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CONSTITUTIVE MODELLING OF WOOD-BASED MATERIALS

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Abstract. Wood, unlike steel and concrete, is an anisotropic material. Because of its inherent characteristics, the mechanical behaviour of wood depends on the grain direction and the load type. Appropriate material models are the fundamental basis of reliable simulations. The constitutive models incorporated in existing general design software packages are often limited, making the software unsuitable for accurately predicting the mechanical behaviour and failure modes of wood-based materials. In this paper, a comprehensive constitutive model, composed of sub-models for describing the elastic properties, strength criterion, post-peak softening for quasi-brittle failure modes, plastic flow and hardening rule for yielding failure modes, and densification perpendicular to grain, was introduced. Modelling considerations on the effects of temperature, moisture content, and loading time were discussed. Advanced and practical modelling methods and key considerations for wood-based products were introduced, aiming to support practicing engineers and researchers to become better acquainted with modelling and analysing timber structures.

1 INTRODUCTION

Computer modelling is an essential part in the analysis and design of residential and commercial buildings. It is also a valuable tool in the development and optimisation of wood-based products, connections, and systems. The global interest in the use of wood-based products as structural and non-structural components in high-rise and long-span structures is gaining momentum, due to their sustainability and low carbon footprint. A survey shows that practicing engineers are typically unfamiliar with timber structure modelling, and researchers generally lack resources for advanced modelling of timber systems [1]. Recently, a first-of-its-kind modelling guide for timber structures [1] has been developed by more than 100 global experts: web.fpinnovations.ca/modelling, for supporting the application of numerical modelling on analysis and design of timber structures, and development and optimisation of wood-based products and systems.

Wood, unlike steel and concrete, is an anisotropic material. Because of its inherent characteristics, the mechanical behaviour of wood depends on the direction of the grain and the load type. The mechanical properties of wood change as temperature, moisture, and loading time change. Moreover, growth characteristics such as slope of grain and knots significantly affect the mechanical behaviour of wood-based products. Appropriate material models are the fundamental basis of reliable simulations. The constitutive models incorporated in existing general design software packages are often limited, making the software unsuitable for accurately predicting the mechanical behaviour and failure modes of wood-based materials.

In this paper, a comprehensive constitutive model, composed of sub-models for describing the elastic properties, strength criterion, post-peak softening for quasi-brittle failure modes, plastic flow, and hardening rule for yielding failure modes, and densification perpendicular to grain, is introduced. Modelling considerations on the effects of temperature, moisture content, and loading time are discussed. Advanced and practical modelling methods and key considerations for wood-based products are introduced, aiming to support practicing engineers and researchers to become better acquainted with modelling and analysing timber structures.

2 CONSTITUTIVE MODELS

The anisotropic characteristics of wood are a result of its fibrous structure and its 3D orthotropic nature (Figure 1). The stiffness and strength of wood vary as a function of the three main grain orientations: longitudinal (L), radial (R), and tangential (T). The failure modes and the stress-strain relationships of wood depend on the direction of the load relative to the grain and the type of load (tension, compression, or shear). As illustrated in Figure 2, for wood subjected to tension or shear, the stress-strain relationship is typically linear elastic and the failure is quasi-brittle; for wood in compression, the stress-strain relationship is typically nonlinear and the failure is ductile [2, 3].

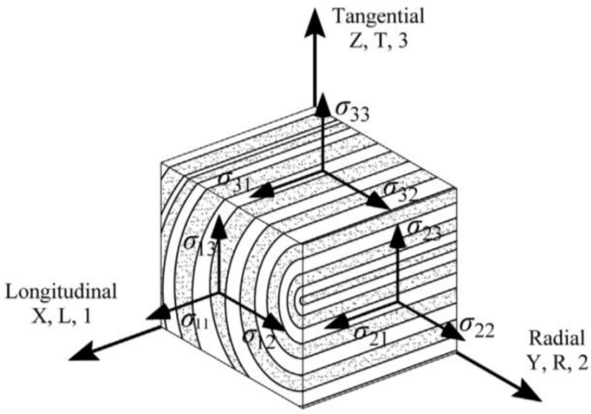

Figure 1: 3D stress components and orthogonal material directions for a timber element [4]

To model the mechanical behaviour of wood-based products under various forces, the constitutive model should include the following components: (a) elastic properties; (b) strength criterion; (c) post-peak softening for quasi-brittle failure modes; (d) plastic flow and hardening rule for yielding failure modes; and (e) second hardening (densification) perpendicular to grain. Depending on the modelling complexities, scenarios, and demands, different constitutive models with various combinations of these components can be adopted. For example, a constitutive model consisting of elastic properties is usually sufficient to determine the deflection and stress distribution of a wood-based element when the load is small; however, strength criterion needs to be included in the constitutive model when the load-carrying capacity, failure mode, or both are required. If the post-strength behaviour is of interest, the post-peak softening, hardening, and yielding – or all – are required in the constitutive model.

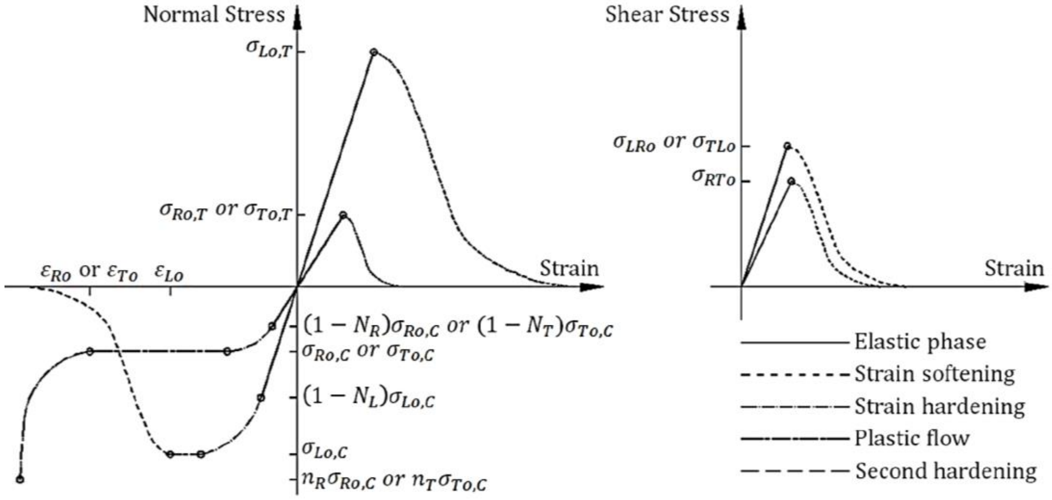

Figure 2: Typical stress-strain behaviour of wood

Note: σ_{io} is the axial strength in the i direction [MPa]; $\sigma_{io,T}$ and $\sigma_{io,C}$ are the tensile strength and the compressive strength in the i direction [MPa]; σ_{ijo} is the shear strength in the i – j plane [MPa]; N_i and n_i are parameters to determine the initial and final ultimate yield surface, respectively; ε_{Lo} is the initial damage strain for compression parallel to grain; ε_{Ro} and ε_{To} are the initial second-hardening strain for compression perpendicular to grain.

2.1 Elastic behaviour

The elasticity of a material defines its strain, ε_{ij}, response to applied stresses, σ_{ij}. Commonly, wood is simplified into orthotropic material, so nine independent material parameters are needed to replicate its orthotropy: three moduli of elasticity (E_L, E_R, and E_T), three shear moduli (G_{LR}, G_{LT}, and G_{RT}), and three Poisson's ratios (ν_{LR}, ν_{LT}, and ν_{RT}). For common species of wood, these parameters have been measured and are recorded in handbooks. The nine material parameters together define a constitutive relation for wood in the form of a 3D generalised Hooke's law.

$$\begin{bmatrix}\sigma_L\\ \sigma_R\\ \sigma_T\\ \sigma_{LR}\\ \sigma_{RT}\\ \sigma_{TL}\end{bmatrix} = \begin{bmatrix} \frac{E_L(1-\nu_{RT}\nu_{TR})}{\Upsilon} & \frac{E_L(\nu_{RL}+\nu_{TL}\nu_{RT})}{\Upsilon} & \frac{E_L(\nu_{TL}+\nu_{RL}\nu_{TR})}{\Upsilon} & 0 & 0 & 0\\ \frac{E_L(\nu_{RL}+\nu_{TL}\nu_{RT})}{\Upsilon} & \frac{E_R(1-\nu_{LT}\nu_{TL})}{\Upsilon} & \frac{E_R(\nu_{TR}+\nu_{LR}\nu_{TL})}{\Upsilon} & 0 & 0 & 0\\ \frac{E_L(\nu_{TL}+\nu_{RL}\nu_{TR})}{\Upsilon} & \frac{E_R(\nu_{TR}+\nu_{LR}\nu_{TL})}{\Upsilon} & \frac{E_T(1-\nu_{LR}\nu_{RL})}{\Upsilon} & 0 & 0 & 0\\ 0 & 0 & 0 & 2G_{LR} & 0 & 0\\ 0 & 0 & 0 & 0 & 2G_{RT} & 0\\ 0 & 0 & 0 & 0 & 0 & 2G_{LT}\end{bmatrix} \begin{bmatrix}\varepsilon_L\\ \varepsilon_R\\ \varepsilon_T\\ \varepsilon_{LR}\\ \varepsilon_{RT}\\ \varepsilon_{TL}\end{bmatrix} \quad (1)$$

$$\Upsilon = 1 - \nu_{LR}\nu_{RL} - \nu_{RT}\nu_{TR} - \nu_{TL}\nu_{LT} - 2\nu_{RL}\nu_{TR}\nu_{LT} \quad (2)$$

$$\nu_{ij} = \nu_{ji}\left(E_i/E_j\right) \quad (3)$$

2.2 Failure models and strength criteria

Unlike steel, which is an isotropic material, wood has significantly different strengths in the longitudinal, radial, and tangential directions. The strength in the longitudinal direction is greater than that in the radial and tangential directions. Moreover, the strengths of wood under tension differ from the strengths under either parallel-to-grain or perpendicular-to-grain compression. Various strength criteria have been developed for predicting localised material failure due to stress caused by static load. A brief comparison of typical strength criteria is given in Table 1 and below:

Table 1: Comparison of various strength criteria

Criterion	Ductile or brittle material	Interaction between the strengths?	Indicate brittle or ductile failure?	Account for different strengths in tension and compression?
Maximum stress	Both	No	Yes	Yes
Coulomb-Mohr	Brittle	No	No	Yes
Von Mises	Ductile	No	No	No
Tresca	Ductile	No	No	No
Hill	Ductile	Yes	No	No
Tsai-Wu	Both	Yes	No	Yes
Hoffman	Both	Yes	No	Yes
Norris	Both	Yes	Yes	Yes
Hashin	Both	Yes	Yes	Yes
Extended Yamada-Sun	Both	Yes	Yes	Yes

The von Mises and Tresca strength criteria apply to very ductile isotropic materials. The Coulomb-Mohr criterion is suitable for brittle materials. The Hill strength criterion can be used for orthotropic materials; however, it is not directly applicable to wood-based materials because it does not model different strengths in tension and compression. The maximum stress criteria consider different strengths in tension and compression, but do not include the interaction between the strengths. The Tsai-Wu and Hoffman interactive strength criteria predict when a given set of stresses will produce failure, but they do not distinguish the failure modes. The Norris approach can predict failure in orthogonal planes rather than orthogonal axes. Both Hashin and extended Yamada-Sun criteria are capable of predicting the failure modes caused by tension or compression in different axes. Compared to these criteria, the Tsai-Wu, Hoffman, Norris, Hashin, and extended Yamada-sum criteria are more suitable for wood-based material. Although the maximum stress, Coulomb-Mohr, von Mises, Tresca, and Hill criteria were originally developed for materials which behave very differently to wood-based material, these criteria could be the only options in some programs. If that is the case, they can still be utilised for wood-based material if using engineering judgments and necessary assumptions.

2.3 Post-strength behaviour of quasi-brittle failure

Tension and shear stresses beyond the elastic limit generate voids and microcracks in the wood matrix which gradually degrade its mechanical properties, including its stiffness. When further loads are applied, the microcracks grow and coalesce, producing macrocrack zones and irreversible damage which eventually leads to failure. As the damage and deformation increase, the material resistance to stress gradually decreases, that is, it softens (Figure 2). This is called quasi-brittle failure.

Continuum damage mechanics have been widely used for modelling the nonlinear behaviour of brittle materials such as concrete, rock, and more recently, timber [2, 5]. Using continuum damage mechanics to model the degradation of properties, a scalar damage parameter, d_i (where $0 \leq d_i \leq 1$), transforms the stress tensor associated with the undamaged state, $\bar{\sigma}_{ij}$, into the stress tensor associated with the damaged state, σ_{ij}:

$$\sigma_{ij} = (1 - d_i)\bar{\sigma}_{ij} = D_d \varepsilon_{ij} \tag{4}$$

where $\boldsymbol{D_d}$ is a reduced stiffness matrix with damage parameters. The damage parameter ranges from zero for no damage to approaching unity for maximum damage. Thus, '$1 - d_i$' is a reduction factor associated with the amount of damage. Figure 3 shows how the damage parameter affects the stress-strain curve.

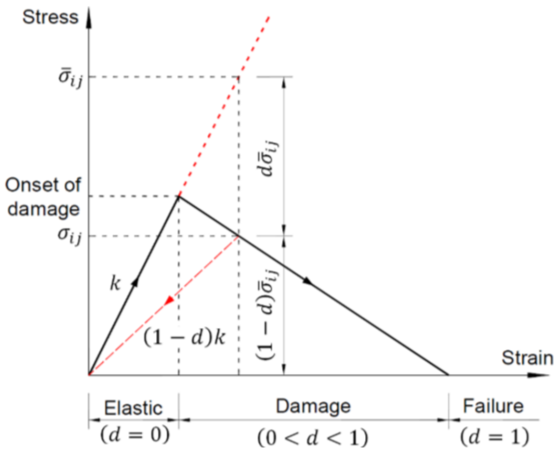

Figure 3: Schematic of continuum damage mechanics

Damage formulations are typically based on strain, stress, or energy. Chen et al. [2] proposed an exponential damage evolution based on the accumulating effect of strains:

$$d_i = 1 - exp\left[\frac{-(\tau_i - \tau_{i,o})}{\tau_{i,o}}\right] \tag{5}$$

where $\tau_{i,o}$ and τ_i are the undamaged elastic strain-energy norms at the time when stresses meet the strength criterion in the i direction and beyond, respectively. The strain-based theory can be used to calculate the undamaged elastic strain energy based on the total strains and the undamaged moduli of elasticity. Assuming that the damage parameters in the three major directions are independent of each other, the undamaged elastic strain-energy norms, τ_i, can be calculated using:

$$\tau_i = \sqrt{\sigma_i^* \varepsilon_i + 2(\sigma_{ij}^* \varepsilon_{ij} + \sigma_{ki}^* \varepsilon_{ki})} \tag{6}$$

where σ_{ij}^* is undamaged stress.

Another issue is strength coupling, in which degradation in one direction affects degradation in another. If failure occurs in the longitudinal modes, all six stress components degrade uniformly. This is because longitudinal failure is catastrophic and renders the wood useless. The wood is not expected to carry load in either the longitudinal or transverse (radial and tangential) direction once the wood fibres are broken. If either the radial or tangential mode fail, only the transverse stress components degrade. This is because neither radial nor tangential failure are catastrophic, and the wood is likely to be able to continue to carry the load in the longitudinal direction.

Based on these assumptions, the reduced stiffness matrix, $\boldsymbol{D_d}$, can be expressed as follows:

$$D_d = \begin{bmatrix} \alpha\frac{E_L(1-\nu_{RT}\nu_{TR})}{\Upsilon} & \beta\frac{E_L(\nu_{RL}+\nu_{TL}\nu_{RT})}{\Upsilon} & \gamma\frac{E_L(\nu_{TL}+\nu_{RL}\nu_{TR})}{\Upsilon} & 0 & 0 & 0 \\ \beta\frac{E_L(\nu_{RL}+\nu_{TL}\nu_{RT})}{\Upsilon} & \beta\frac{E_R(1-\nu_{LT}\nu_{TL})}{\Upsilon} & \eta\frac{E_R(\nu_{TR}+\nu_{LR}\nu_{TL})}{\Upsilon} & 0 & 0 & 0 \\ \gamma\frac{E_L(\nu_{TL}+\nu_{RL}\nu_{TR})}{\Upsilon} & \eta\frac{E_R(\nu_{TR}+\nu_{LR}\nu_{TL})}{\Upsilon} & \gamma\frac{E_T(1-\nu_{LR}\nu_{RL})}{\Upsilon} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\beta G_{LR} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\eta G_{RT} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\gamma G_{LT} \end{bmatrix} \quad (7)$$

where $\alpha = 1 - d_L$, $\beta = 1 - d_R$, $\gamma = 1 - d_T$, $\eta = 1 - max(d_R, d_T)$, $d_L = d(\tau_L)$, $d_R = max[d(\tau_L), d(\tau_R)]$, and $d_T = max[d(\tau_L), d(\tau_T)]$. These factors consider the coupling effect of damage. Once the scalar damage parameter, d_i, reaches its maximum value, the corresponding element can be removed using the element deletion techniques available in advanced modelling tools or software.

2.4 Post-strength behaviour of ductile failure

Ductile yielding of wood under either parallel-to-grain or perpendicular-to-grain compression occurs once the strength criterion is satisfied, based on the plastic flow and hardening evolution rule. From a physical point of view, plasticity in compression occurs in the material matrix, between voids and cracks, leading to local hardening behaviour. Softening, similar to that described in Section 2.3, is triggered when the compression strain parallel to grain reaches a specific criterion, ε_{Lo}. A second hardening happens once the compression strain perpendicular to grain reaches a specific criterion, ε_{Ro} or ε_{To} (see Figure 2).

The plasticity algorithms limit the stress components once the strength criterion is satisfied. This is achieved by bringing the stress state back to the yield surface. A typical approach for modelling plasticity is to partition the stress and strain tensors into elastic and plastic parts:

$$\Delta\varepsilon_{ij} = \Delta\varepsilon_{ij}^e + \Delta\varepsilon_{ij}^p = \Delta\varepsilon_{ij}^e + \Delta\lambda\frac{\partial g}{\partial\sigma_{ij}} \quad (8)$$

where $\Delta\varepsilon_{ij}$, $\Delta\varepsilon_{ij}^e$, and $\Delta\varepsilon_{ij}^p$ are total, elastic, and plastic strain increments, respectively; $\Delta\lambda$ is the plastic flow parameter increment; and g is a plastic potential function. Partitioning is conducted with return mapping algorithms that enforce the plastic consistency conditions. Such algorithms allow for the control of plastic strain generation. In addition, return mapping algorithms with associated flow satisfy the second law of thermodynamics. Radial-return algorithm is typically used, and the plastic potential function, g, can be simplified to yield or strength function. The plastic flow parameter increment, which depends on the yield function, can be derived by solving the plastic consistency condition. Where extended Yamada-Sun criteria are adopted as the yield function [2], the yield function and the corresponding increment of the plastic flow parameter $\Delta\lambda$ can be expressed as follows:

$$f_i(\sigma_i, \sigma_{ij}, \sigma_{ki}) = \frac{\sigma_i^2}{\sigma_{io,C}^2} + \frac{\sigma_{ij}^2}{\sigma_{ijo}^2} + \frac{\sigma_{ki}^2}{\sigma_{kio}^2} - 1 \tag{9}$$

$$\Delta\lambda_i = \frac{f_i^*/4}{\frac{E_i(1-\nu_{jk}\nu_{kj})}{\Upsilon}\frac{\sigma_i^2}{\sigma_{io,C}^4} + 4G_{ij}\frac{\sigma_{ij}^2}{\sigma_{ijo}^4} + 4G_{ik}\frac{\sigma_{ij}^2}{\sigma_{kio}^4}} \tag{10}$$

where f_i^* is the value of the failure criterion calculated from the trail elastic stresses.

Wood exhibits pre-peak nonlinearity in parallel-to-grain and perpendicular-to-grain compression. A translating yield-surface approach that simulates a gradual change in modulus of elasticity is typically adopted to describe the post-yield portion of the stress-strain curve. The approach is to define initial yield surfaces that harden (translate) until they coincide with the ultimate yield surfaces, as demonstrated in Figure 4 for the longitudinal hardening. The location of the initial yield surface determines the onset of plasticity. The rate of translation determines the extent of nonlinearity.

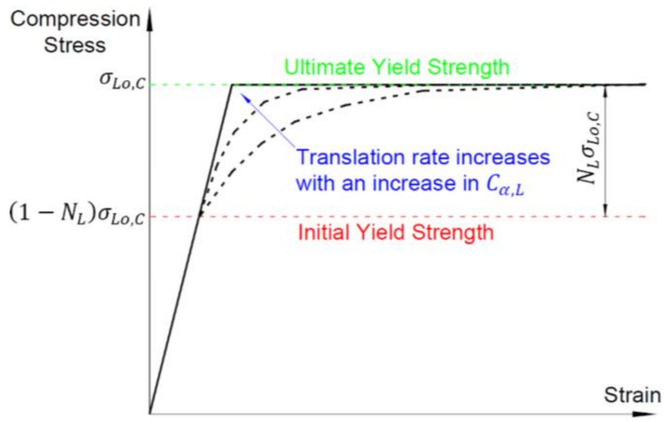

Figure 4: Hardening in longitudinal direction

The state variable that defines the translation of the yield surface is known as the backstress. The value of the backstress is zero upon initial yield and is the total translation of the yield surface in stress space at ultimate yield (in uniaxial compression). Either isotropic or kinematic hardening rule can be adopted to define the growth of the backstress based on the stress and plastic strain. This is accomplished by defining the incremental backstress.

When extended Yamada-Sun criteria are adopted as the strength function, the ultimate yield surface is described in Equation 9 and the initial yield surface is described as:

$$f_i(\sigma_i, \sigma_{ij}, \sigma_{ki}) = \frac{\sigma_i^2}{\sigma_{io,C}^2(1-N_i)^2} + \frac{\sigma_{ij}^2}{\sigma_{ijo}^2} + \frac{\sigma_{ki}^2}{\sigma_{kio}^2} - 1 \tag{11}$$

where N_i is a parameter to determine the initial yield surface. The corresponding incremental backstress, $\Delta\alpha_i$, can be expressed as follows:

$$\Delta\alpha_i = C_{\alpha,i} G_{\alpha,i} (\sigma_i - \alpha_i) \Delta\varepsilon_i^{eff} \tag{12}$$

where $\Delta\alpha_i$ and $\Delta\varepsilon_i^{eff}$ are the increment of backstress and effective strain, respectively, in the i direction, and $C_{\alpha,i}$ and $G_{\alpha,i}$ are hardening parameters in the i direction. The parameter $C_{\alpha,i}$ determines the rate of hardening and must be calibrated from test data. The parameter $G_{\alpha,i}$ restricts the motion of the yield surface so that it cannot translate outside the ultimate surface.

2.5 Densification perpendicular to grain

Unlike for parallel-to-grain compression, where the stress drops (strain softening), as illustrated in Figure 2, the stress increases sharply with strain in wood under perpendicular-to-grain compression beyond the plastic region. This rapid increase in stress is due to the elimination of air voids and compression of the solid wood structure; hence, this region is termed the densification region. Accordingly, when the perpendicular-to-grain compression strain reaches the criterion ε_{Ro} or ε_{To}, a second hardening occurs, as shown in Figure 5. Like the hardening discussed in Section 2.4, the translating yield-surface approach can also be used for second hardening, in which the ultimate yield surface will harden (translate) to the final ultimate yield surface.

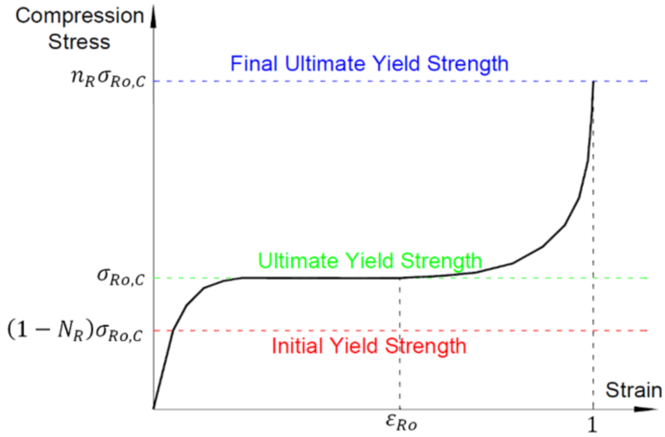

Figure 5: Hardening in perpendicular direction

For example, when Chen et al. [2] adopted extended Yamada-Sun criteria as the strength function, the ultimate yield surface (Equation 9) hardens (translates) to final ultimate yield surface, as described as:

$$f_{i,fn}(\sigma_i, \sigma_{ij}, \sigma_{ki}) = \frac{\sigma_i^2}{n_i^2 \sigma_{io,C}^2} + \frac{\sigma_{ij}^2}{\sigma_{ijo}^2} + \frac{\sigma_{ki}^2}{\sigma_{kio}^2} - 1 \tag{13}$$

where n_i is a parameter to determine the final ultimate yield surface. A strain-based hardening evolution is developed to define the second backstress, β_i:

$$\beta_i = -(n_i - 1)\sigma_{io,C}\sqrt{1-\left(\frac{\sigma_{ij}^2}{\sigma_{ijo}^2}+\frac{\sigma_{ki}^2}{\sigma_{kio}^2}\right)}\left(\frac{\varepsilon_i^{eff}+\varepsilon_{io}}{1+\varepsilon_{io}}\right)^2 \tag{14}$$

2.6 Typical constitutive models

The constitutive models incorporated in existing finite element software packages are often limited, making the general finite element software unsuitable for accurately predicting the mechanical behaviour and failure modes of wood-based materials. Some researchers have developed specific constitutive models for wood-based members, e.g., Chen et al. [2, 3], Danielsson & Gustafsson [6], Khennane et al. [7], Sandhaas et al. [5], Schmidt & Kaliske [8], and Zhu et al., [9].

Wood^{S} model, developed by Chen et al. [2], is a structural orthotropic elastoplastic-damage constitutive model for wood. The model incorporates the effects of orthotropic elasticity and linear softening (damage), anisotropic plasticity with kinematic hardening, large plastic deformations, and densification. The constitutive model considers eight types of brittle and ductile failure modes, each of which is associated with a different failure criterion. Wood^{S} is one of the first constitutive models capable of simulating the complete stress-strain behaviour and various failure modes of wood-based members under different loading conditions, thus providing an important approach for the numerical modelling of wood. A similar constitutive model was developed by Sandhaas et al. [5]. The Wood^{S} model has been recently upgraded to WoodST [3] for simulating the structural response of wood-based members and connections exposed to fire.

As tall or large timber structures are becoming a viable option in the construction industry, the structural elements and connections are becoming more complex, and the corresponding design is beyond the compatibility of general design software packages. Designers can still carry out the design using any tools by making more assumptions. The design as well as the assumptions must be verified by testing, numerical simulation, or both. In such a scenario, general purpose finite element software with a comprehensive constitutive model of wood-based material is the first choice for the simulation. A comprehensive constitutive model can predict potential failure modes, including those that may be overlooked in design, providing more reliable analysis results to support the design. When the chosen software does not have comprehensive constitutive models, engineers can select available constitutive models, whichever model is most suitable for the specific design case based on suitable assumptions. Selecting constitutive models depends on the modelling scope and objectives. With the appropriate constitutive model, the strength, stability, and deflection problems of wood-based members can be investigated and evaluated. The outputs, however, need to be interpreted more carefully using engineering judgment.

3 KEY INFLUENCING FACTORS

Growth characteristics such as slope of grain and knots significantly affect the mechanical behaviour of wood-based products. Moreover, the mechanical properties of wood change as temperature, moisture, and loading time change.

3.1 Growth characteristics

Wood is a non-homogeneous material, containing slope of grain, knots, etc. These growth characteristics, which serve the needs of the tree, usually significantly reduce the strength of cut timber, and it is used for other purposes. The slope of grain can be simulated by adjusting the local coordinate system of material to follow the wood grain. Usually, a rectangular Cartesian axis system is sufficient. A cylindrical axis system can be adopted to consider the location of pith. Two typical approaches consider the effect of knots and other local growth characteristics: an efficient approach is to use test results as material input; these can be either be the results of full-scale tests conducted using representative materials or the results of converting the design values. A more sophisticated approach is to build the growth characteristics into the models using a stochastic simulation method.

3.2 Temperature and fire

In general, the moduli of elasticity, shear moduli, and strengths of wood decrease when heated or increase when cooled. The change in properties that occurs when wood is quickly heated or cooled is termed an 'immediate' effect. At temperatures below 100 °C, the immediate effect is essentially reversible, meaning the property will return to the state at the original temperature if the temperature change is rapid. At elevated temperatures, the effect is irreversible. This permanent effect is caused by degradation of the wood, resulting in loss of weight and strength. The effects in most fire events are irreversible. Above 300 °C, wood is fully converted into char and has no strength or stiffness. Deriving realistic temperature-dependent mechanical properties requires considering complicated algorithms, such as thermal transport by mass flow (e.g., moisture/air movement), the constantly changing geometry, and the formation of cracks in charcoal by thermal stresses. The complexity of these problems leads to a huge input effort, coupled simulations, and lengthy calculations. Simplified relationships between the mechanical properties and the temperature are conventionally adopted to implicitly account for the complex physical and chemical phenomena. The local values of mechanical properties for wood-based members should be multiplied by a temperature-dependent reduction factor. A multilinear reduction model is usually used to describe the effect of temperature on the modulus of elasticity, shear modulus, and strength of wood, e.g., WoodST [3].

3.3 Moisture content

Wood can be characterised as a natural, cellular, polymer-based, hygrothermal viscoelastic material. The effect of moisture content on the mechanical behaviour of timber structures is seen as swelling and shrinkage. Swelling and shrinkage create internal stresses that can lead to shape distortion. The moisture content also affects the mechanical and rheological properties in which the influence of moisture prevails. With increasing moisture content, the stiffness and strength properties first increase slightly and then decrease until fibre saturation is reached. It is generally assumed that the mechanical properties do not change above the saturation point. In normal service environments (e.g., temperature below 50 °C), the effect of humidity on the mechanical properties of timber structures is much more significant than temperature. Like the effect of temperature, the influence of moisture content, for example, reduction factors, can be represented using a quadratic curve or a multi-segment straight line derived by fitting the test results. The timber structures under changing moisture content conditions may be simply simulated by conducting thermal-moisture analogy analysis. The moisture content [%] and the shrinkage coefficient [% length/% moisture content] can be analogised as temperature [°C] and thermal expansion coefficient [% length/°C].

3.4 Creep

Wood is a rheological material and deforms depending upon the loading history and the elapsed time. The creep of wood is a highly nonlinear. In addition to the stress level, types and directions of loading significantly influence creep. In broad terms, the rheological behaviour is also a function of the thermal and moisture histories and of their interaction with the loading history. The dependency of viscoelastic creep behaviour on temperature is often considered by applying a time-temperature-equivalence hypothesis. A change in Young's modulus with respect to temperature is equivalent to a shift in time on the logarithmic timescale. However, the largest impact on the process of creep is caused by changing moisture content, the so-called mechanosorptive effect. This effect is often assumed to be time-independent since the influence of time is much smaller. Models developed to describe the creep process can be categorised into three groups: structural models, mechanical (rheological) models, and purely mathematical models. Rheological models, providing a simpler physical interpretation than other models, are commonly used.

3.5 Duration of load

Strength of wood under constant loading decreases with time. This phenomenon is known as duration of load (DOL). The DOL feature distinguishes timber engineering from other commonly used materials (e.g., steel), whose strength is little affected by the load history. DOL also depends on the current climate and climate history. The combined action of load and change in moisture content further reduce load-carrying capacity. Six types of models have been developed to predict the DOL effect of an arbitrary load on the strength of a structural wood member. These models can be incorporated into the material constitutive models. Alternatively, the strengths are scaled up or down using one of the above-mentioned models before inputting them into the material models.

4 SUMMARY

Appropriate material models are the fundamental basis of reliable simulations, especially for timber structures. In this paper, sub-models for describing the elastic properties, strength criterion, post-peak softening for quasi-brittle failure modes, plastic flow and hardening rule for yielding failure modes, and densification perpendicular to grain were discussed. Depending on the modelling complexities, scenarios, and demands, however, different constitutive models with various combinations of the sub-models can be adopted. Typical constitutive models developed for wood-based materials were introduced. The key influencing factors, including growth characteristics, temperature and fire, moisture content, creep, and DOL were discussed along with corresponding modelling recommendations. The information presented in this paper is intended to help engineers and researchers become more acquainted with the constitutive models of timber structures.

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