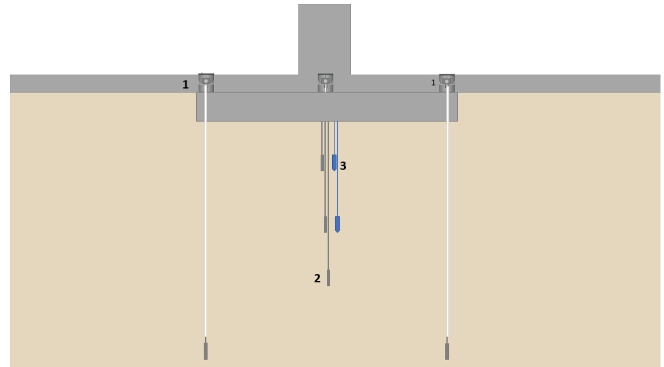


Figure 13: Section View Showing Footing Instrumentation

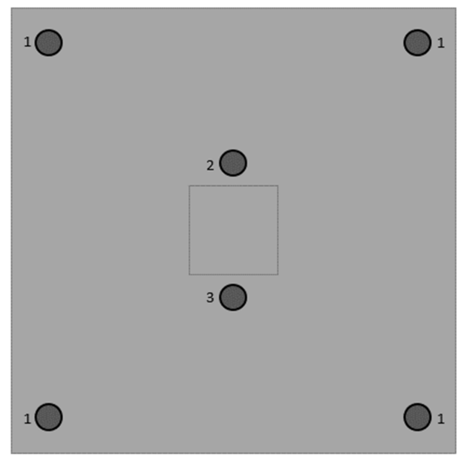


Figure 14: Plan View of Footing Instrumentation Layout

Key numbers in the Schematic:

No. 1. A single point anchor rod extended below the zone of influence. A shallow borehole would be drilled prior to footing construction and the rod and anchor inserted. The anchor depth would be well below the predicted level of stress. A sleeve would be installed through the footing and slab above to allow the travel of the anchor rod. A canister would be inserted in the slab to house the anchor fixity along with a glass target prism. An opening in the canister lid would allow observation of the prism by an automated motorized total station (AMTS). The AMTS measurement would serve as a redundant measurement and a check on the stability of the anchor. The overall intent of the four corner instrument assemblies would be to monitor possible footing displacements as the remaining instruments are utilized to monitor soil movement.

No. 2. This instrument is a multipoint extensometer with anchors at depths within the zone of predicted soil stress. A shallow borehole would be drilled prior to footing construction and the rod assemblies inserted with anchors. As with the single point rod anchor, the extensometer rods would be brought through the footing and slab in sleeves with sensors in a cannister inserted in the slab. An AMTS prism would also be fixed to the top of the sensor array to provide a redundant measurement. The overall intent of the assembly will be to detect and observe the movement differentials between the individual anchors and the slab as well as observe the comparative differentials between the rod anchors themselves.

No. 3. Instrument three is a bi-axial pressure cell assembly installation intended to monitor soil pressure differentials in the horizontal plane under the footing as the soil is compressed. These instruments can be installed in a drilled borehole or pushed into the soil with a direct push CPT/DMT rig prior to footing construction. In either case cables would be brought through the footing and slab with data readout housed in a cannister inserted in the slab.

To measure stresses below the footing, the engineer should use strain gauges, installed within the footing, either embedded in the concrete or attached to the reinforcing steel as sister bar strain gauges. Within cohesive soil, the engineer should install piezometers to monitor changes in pore water pressure under the footing to determine the time rate of change of the stresses and deformations.

In summary, when using geotechnical instruments along with data acquisition through on site or wireless communication, the engineer can understand the behavior of the soil under spread footings and the distribution of vertical and horizontal stresses allowing him/her to compare measured and predicted stresses and deformations.

9. Conclusions

1. Harr k_o method based on horizontal stress and the normal probability distribution provides an accurate method to predict the stress distribution in the supporting soil from applied stresses.
2. Boussinesq method based on the soil behaving as an elastic material and has no geotechnical input parameters provides a stress distribution that approximates the Harr solution with a k_o factor of about 0.4.
3. Westergaard method has similar stress distributions as Boussinesq for Poisson's ratio of 0.3.
4. Geotechnical engineers can use dilatometer and pressuremeter tests for accurately computing settlement because both tests predict horizontal stresses and deformation moduli.
5. Future research should include geotechnical instrumentation to measure the new stresses and deformations from loaded footing and compare these measurements with predicted ones.

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