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Suspension of High Flexible Lines such as Pipes or Cables

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ABSTRACT

Determining the deformed configuration and the induced stress of a suspended slender element is a felt problem in many engineering applications. To recall just a few, suspended high voltage cables, suspension bridge cables, the lifting and lowering of onshore pipelines or the laying (J or S layout) of offshore subsea pipes. All these cases have in common a high length if compared to their thickness and can be modelled as very slender beams. The high flexibility implies solving a non-linear problem, inasmuch involves the phenomenon of geometric nonlinearity in which the stiffness of the structure in the deformed configuration is unknown due to the large displacements.
Many authors have worked on this issue; a widely used procedure consists of a two-field model that alternates the use traits of linear beams (small displacements Eulero-Bernoulli theory) and cables (large displacements catenary solution) to model the region with high and small curvature rates respectively.
To avoid this intricacy, we propose a new procedure to directly address the non-linear large displacements problem of slender beams in this paper. The idea comes from the observation that the catenary, the simplest problem involving large displacements, is fully governed by only one variable, the stress at the vertex. This concept it is here extended to the beam case in which bending strain is considered dominant. The method turns out to be very simple and fast and can manage the cases of cable-lines or pipelines loaded by multi-hooking points. The solution algorithm is presented with some numerical examples which concern the end-lifting of pipes initially in contact with the soil and the fully suspended pipes by some loading points.

Keywords: Pipes; Cables; Lifting and Laying; Nonlinear Bending; Large Displacements and Rotations.

NOMENCLATURE

D_e	External diameter of the pipe $[m]$
D_i	Internal diameter of the pipe $[m]$
E	Young's modulus $[N/m^2]$
I	Area Inertia moment $[m^4]$
L	Length of the pipe/cable $[m]$
M	Bending moment $[Nm]$
N	Axial (normal) force $[N]$
q_y	Distributed force parallel to y axis $[N/m]$
R	Curvature radius $[m]$
T	Shear force $[N]$
ρ	Volume density $[kg/m^3]$
ψ	Slope angle $[rad]$

1. INTRODUCTION

Pipelines are essential equipments used for transferring and distributing oil and gas. Their installation is a crucial operation. During the lifting and lowering for onshore installation or the laying of subsea offshore lines, the pipes experience the most considerable induced stress during their lifetime (Scott *et al.*, 2008; Sen and Zhou, 2008; Duan *et al.*, 2014). For this reason, a reliable preventive structural analysis of the installation sequence should be performed to ensure structural safety and to avoid any damage, such as cracking, yielding, distortions and buckling (Cai *et al.*, 2017; Cao and Zhang, 2018).
A pipeline is a slender structure if compared to its length; therefore, a faithful simulation must consider that its structural behaviour is nonlinear due to large displacements (Frisch-Fay, 1962; Iandiorio and Salvini, 2018, 2020a). The load conditions to analyse involve modelling the pipeline as a slender beam, where the deformed configuration is far from the undeformed one, and the main stress is induced by bending. At the same time, extensional and shear effects turn out to be irrelevant. The significant difficulties in studying the lifting or laying of a slender pipe are the nonlinearity due to large displacements and the not known (a priori) lifted length.
It is possible to examine the behaviour of a pipeline in two ways.
The first, analytical, implies solving a set of nonlinear differential equations where the boundary conditions are of Boundary Value Problem type, and the lifted length is another unknown of the problem (Wang, 1983a, 1983b, 1990; Iandiorio and Salvini, 2020b, 2022; Marotta *et al.*, 2021).
The second, pure numerical, is to carry out a Finite Element Analysis (Felippa and Chung, 1981[a],1981b; Humer and Irschik, 2011). Performing the structural analysis of a lifting implies managing the contact between the pipe and the soil for the whole length of the pipe to lift. The consequence is that it is not known (a priori) the effective lifted length; this implies simulating a much greater length of pipe if compared to the portion effectively lifted (e.g. of *J*- or *S*-laying of subsea pipes (Guo *et al.*, 2013) with an indeterminate contact location with the marine soil).
Downstream of the issues just mentioned the evaluation of the configuration and the stress during the lifting, lowering or laying of onshore or offshore pipes installation has been addressed in different ways by many authors. The first works date back to 1967; Wilhoit and Merwin (1967) have proposed a highly simplified model in which the pipe is subdivided into several small pieces treated with the small displacements theory and where the horizontal forces are omitted; Plunkett (1967) treated the pipeline's non-linear ODE, approximating it with asymptotic expansions using the perturbation methods of Wasow (1956). Plunkett’s approach addresses the nonlinear bending of pipes as a sort of stiffened catenary. Palmer (1974) has numerically solved the geometric nonlinear equations which govern the S-laying here; he investigated the error made if the pipe is modelled as a catenary, emphasising that the discrepancy is mostly important around the boundary regions in which high changes of curvature occur. All previous works gave a solid foundation to current analytical models. An approach followed by many authors is the two-fields models, e.g. Lenci and Callegari (2005), in which the first part of the pipe, in contact with the soil, is modelled according to small displacements Eulero-Bernoulli beam theory on elastic Winkler’s foundation (constant soil stiffness), while the remaining part in which the curvature rate is moderate is modelled as a simple catenary. Referring to the latter mentioned subdivided model, many authors studied the deformed configuration and the stresses due to the lifting and lowering of onshore and offshore pipes by multi-hooking points, also trying to optimize the lifting (Liu *et al.*, 2017; Hwang, and Lee, 2017; Guo *et al.*, 2020). Unfortunately, this latter approach suffers from the limitations of the linear theory and cannot consider large displacements unless continuously checks the inflection angle and curvature rate and alternating parts of beam and catenary.
In the present work, we propose a new fast procedure to evaluate the configuration and stress of pipeline liftings by multi-hooking points, considering a full non-linear approach.
The idea comes from the observation that the catenary model is fully determined only by static equilibrium. A (first-order) differential equation may be explicitly integrated so that the solution depends

only on one parameter. If bending stiffness is introduced (as in the pipeline case), the analytical formulation presents a two-order ODE, causing the previously mentioned complications.
The issue addressed in this paper is: “Is it possible to explicitly integrate (i.e. simply as the catenary case) the nonlinear bending problem of the pipeline?”. Or, in other words, “is the solution still dependent on a single parameter?”.
The answer is “yes”, and this work aims to explain the proposed fast and simple procedure to follow. This alternative strategy is applied in the paper to examine the stress and configuration occurring during the multi-point lifting of long pipes and during the laying installation of subsea pipeline.

2. MATHEMATICAL MODEL

2.1 The Catenary model

The Catenary is an example of a one-dimensional element that, while being governed by a non-linear differential equation, gives a closed-form solution experiencing large displacements (Loney, 1956). The problem concerns an ideal chain (axially undeformable, i.e. it cannot change its length, presenting no bending stiffness) suspended through 2 end points (Fig. 1).

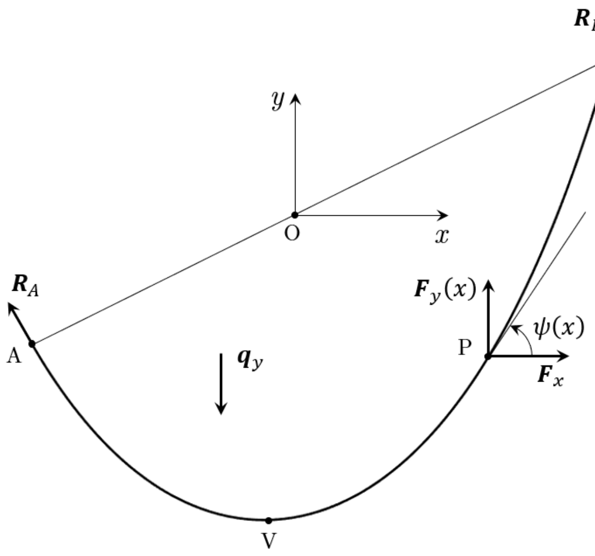

Figure 1. Representation of the Catenary

The chain suffers from a distributed load due to its weight and the reaction forces at the constraints in A and B. The differential equation that governs the deformed shape only accounts of the force balance. Let us consider a reference system centred on the middle point between the constraints A and B (Fig.1). Being P a generic point along the arch AB, the force balance of the piece AP is considered. Two loads, horizontal F_x and vertical F_y occur at the P point. The F_x value only depends on support A and remains unchanged whatever is the P location. Inversely, the F_y varies along the abscissa x. However, since the chain presents no bending stiffness, the resulting load can only be oriented as the local slope, therefore:

$$F_y(x) = F_x \tan\psi(x) = F_x \frac{dy(x)}{dx} \tag{1}$$

On the other hand, F_y can be computed considering that it absorbs the whole weight of the chain part between the vertex V and the point P. Therefore, being q_y the force per unit length acting along the catenary:

$$F_y(x) = q_y \int_{x(V)}^{x} \sqrt{1 + [y'(\tilde{x})]^2}\, d\tilde{x} \tag{2}$$

Where $\tilde{x}$ is a dummy variable.
Setting equal the Eq.s(1,2) and differentiating by x, one obtains the following second-order differential equation:

$$\frac{y''(x)}{\sqrt{1 + [y'(x)]^2}} = \frac{q_y}{F_x} \tag{3}$$

The previous equation admits the solution:

$$y(x) = \frac{1}{k} cosh[k(x + c_1)] + c_2 \tag{4}$$

where $k = q_y/F_x$. Coming back to Fig. 1, the reference is centred on the midpoint of the segment *AB*, so that the knowledge of the coordinates $B = (x_B, y_B)$ (symmetric to point *A* by respect to the vertical axes) provides the length L of the catenary:

$$y(x_B) - y(-x_B) = 2y_B \quad ; \quad L = \int_{-x_B}^{x_B} \sqrt{1 + [y'(x)]^2}\, dx \tag{5}$$

Applying the Eq.(4) into Eq.(5) one obtains:

$$y_B = \frac{1}{k}[sinh(kc_1) \cdot sinh(kx_B)] \quad ; \quad \frac{L}{2} = \frac{1}{k}[cosh(kc_1) \cdot sinh(kx_B)] \tag{6}$$

The ratio between the Eq.s(6) turn out:

$$kc_1 = artanh\left(\frac{2y_B}{L}\right) \tag{7}$$

Furthermore, being:

$$sinh(kc_1) = \frac{tanh(kx_B)}{\sqrt{1 - tanh^2(kx_B)}} = \frac{2y_B}{\sqrt{L^2 - 4{y_B}^2}} \tag{8}$$

Using Eq.(8) into the first of Eq.s(6):

$$sinh(kx_B) = \frac{k}{2}\sqrt{L^2 - 4{y_B}^2} \tag{9}$$

If the values of x_B, y_B, L are known, the k value can be numerically obtained from Eq.(9).

Evaluating Eq.(4) in $y(x_B) = y_B$ the constant c_2 turns out:

$$c_2 = y_B - \frac{1}{k} cosh[k(x_B + c_1)] \tag{10}$$

Therefore, using Eq.s(7,10) into the Eq.(4) the general solution of the catenary is found:

$$y(x) = y_B + \frac{2}{k} \sinh\left[\frac{k}{2}(x - x_B)\right] \sinh\left[\frac{k}{2}(x + x_B) + artanh\left(\frac{2y_B}{L}\right)\right] \tag{11}$$

where $x \in [-x_B, x_B]$.
From Eq.(11), the axial force at any point of the catenary is obtained:

$$N(x) = \sqrt{(F_x)^2 + [F_y(x)]^2} = \frac{q_y}{k} cosh\left[kx + artanh\left(\frac{2y_B}{L}\right)\right] \tag{12}$$

The coordinates of the vertex *V* results by equating to zero the first derivative of Eq.(11), obtaining:

$$V = (x_V, y_V) = \left(-\frac{1}{k} artanh\left(\frac{2y_B}{L}\right), \frac{1}{k} + c_2\right) \tag{13}$$

At the vertex *V* the axial force assumes the value:

$$N(x_V) = F_x = \frac{q_y}{k} \tag{14}$$

It is useful to highlight an interesting property of the catenary solution. If we consider the solution of a catenary expanded on the left or right extremes, the geometrical shape is always the same whatever is the location of the points A and B on the plane. In other words, given two arbitrary points in a two-dimensional plane, the geometrical shape can be found translating the curve given so that the two points lies on the curve itself. Thus, given q_y any catenary is only the function of the F_x value.
However, the previous closed-form solution refers to the case of constant self-weight. Using the result that the catenary is fully determined if the F_x value is fixed, a numerical approach is followed to solve the general case in which the self-weight may vary along the chain length.
Now, taking into account a catenary line segment of small finite length Δs. The two equilibrium equations may be written, considering that the slope angle monotonically changes along the chain. Neglecting the variation of the curvature radius $R(\psi + \Delta\psi)$ within the small element, equilibrium conditions give (Fig.2):

$$\begin{cases} (N + \Delta N)\, cos(\psi + \Delta\psi) - N\, cos\,\psi = 0 \\ (N + \Delta N)\, sin(\psi + \Delta\psi) - N\, sin\,\psi = q_y R \Delta\psi \end{cases} \tag{15}$$

From the first of Eq.(15), the axial force increment is given:

$$(N + \Delta N) = \frac{cos\,\psi}{cos(\psi + \Delta\psi)}\, N \tag{16}$$

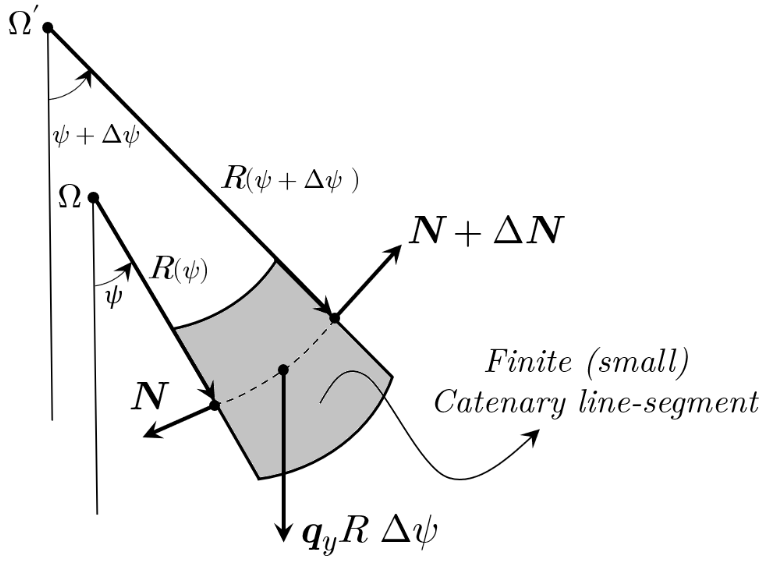

Figure 2. Small Finite Catenary line-segment

Applying Eq.(16) into the second of (14) R is given:

$$R = \frac{N}{q_y \Delta\psi}[tan(\psi + \Delta\psi)\, cos\,\psi - sin\,\psi] \tag{17}$$

And the increment of curvilinear abscissa results:

$$\Delta s = R\ \Delta\psi \tag{18}$$

The position of the new point of the catenary results:

$$\begin{cases} x(\psi + \Delta\psi) = x(\psi) + R[sin(\psi + \Delta\psi) - sin\,\psi] \\ y(\psi + \Delta\psi) = y(\psi) + R[cos\,\psi - cos(\psi + \Delta\psi)] \end{cases} \tag{19}$$

If the value of N at the initial point is known and $\Delta\psi$ is assumed small, the previous equation form an explicit algorithm that allows to get a quick solution of any catenary, even with a varying $q_y(s)$.
A simple comparison of the analytical solution towards the numerical one indicates that there is no need to assume particular small increments of the slope angle to get very reliable solutions.
Clearly, no need to reproduce a numerical solution if a closed form one exists. However, the previous approach extends the capability to have a quick solution even if the self-weight modifies along the chain and if additional discrete weights are added at some points of the catenary. In this last case, when the cumulative s position identifies the location of an additional force, it is sufficient to just add it into the right hand term of the second of Eq.(15).

2.2 Extension to Beams subjected to Non-Linear Bending

The same approach described for the catenary may be extended to beams if the bending stiffness and bending moment are considered at every small part of the suspended beam.

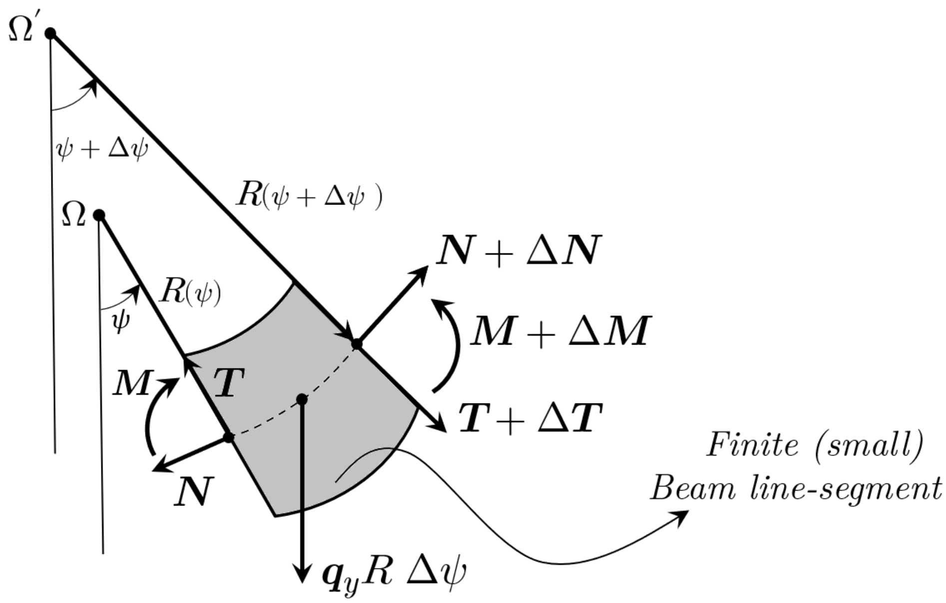

Figure 3. Small Finite Beam line-segment

Now three equilibrium equations appear, horizontal and vertical direction and moment equilibrium on one pole, here the right pole is a good choice (Fig.3):

$$\begin{cases} (N+\Delta N)\,cos(\psi+\Delta\psi)+(T+\Delta T)\,sin(\psi+\Delta\psi)-N\,cos\,\psi - T\,sin\,\psi = 0 \\ (N+\Delta N)\,sin(\psi+\Delta\psi)-(T+\Delta T)\,cos(\psi+\Delta\psi)-N\,sin\,\psi + T\,cos\,\psi - q_y R\Delta\psi = 0 \\ (M+\Delta M)-M-NR(1-cos\,\Delta\psi)-TR\,sin\,\psi+q_y R^2 \sin\left(\frac{\Delta\psi}{2}\right)d\psi = 0 \end{cases} \quad (20)$$

Considering that the unknown elastic bending moment is a function of the curvature radius as $(M+\Delta M)=EI/R$, substituting this equation in the third of Eq.(20), a third-degree equation in R turns out:

$$-\left[q_y \sin\left(\frac{\Delta\psi}{2}\right)d\psi\right]\ R^3+\left[N(1-cos\,\Delta\psi)+T\sin(\Delta\psi)\right]R^2+MR-EI=0 \quad (21)$$

Eq.(21) gives three solutions for R; the right one sign is in accordance with the sign of incremental $\Delta\psi$, and it is the closest value to the radius of curvature of the previous line segment. When an inflection point occurs, this sign should be inverted. This eventuality is easy to highlight since the three solutions become non-reals or cause a change of sign in the radius of curvature. In such a case the solution of the Eq.(21) is performed again using the opposite incremental sign of $\Delta\psi$ (reversed from the previous one). When the new value of R is given, the new values of $(N+\Delta N)$ and $(T+\Delta T)$ are obtained by Eq.(20):

$$\begin{Bmatrix}(N+\Delta N)\\(T+\Delta T)\end{Bmatrix}=\begin{bmatrix}cos(\psi+\Delta\psi) & sin(\psi+\Delta\psi)\\ sin(\psi+\Delta\psi) & -cos(\psi+\Delta\psi)\end{bmatrix}\begin{Bmatrix}N\cos\psi+T\sin\psi\\ N\sin\psi-T\cos\psi+q_y R\Delta\psi\end{Bmatrix} \quad (22)$$

The position of the new point of the beam can be obtained using Eq.(19).

The solution system described here corresponds to searching the solution from left to right, considering as known the acting loads at the start point. These latter correspond to the essential condition in which the position of the first point corresponds to the origin of the reference system, and the natural conditions concern normal, shear and bending loads applied to the initial point.
For the case in which a pipe is fully suspended, the initial slope is not given, the above natural conditions are nulls, and it is the unique variable to identify to get the balanced solution. Therefore, the procedure comes as iterative. When a first guess of the initial slope is set, the solution is performed up to the final end. At this end, some information does exist: e.g. if it has a free end or is simply supported the curvature at the end is null. The search so consists of the finding of the initial angle that allows the satisfaction of this final condition.

3. NUMERICAL RESULTS

In this section, some numerical applications of the previously shown method. All the analyzed cases consider an elastic pipe with $E = 209\ GPa$, $\rho = 7763\ kg/m^3$, $D_e = 0.5080\ m$ and $D_i = 0.4760\ m$.
First is shown the case of the lifting of heavy pipes. The pipe is supported by the soil at the first end (suppose left) and simply lifted (vertical lift and zero bending) at the right end. In this case, the initial condition is given by the angle set to zero, and increasing the solution is governed by the initial lift T_0 transmitted to the soil. Since no bending is applied at both ends, the actual supporting force T_0 on the left governs the whole deformed configuration, and it is the unique independent variable. The increasing of T_0 value causes the lifted part of the pipe grows. Fig.s 4,5,6 shows the evolution of the lifting in the case of $0, -20\ and\ 10$ degrees initial slopes, respectively. All the solutions in Fig.s 4,5,6 are given by setting an initial T_0 value of $20\ kN$ and increasing if of $2.5\ kN$ to obtain the successive configuration.

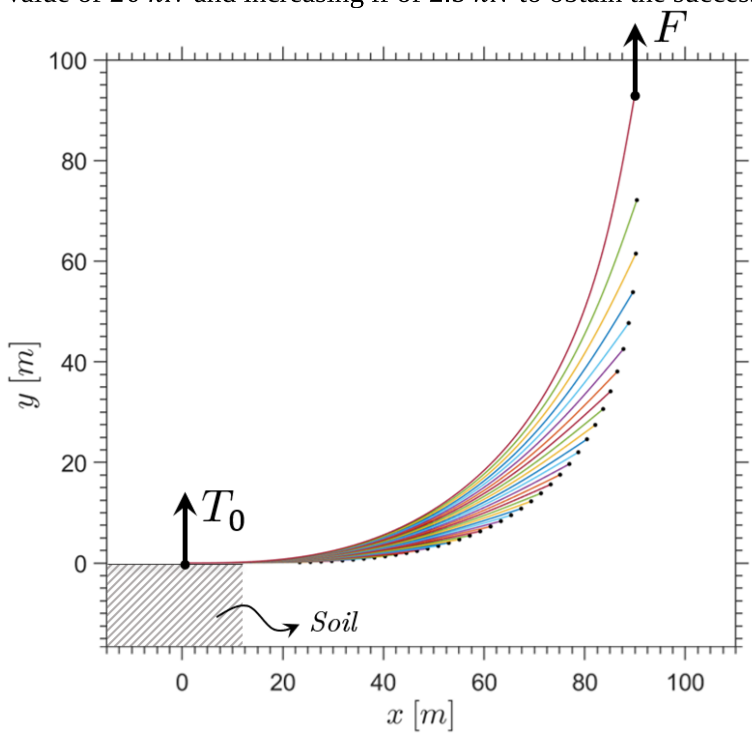

Figure 4. Lifting of a heavy pipe in contact with the soil with zero initial slope

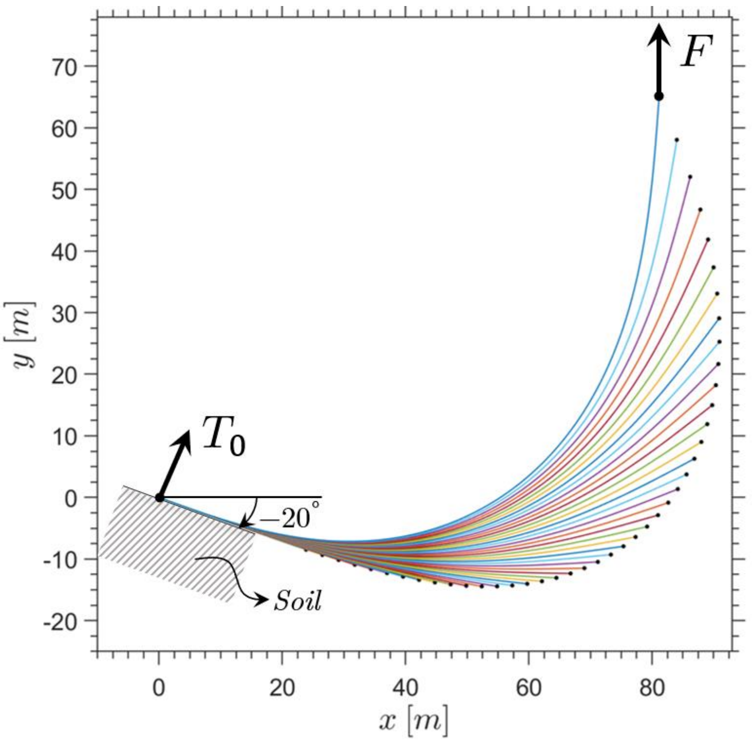

Figure 5. Lifting of a heavy pipe in contact with the soil with an initial slope of -20 deg

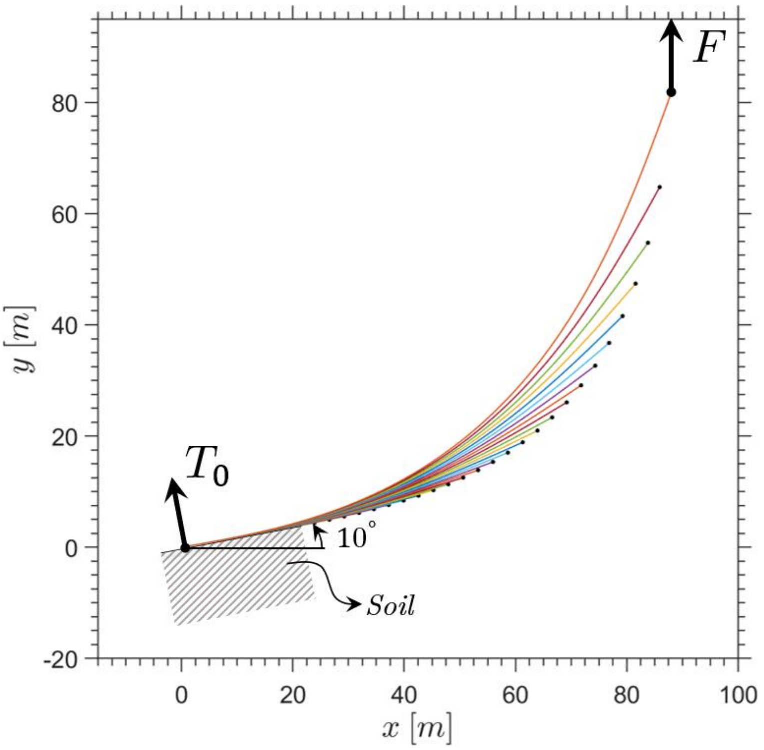

Figure 6. Lifting of a heavy pipe in contact with the soil with an initial slope of 10 deg

Another interesting application regards the case of a fully lifted pipe. More precisely, the capability to sustain a pipe suspended by known vertical forces placed in some location around the beam length. In this case, the loads can be whatever, but their sum should correspond to the lifted weight. In the application presented here, the initial sustain loads are computed through a linear (small displacements) analysis, such that the pipe keeps almost straight when suspended. The unique unknown, in this case, is the initial angle, inasmuch all the loads at the left end are nulls.

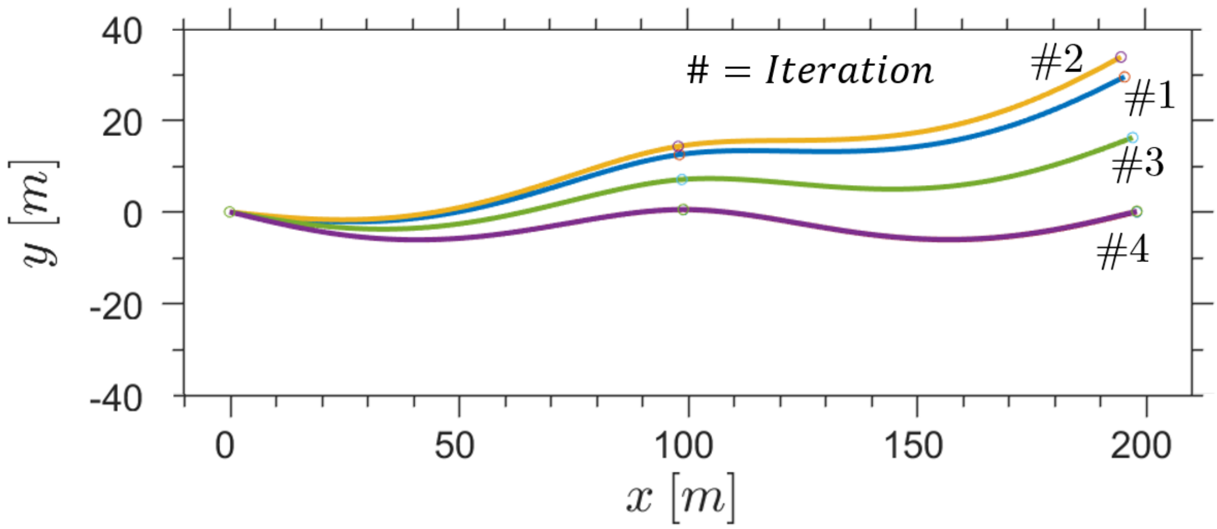

Figure 7. Lift of a pipe by three hooking points: convergence history

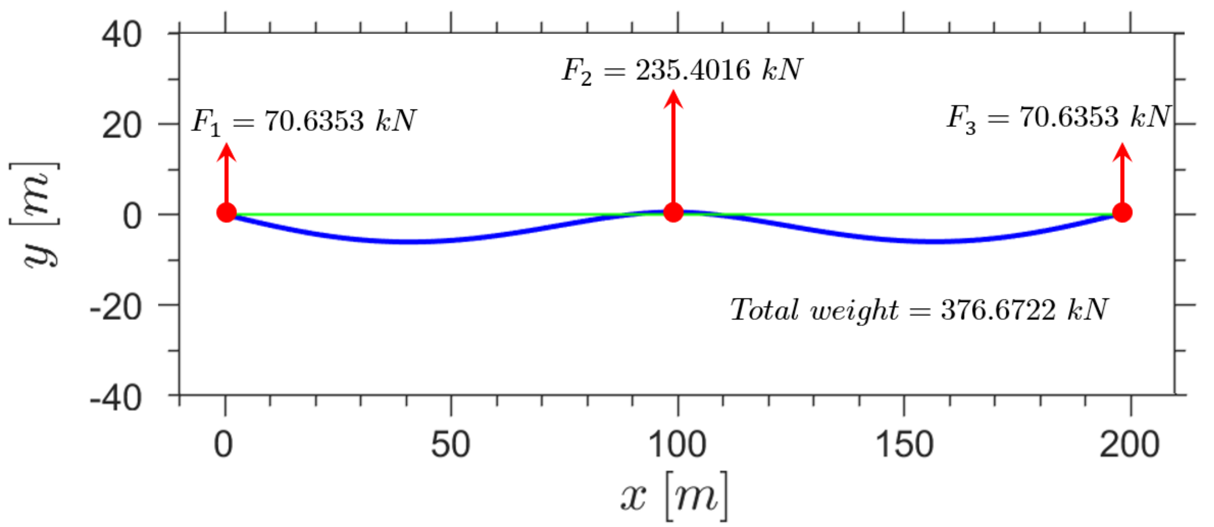

Figure 8. Lift of a pipe by three hooking points: converged configuration

Fig. 7 shows the convergence history for a long pipe of $L = 200\ m$ suspended on three equidistant points along its length. The solution convergence after 4 iterations. The converged solution is shown in Fig. 8.

The same method also works on more complex situations where the lifting points increase and are not symmetrically placed. Let us consider a case of four lifting points irregularly disposed. Again, the lifting loads are obtained by a linear solution that also gives a first estimate of the initial unknown angle. The result for a long pipe $L = 400\ m$ is shown in Fig.9. It is interesting to observe that in this last case the right end is not sustained, but it results only driven by its self-weight.
In this case, it is evident that the applied loads cause a different height of the sustained points, marked with the symbol Δ in the Fig.9. It is interesting to try to modify the loads to attain the same level (straight green line in the picture).
This can be done by a non-linear correction algorithm that decreases loads of the point above the green line and increases those above the same line. All the computations are affected by an error since the truncation makes a sum of the error during the computation on successive elements. The final result, obtained after 10 iterations, is given in Fig.10; the maximum shift, experienced by the second hooking point, is of 1.03 m by respect a total length of 400 m.

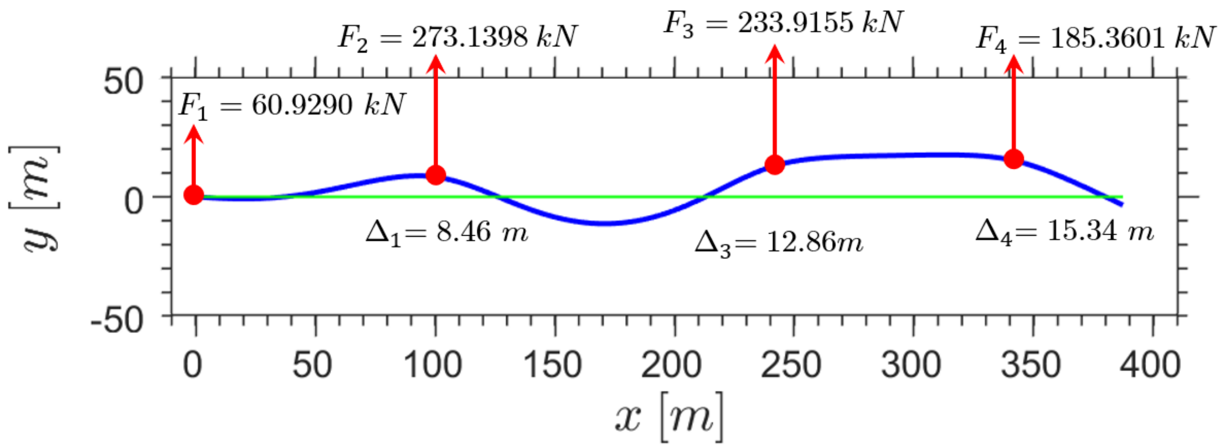

Figure 9. Lift of a pipe by four hooking points

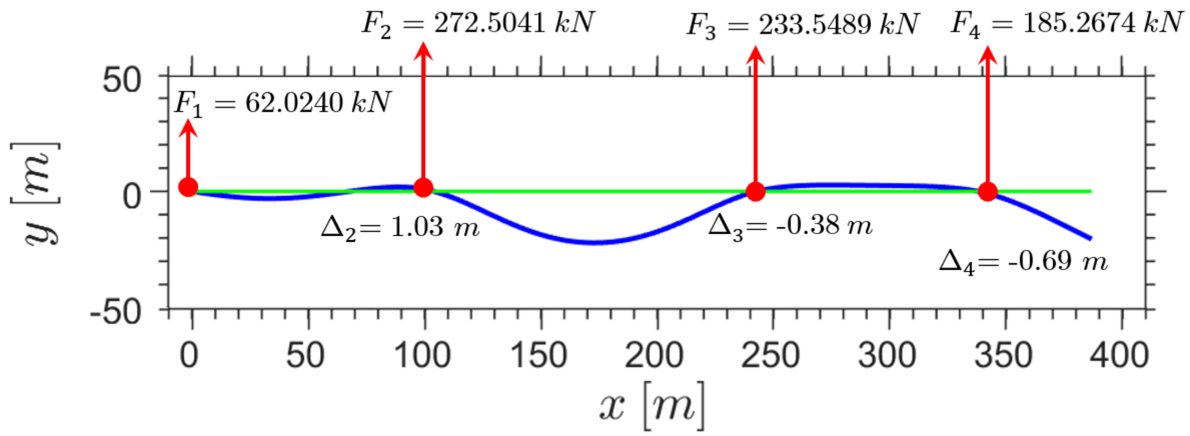

Figure 10. Lift of a pipe by four aligned hooking points

4. CONCLUSIONS

This paper discusses a new method to study the deformed configuration (displacements, stresses and strains) of slender elements as cables or pipes, subjected to large displacements. The proposed method is very simple to implement and quick to converge inasmuch it works with an explicit algorithm that does not require particular checks or costly matrix inversions. The basic idea starts from the well-known catenary (undeformable cable) model, a large-displacements problem in which the closed-form solution is governed by a single quantity, i.e. the tension at the vertex. This is extended to the case of elastic beam subjected to large displacements, in which the bending strain is assumed to be dominant. This last assumption fits well with the cases here taking into account, i.e. pipes' lifting and suspension problems, in which axial and shear strains can be neglected.

Some numerical examples are shown: the end-lifting of a pipe initially in contact with the soil for three initial slope angles, and the full suspension of pipes by multi-hooking points. In the latter case, it is shown how, starting from a converged solution, it is possible to obtain a combination of lifting forces, such as having all the hooking points aligned, just as it would be desirable for it to happen in a practical case.

Furthermore, this idea could be used to study the lifting of heavy slender elements also involving other non-linear phenomena, which can often happen, such as plasticity or buckling.

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