Combining penetration resistance and shear-wave velocity to quantify soil microstructure for liquefaction assessment

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ABSTRACT

This paper discusses two ratios involving penetration resistance and shear-wave velocity (V_S) that have been proposed for quantifying the influence of microstructure in aged and cemented soils for liquefaction assessment. The first ratio is the small-strain shear modulus (G_{max}) divided by the cone penetration test tip resistance (q_c). Because G_{max}/q_c is dimensionless, it can be expressed as a ratio of measured V_S divided by a function of q_c with velocity units. The second ratio is the measured V_S divided by an estimated V_S from penetration resistance-V_S relationships for relatively young sand deposits ($MEVR$). The advantages and limitations of both ratios are discussed. The influence of various fines content (FC) corrections on V_S, penetration resistance-V_S relationships, and a relationship between $MEVR$ and the liquefaction cyclic resistance ratio correction factor for microstructure (K_{DR}) is evaluated using two published datasets. The results show the FC correction to V_S is minimal in the range for which the correction was derived. The FC corrections to q_c and standard penetration test blow count are significant for silty soils, having a net effect of lowering the penetration resistance-V_S relationships and increasing the slope of the $MEVR$-K_{DR} predictive relationship.

Keywords: cemented sediments; liquefaction; penetration test; shear-wave velocity.

1. Introduction

Microstructure in soils caused by aging and cementation can increase stiffness, strength, and liquefaction resistance. On the other hand, all or part of the microstructure can be erased by a disturbing event such as liquefaction or construction excavation. A promising approach to quantifying the influence of soil microstructure is combining field measurements of penetration resistance and shear-wave velocity.

Shear-wave velocity (V_S) and small-strain shear modulus (G_{max}) are directly related by:

$$G_{max} = \rho {V_S}^2 \quad (1)$$

where ρ is the mass density (i.e., the total unit weight divided by the acceleration of gravity). Eslaamizaad and Robertson (1996), Schneider et al. (2004), and Schnaid (2009) observed that aged and cemented soils tend to exhibit greater ratios of G_{max} to cone penetration test (CPT) tip resistance (q_c). The ratio G_{max}/q_c can be rewritten in terms of V_S and expressed as:

$$\frac{G_{max}}{q_c} = \left[\frac{V_S}{(q_c/\rho)^{0.5}}\right]^2 \quad (2)$$

Because Eq. 2 is a dimensionless ratio, $(q_c/\rho)^{0.5}$ must be in velocity units.

An alternative to G_{max}/q_c is the ratio of measured shear-wave velocity to estimated shear-wave velocity ($MEVR$) defined as (Andrus et al. 2009):

$$MEVR = \frac{V_{SM}}{V_{SE}} \quad (3)$$

where V_{SM} is the measured shear-wave velocity; and V_{SE} is the estimated shear-wave velocity. Andrus et al. (2009) assumed one form of V_{SM} to be:

$$V_{SM} = V_{S1cs} = K_{cs} V_S \left(\frac{P_{ref}}{\sigma'_{vo}}\right)^{0.25} \quad (4)$$

where V_{S1cs} is the shear-wave velocity corrected for overburden stress and fines content; K_{cs} is a correction factor for fines content; P_{ref} is a reference stress equal to 100 kPa; and σ'_{vo} is the initial vertical effective stress. Fines content (FC) in this paper is defined as percent of mass passing sieve 0.075 mm (or No. 200).

For computing V_{SE} corresponding to Eq. 4, Andrus et al. (2009) assumed the following (Andrus et al. 2004):

$$V_{SE} = V_{S1cs} = 62.6(q_{c1Ncs})^{0.231} \quad (5)$$

$$V_{SE} = V_{S1cs} = 87.8[(N_1)_{60cs}]^{0.253} \quad (6)$$

where q_{c1Ncs} is the normalized overburden stress-corrected clean-sand equivalent CPT tip resistance; and $(N_1)_{60cs}$ is the overburden stress-corrected clean-sand equivalent standard penetration test (SPT) blow count computed using the procedures described in Youd et al. (2001). Considering Eqs. 5 and 6 and an alternative form of Eq. 5 (i.e., uncorrected V_S and q_c), Andrus et al. (2009) estimated them to correspond to 6 to 20-year-old sands.

Table 1 summarizes the features of G_{max}/q_c and $MEVR$. One advantage of G_{max}/q_c is its simplicity (i.e., no corrections are involved). The advantages of $MEVR$ include the ability to estimate V_S from CPT and SPT measurements, the best-fit exponent, and the knowledge that V_{SE} is for 6 to 20-year-old predominantly quartz sand. Additionally, when $MEVR$ and G_{max}/q_c are used for predicting the liquefaction cyclic resistance ratio aging/cementation correction factor (K_{DR}), $MEVR$ provides the highest coefficient of determination (R^2) and the lowest root mean squared error ($RMSE$) of the two ratios (Bwambale and Andrus 2019).

Table 1. Comparison of two dimensionless ratios involving penetration resistance and V_S

Feature	G_{max}/q_c	$MEVR$
Type of penetration test(s)	CPT	CPT or SPT
Reference or estimated V_S	$\left(\frac{q_c}{\rho}\right)^{0.5}$	$a(q_{c1Ncs})^b$ or $a[(N_1)_{60cs}]^b$
Coefficient	Assume $\left(\frac{1}{\rho}\right)^{0.5}$	Best fit a
Exponent on penetration resistance	Assume 0.5	Best fit b
Meaning of reference or estimated V_S	Needs to be determined	V_S for 6 to 20-year-old predominantly quartz sand
Fines content and overburden stress corrections	No	Yes
Other required site properties	Total unit weight at given depth	Total unit weight above given depth; depth to groundwater table; fines content at given depth
K_{DR} as a predictor variable	Lower R^2, higher $RMSE$	Higher R^2, lower $RMSE$

Since the development of Eqs. 4, 5, and 6, updates to the FC corrections to V_S, q_c and SPT blow count for use in liquefaction evaluations have been proposed. This paper aims to evaluate the influence of the FC corrections on V_S, q_c, SPT blow count, penetration resistance-V_S relationships, and the $MEVR$-K_{DR} relationship.

2. Selected fines content corrections

Various fines content corrections have been proposed for the overburden stress-corrected shear-wave velocity (V_{S1}), the overburden stress-corrected SPT blow count [$(N_1)_{60}$], and the normalized overburden stress-corrected CPT tip resistance (q_{ciN}) in liquefaction evaluations (e.g., Robertson and Wride 1998; Youd et al. 2001; Juang et al. 2002; Idriss and Boulanger 2008; Boulanger and Idriss 2016). The corrections were derived by compiling liquefaction and no liquefaction case histories into bins and comparing them with the baseline liquefaction cyclic resistance ratio (CRR) curves for clean sands. Five fines content corrections are discussed below.

2.1. Shear-wave velocity

Juang et al. (2002) suggested the following relationship for estimating K_{cs} (see Eq. 4):

$$K_{cs} = 1.0 \quad \text{for } FC \leq 5\% \tag{7a}$$

$$K_{cs} = 1.0 + (FC - 5)T \quad \text{for } 5\% < FC < 35\% \tag{7b}$$

$$K_{cs} = 1.0 + 30T \quad \text{for } FC \geq 35\% \tag{7c}$$

$$T = 0.009 - 0.0109(V_{S1}/100) + 0.0038(V_{S1}/100)^2 \tag{8}$$

where FC is in percent; and V_{S1} is in m/s. Values of K_{cs} based on Eq. 7c increase from 1.06 at V_{S1} = 100 m/s to 1.07 at V_{S1} = 200 m/s to 1.32 to V_{S1} = 300 m/s. It is important to note that Eqs. 7 and 8 were derived using CRR curves from Andrus and Stokoe (2000) and liquefaction cases limited to V_{S1} = 105 to 205 m/s.

Figure 1 compares the V_{S1} values from the dataset of Holocene soil with CPT soil behavior type index (I_c) < 2.25 compiled by Andrus et al. (2004) with V_{S1cs} values based on the FC correction by Juang et al. (2002). It can be seen in Fig. 1 that the FC correction is minor for $V_{S1} \leq$ 205 m/s. For V_{S1} > 205 m/s, the range for which little or no data support Eqs. 7 and 8, the FC correction is as much as 9%. The finding that the FC correction is minor for V_{S1} ≤ 205 m/s agrees with the conclusion of Kayen et al. (2013), who, based on an analysis of liquefaction case history data, concluded that the K_{cs} correction can be ignored in liquefaction triggering evaluations.

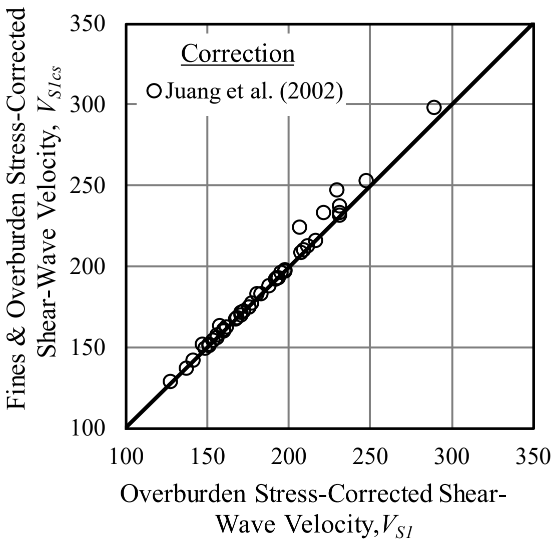

Figure 1. Comparison of V_{S1} in Andrus et al. (2004) for Holocene soil with I_c < 2.25 with V_{S1cs} values based on the FC correction by Juang et al. (2002).

2.2. CPT tip resistance

Youd et al. (2001) recommended the Robertson and Wride (1998) I_c (or FC) correction of measured CPT tip resistance to an equivalent clean sand value. The Robertson and Wride (1998) correction is expressed as:

$$q_{c1Ncs} = K_c q_{ciN} \tag{9}$$

$$K_c = 1.0 \quad \text{for } I_c \leq 1.64 \tag{10a}$$

$$K_c = -0.403I_c^{\ 4} + 5.581I_c^{\ 3} - 21.63I_c^{\ 2} + 33.75I_c - 17.88 \quad \text{for } I_c > 1.64 \tag{10b}$$

Boulanger and Idriss (2016) recommended the following correction of CPT tip resistance to an equivalent clean sand value:

$$q_{c1Ncs} = q_{c1N} + \Delta q_{c1N} \tag{11}$$

$$\Delta q_{c1N} = \left(11.9 + \frac{q_{c1N}}{14.6}\right) exp\left[1.63 - \frac{9.7}{FC+2} - \left(\frac{15.7}{FC+2}\right)^2\right] \quad (12)$$

In the absence of FC information, Boulanger and Idriss (2016) recommended using the following:

$$FC = 80(I_c + C_{FC}) - 137 \quad (13)$$

where C_{FC} is a fitting parameter (default value of 0.0) that depends on many factors (e.g., material, deposition, ground densification) and should be calibrated for individual geologic strata. Equation 13 was recommended by Boulanger and Idriss (2016) for $0\% \leq FC \leq 100\%$.

Figure 2 compares the q_{ciN} values in the dataset by Andrus et al. (2004) for Holocene soil with I_c < 2.25 with q_{ciNcs} values based on the Robertson and Wride (1998) and Boulanger and Idriss (2016) FC corrections. It can be seen in Fig. 2 that the Robertson and Wride (1998) correction is sometimes more significant than the Boulanger and Idriss (2016) correction.

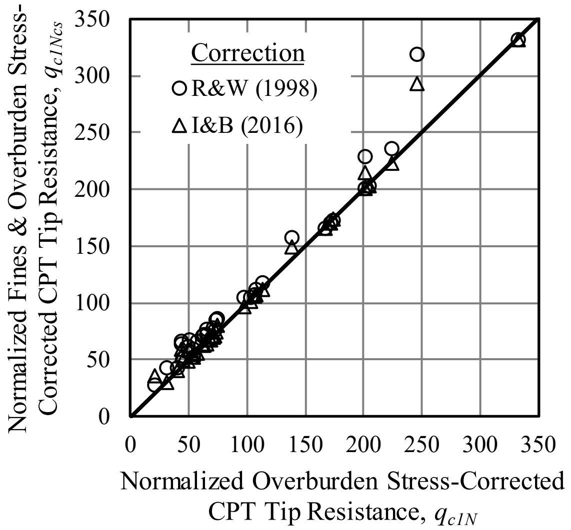

Figure 2. Comparison of q_{ciN} values in the Andrus et al. (2004) for Holocene soil with I_c < 2.25 with q_{ciNcs} values based on the Robertson and Wride (1998) and Idriss and Boulanger (2016) FC corrections.

2.3. SPT blow count

Youd et al. (2001) recommended the following FC correction to SPT blow count which I. M. Idriss developed with the assistance of R. B. Seed:

$$(N_1)_{60,cs} = \alpha + \beta(N_1)_{60} \quad (14)$$

$$\alpha = 0.0 \quad \text{for } FC \leq 5\% \quad (15a)$$

$$\alpha = exp(1.76 - 190/FC^2) \quad \text{for } 5\% < FC < 35\% \quad (15b)$$

$$\alpha = 5.0 \quad \text{for } FC \geq 35\% \quad (15c)$$

$$\beta = 1.0 \quad \text{for } FC \leq 5\% \quad (16a)$$

$$\beta = 0.99 + FC^{1.5}/1000 \quad \text{for } 5\% < FC < 35\% \quad (16b)$$

$$\beta = 1.2 \quad \text{for } FC \geq 35\% \quad (16c)$$

where FC is in percent.

Idriss and Boulanger (2008) reformulated Eqs. 14, 15 and 16 and considered additional data from Cetin et al. (2000). Their revised correction for $5\% < FC < 35\%$ is computed as:

$$(N_1)_{60cs} = (N_1)_{60} + \Delta(N_1)_{60} \quad (17)$$

$$\Delta(N_1)_{60} = exp\left[1.63 + \frac{9.7}{FC+0.01} - \left(\frac{15.7}{FC+0.01}\right)^2\right] \quad (18)$$

where FC is in percent. The values of $\Delta(N_1)_{60}$ are 0.0 for $FC \leq 5\%$ and 5.5 for $FC \geq 35\%$.

Figure 3 compares the $(N_1)_{60}$ values in the dataset by Andrus et al. (2004) for Holocene soil with I_c < 2.25 with $(N_1)_{60,cs}$ values based on the Youd et al. (2001) and Idriss and Boulanger (2008) FC corrections. It can be seen in Fig. 3 that the correction is significant for six data pairs. The Idriss & Boulanger (2008) procedure provides the smaller corrections. Because Eqs. 17 and 18 represent an update to the Youd et al. (2001) recommended correction, the Idriss and Boulanger (2008) correction is considered in this paper going forward. The fact that the FC correction is small for many of the cases in Fig. 3 (and Figs. 1 and 2) suggests that only small changes are likely in updates to the penetration resistance-V_S relationships proposed by Andrus et al. (2004).

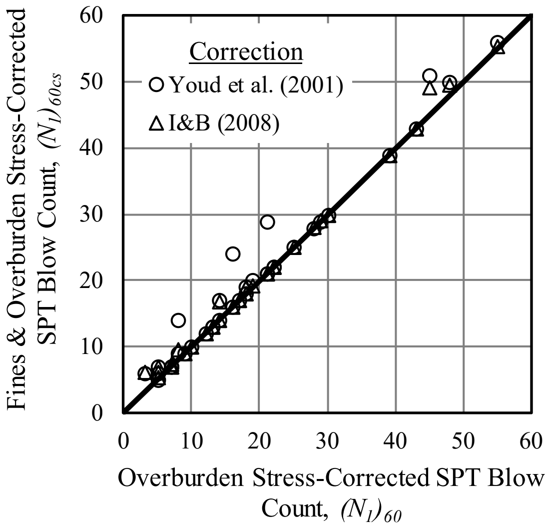

Figure 3. Comparison of $(N_1)_{60}$ values in the dataset by Andrus et al. (2004) for Holocene soil with I_c < 2.25 with $(N_1)_{60,cs}$ values are based the Youd et al. (2001) and Idriss and Boulanger (2008) FC corrections.

3. Updated penetration resistance-V_S relationships

Figures 4a, 4b and 4c present the Andrus et al. (2004) Holocene soil with I_c < 2.25 data with no FC correction applied to V_{S1} and the Robertson and Wride (1998) FC correction applied to q_{ciN}, the Boulanger and Idriss (2016) FC correction applied to q_{ciN}, and the Idriss and Boulanger (2008) FC correction applied to $(N_1)_{60}$, respectively. Also presented are regression curves obtained in this study based on the plotted data. It can be seen in Figs. 4a, 4b, and 4c that this study curves plot somewhat below the Andrus et al. (2004) relationships, particularly at the higher blow counts.

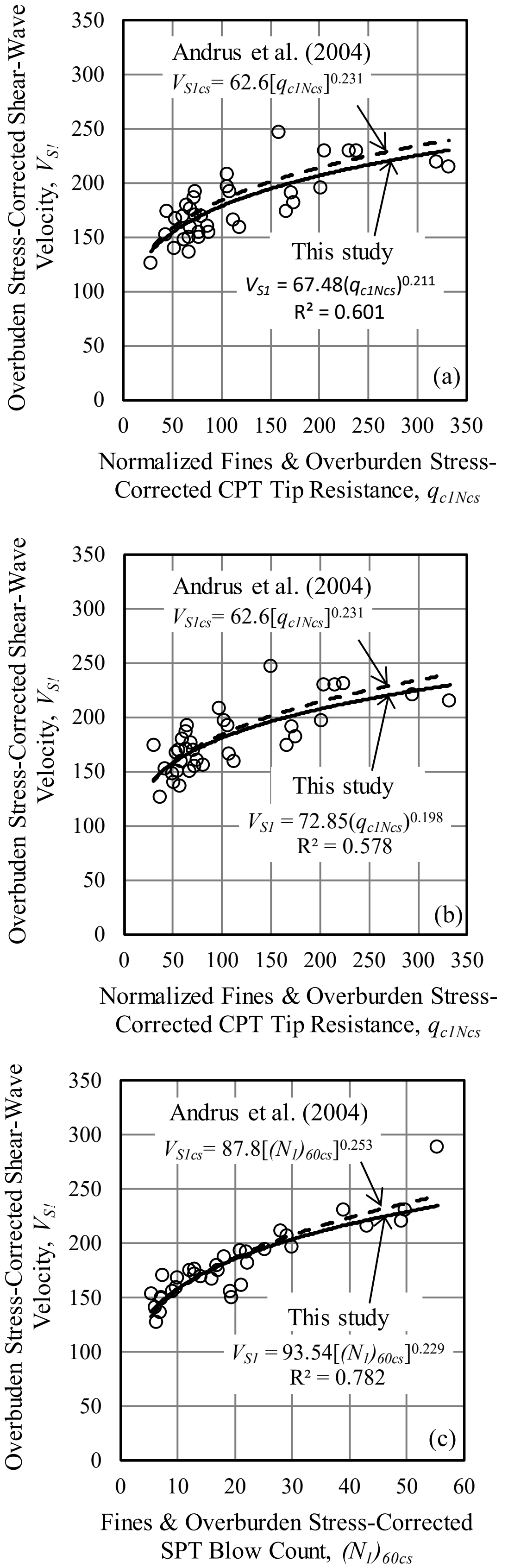

Figure 4. Comparison of two regression curves and the Andrus et al. (2004) Holocene soil data based on no FC correction applied to V_{S1} and (a) the Robertson and Wride (1998) FC correction applied to q_{ciN}; (b) the Boulanger and Idriss (2016) FC correction applied to q_{ciN}; and (c) the Idriss and Boulanger (2008) FC correction applied to $(N_1)_{60}$.

The relationships in Fig. 4 labeled as "this study" are defined by:

$$V_{SE} = 67.48(q_{c1Ncs})^{0.211} \quad (19)$$

$$V_{SE} = 72.85(q_{c1Ncs})^{0.198} \quad (20)$$

$$V_{SE} = 93.54[(N_1)_{60cs}]^{0.229} \quad (21)$$

Equations 19, 20 and 21 are updates to Eqs. 5 and 6 based on the more recent FC corrections. V_{SE} values provided by Eq. 20 are < 4% higher than V_{SE} values provided by Eq. 19, well within the range of the plotted data.

4. Influence of microstructure on soil liquefaction triggering

The resistance of soils to liquefaction triggering can be expressed by the cyclic resistance ratio (CRR). CRR is typically determined from semi-empirical charts based on field tests (e.g., Seed et al. 1985; Youd et al. 2001; Idriss and Boulanger 2008; Boulanger and Idriss 2016). The commonly used CRR charts are based primarily on case histories from level ground, shallow, saturated, uncemented, and young deposits. Thus, the following corrections to CRR for differing site conditions have been proposed:

$$CRR_{corrected} = K_\sigma K_\alpha K_S K_{DR} CRR \quad (22)$$

where K_σ is the correction factor for effective overburden stress; K_α is the correction factor for sloping ground or initial static shear stress; K_S is the correction factor for unsaturated conditions below the groundwater table; and K_{DR} is the correction factor for the effect of microstructure due to aging and cementation.

K_{DR} is defined as the CRR of the intact soil divided by the CRR of the same soil at an assumed reference condition. As Hayati and Andrus (2009) discussed, for K_{DR} based solely on laboratory cyclic testing, the reference condition is the reconstituted specimen, typically tested within a few days of its formation. On the other hand, commonly used CRR charts correspond to the average age of the field case histories used in their development, which is on the order of 10 to 20 years. Thus, if K_{DR} based solely on laboratory cyclic testing is used to correct CRR from charts, K_{DR} must first be corrected for the differing reference condition (Andrus and Boland 2024).

Hayati and Andrus (2009) proposed the first $MEVR$-K_{DR} relationships. Based on a critical review of the available field case histories and additional data, Bwambale and Andrus (2019) updated the Hayati and Andrus (2009) relationship for use with commonly used CRR charts. The updated relationship is expressed as follows:

$$K_{DR,chart} = 0.92\, MEVR + 0.12 \quad (23)$$

where $K_{DR,chart}$ is the soil microstructure correction factor for commonly used CRR charts. Equations 19, 20 and 21 provide the opportunity to update Eq. 23.

Table 2 summarizes the cases compiled by Bwambale and Andrus (2019). Cases 15A and 15B are from the study by Hayati and Andrus (2008) and were omitted accidentally from the summary table in Bwambale and Andrus (2019). The cases in Table 2 are limited to FC < 67 %, I_c < 2.53, q_{c1N} < 131, V_{S1} < 290 m/s, and K_{DR} < 2.5.

Table 2. Summary of $MEVR$-$K_{DR,chart}$ cases (adapted from Bwambale and Andrus 2019)

Case	FC (%)	I_c	q_{c1N}	V_{S1M} (m/s)	$K_{DR,chart}$
12A	10	1.99	71	156	1.14
12B	5	1.70	51	168	0.96
12C	5	1.69	66	177	0.76
12D	15	2.54	19	127	0.92
12E	8	1.72	37	153	1.38
12F	10	1.72	42	141	1.28
12G	5	≤1.64[a]	58	185	1.22
15A	32[a]	2.53[b]	29[b]	148[b]	1.00
15B	11[a]	1.93[b]	83[b]	256[b]	1.70
Gainsborough	32	2.07	50	142	1.19
Kilmore Ch. Fm.	3	1.95	69	168	1.07
Kilmore Sp. Fm.	35	2.07	53	154	0.91
Port Hills	32[b]	2.45[b]	33[b]	289[b]	2.41
Riccarton	66	2.31	37	154	1.37
Yodo	2	≤1.64[a]	131	202	0.62
Toro	4	≤1.64[a]	110	203	1.08
Edo	<1	≤1.64[a]	88	164	1.36
MES	<1	≤1.64[a]	120	141	0.67
FHS	10	1.88	58	140	0.76
HWD	12	1.95	80	142	0.46
SAM	4	≤1.64[a]	79	223	0.89

[a]Estimated from $FC = 1.75 I_c^{3.25} - 3.7$ for $1.26 \le I_c \le 3.5$ (Robertson and Wride 1998).
[b]Based on multiple seismic CPT.

Figure 5 presents the $MEVR$-$K_{DR,chart}$ relationships based on the cases summarized in Table 2 and the Robertson and Wride (1998) and Boulanger and Idriss FC corrections. It can be seen in Fig. 5 that the Robertson and Wride (1998) correction provides slightly higher $MEVR$ values than the Boulanger and Idriss correction. The relationship based on the Robertson and Wride (1998) correction is expressed as:

$$K_{DR,chart} = 1.24\ MEVR - 0.15 \quad (24a)$$

The relationship based on the Boulanger and Idriss (2016) correction is expressed as:

$$K_{DR,chart} = 1.25\ MEVR - 0.18 \quad (24b)$$

The difference between Eqs. 24a and 24b is 3.3% at $MEVR$ = 0.7 and 0.6% at $MEVR$ = 1.8. Thus, for practical purposes, Eqs. 24a and 24b can be assumed equal within the limits of the data plotted in Fig. 5.

Equations 24a and 24b are supported by data pairs with $MEVR$ between 0.7 and 1.4. More data are needed to constrain better the $MEVR$-$K_{DR,chart}$ relationship above $MEVR$ > 1.2.

Figure 6 compares the average of Eqs. 24a and 24b with the Hayati and Andrus (2009) and Bwambale and Andrus (2019) relationships. As seen in Fig. 6, this study relationship compares well with the two earlier relationships at $MEVR$ = 0.7 but exhibits a more significant increase at higher values of $MEVR$. Much of the difference in slopes can be explained by the Port Hills case with V_{S1} = 289 m/s and V_{S1cs} = 409 m/s (Bwambale and Andrus 2019). It is important to note that the V_{S1} value of 289 m/s is well outside the data limits for which Eqs. 7 and 8 are supported.

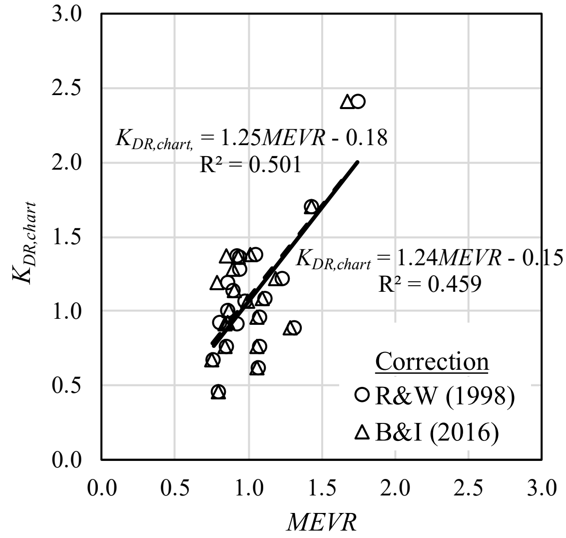

Figure 5. Relationships between $MEVR$ and $K_{DR,chart}$ based on the cases summarized in Table 2 and the Robertson and Wride (1998) and Boulanger and Idriss (2016) FC corrections.

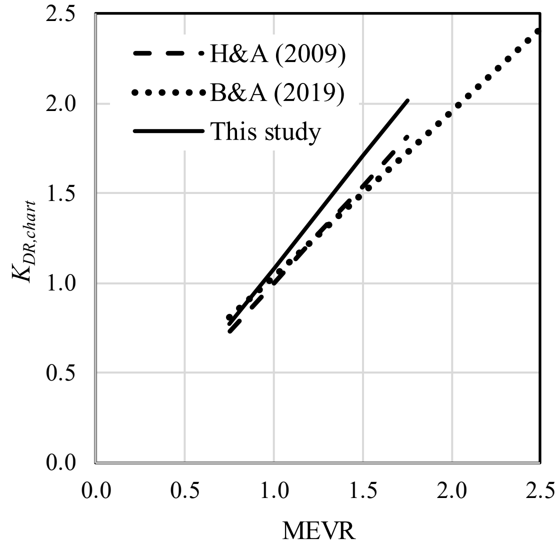

Figure 6. Comparison of three $MEVR$-$K_{DR,chart}$ relationships. This study relationship is an average of Eqs. 24a and 24b.

5. Conclusions

The influence of FC corrections on V_{S1}, q_{c1N}, $(N_1)_{60}$, the penetration resistance-V_S relationships, and the $MEVR$-$K_{DR,chart}$ relationship were evaluated using two published datasets. Figure 1 showed the FC correction to V_{S1} is minimal in the range for which the correction is valid (V_{S1} < 205 m/s). This observation agrees with the analysis of Kayen et al. (2013), who concluded that the FC correction to V_{S1} can be ignored in liquefaction triggering evaluations. The FC corrections to q_{c1N} and $(N_1)_{60}$ are significant for some. The net effect of the FC corrections to q_{c1N} and $(N_1)_{60}$, with respect to the previous studies, is to lower the penetration resistance-V_S relationships and to increase the slope of the $MEVR$-$K_{DR,chart}$ relationship. Much of the difference in slopes of the $MEVR$-$K_{DR,chart}$ relationships can be explained by the FC corrrection applied to the Port Hills V_{S1} value by Bwambale and Andrus (2019). More data are needed to constrain better the $MEVR$-$K_{DR,chart}$ relationship above $MEVR > 1.2$. The revised $MEVR$-$K_{DR,chart}$ relationship provides a rational and cost-effective approach for quantifying soil microstructure's influence on soil liquefaction triggering.

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References

Andrus, R., and Boland, K. "Quantifying the influence of microstructure on liquefaction resistance in aged and cemented sands," *Proc, 18th World Conference on Earthquake Engrg* to be held in Milan, Italy, 30th June – 5th July 2024.

Andrus, R. D., and Stokoe, K. H. II. "Liquefaction resistance of soils from shear-wave velocity," *J Geotechnical and Geoenvironmental Engrg,* 126(11), pp. 1015-1025, 2009. https://doi.org/10.1061/(ASCE)1090-0241(2000)126:11(1015)

Andrus, R. D., Piratheepan, P., Ellis, B. S., Zhang, J., and Juang, C. H. "Comparing liquefaction evaluation methods using penetration-V_S relationships," *Soil Dynamics and Earthquake Engrg,* 24(9), pp. 713-721, 2004. https://doi.org/10.1016/j.soildyn.2004.06.001

Andrus, R. D., Hayati, H., and Mohanan, N. P. "Correcting liquefaction resistance of aged sands using measured to estimated velocity ratio," *J Geotechnical and Geoenvironmental Engrg,* 135(6), pp. 735-744, 2009. https://doi.org/10.1061/(ASCE)GT.1943-5606.0000025

Boulanger, R. W., and Idriss, I. M. "CPT-based liquefaction triggering procedure," *J Geotechnical and Geoenvironmental Engrg,* 142(2), 2016. https://doi.org/10.1061/(ASCE)GT.1943-5606.0001388

Bwambale, B., and Andrus R. D. "State of the art in the assessment of aging effects on soil liquefaction," *Soil Dynamics and Earthquake Engrg*, 125(10), 2019. https://doi.org/10.1016/j.soildyn.2019.04.032

Eslaamizaad, S., and Robertson, P. K. "Seismic cone penetration test to identify cemented sands," *Proc, 49th Canadian Geotechnical Conference,* Canadian Geotechnical Society, Vol. 1, pp. 352-360, 1996.

Hayati, H., and Andrus, R. D "Liquefaction potential map of Charleston, South Carolina based on the 1886 earthquake," *J Geotechnical and Geoenvironmental Engrg,* 134(6), 2008. https://doi.org/10.1061/(ASCE)1090-0241(2008)134: 6(815)

Hayati, H., and Andrus, R. D "Updated liquefaction resistance correction factors for aged sands," *J Geotechnical and Geoenvironmental Engrg,* 135(11), 2009. https://doi.org/10.1061/(ASCE)GT.1943-5606.0000118

Idriss, I. M., and Boulanger, R. W. *Soil Liquefaction during Earthquakes*, Earthquake Engineering Research Institute, Oakland, CA, 2008.

Juang, C. H., Jiang, T., and Andrus, R. D. "Assessing probability-based methods for liquefaction potential evaluation," *J Geotechnical and Geoenvironmental Engrg*, 128(7), pp. 580-589, 2002. https://doi.org/10.1061/(ASCE)1090-0241(2002)128:7(580)

Kayen, R., Moss, R. E. S., Thompson, E. M., Seed, R. B., Cetin, K. O., and Der Kiureghian, A. "Shear-wave velocity-based probabilistic and deterministic assessment of seismic soil liquefaction potential," *J Geotechnical and Geoenvironmental Engrg,* 139(3), 2013. https://doi.org/10.1061/(ASCE)GT.1943-5606.0000743

Robertson, P. K., and Wride, C. E. "Evaluating cyclic liquefaction potential using the cone penetration test," *Canadian Geotechnical J,* 35(3), pp. 442-459, 1998. https://doi.org/10.1139/t98-017

Schneider, J. A., McGillivray, A. V., and Mayne, P. W. "Evaluation of SCPTU intra-correlations at sand sites in the lower Mississippi River Valley, USA," In: A. Viana da Fonseca & P. W. Mayne, editors, *2nd International Conference on Geotechnical and Geophysical Site Characterization (ISC-2),* Millpress, Rotterdam, pp. 1003-1010, 2004.

Schnaid, F. *In-Situ Testing in Geomechanics – The Main Tests*, Taylor & Francis Group, London, 2009.

Youd, T. L., Idriss, I. M., Andrus, R. D., Arango, I., Castro, G., Christian, J. T., Dobry, R., Finn, W. D. L., Harder, L. F., Jr., Hynes, M. E., Ishihara, K., Koester, J. P., Liao, S. S. C., Marcuson, W. F., III, Martin, G. R., Mitchell, J. K., Moriwaki, Y., Power, M. S., Robertson, P. K., Seed, R. B., and Stokoe, K. H., II. "Liquefaction resistance of soils: Summary report from the 1996 NCEER and 1998 NCEER/NSF workshops on evaluation of liquefaction resistance of soils," *J Geotechnical and Geoenvironmental Engrg*, 127(10), pp. 817-833, 2001. https://doi.org/10.1061/(ASCE)1090-0241(2001)127:10(817)