

SUSTAINABLE MANUFACTURING VIA ROBUST OPTIMIZATION AND TAILORED SCATTER

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Summary. Manufacturing industries contribute to 16.7% of global CO₂ emissions. Any waste generated during manufacturing processes such as defective or rejected products due to poor tolerances or quality issues, results in excessive energy consumption and unnecessary carbon emission. Most manufacturing waste and quality fluctuations arise from uncertainty and variation in raw materials and process conditions. Moreover, manufacturing with recycled materials introduces challenges related to quality control. For this purpose, uncertainty quantification via surrogate models and robust optimization are among the promising methods to reduce waste during manufacturing processes. Inverse robust optimization, also known as tailored scatter, tailored variation, or tailored uncertainty is a recently-introduced method that explores designing uncertainty for a given robust performance. These inverse problems face computational complexities due to process non-linearities, correlations and problem dimensionality. In this work, an efficient implementation of robust optimization using Robustimizer software is applied and its potential for tailored variation is presented. An additive manufacturing process, laser powder bed fusion, is used to demonstrate the potential of presented methods in increasing efficiency, and reducing environmental footprints in a more accurate and computationally efficient way. Robust process settings are achieved leading to minimal variation in melt pool size in the presence of uncertainties of material and process. In addition, the tailored scatter approach is implemented to provide a methodology for tighter control of noise variables in the laser powder bed fusion process considering uncertainties.

1 INTRODUCTION

Manufacturing industries are responsible for 16.7% of global CO₂ emissions [1]. The 16.7% figure includes contributions from components which are defective, have poor tolerances, or do not meet the performance criteria due to quality issues. This excessive energy consumption and unnecessary carbon emissions, which have its root in variations of raw material and process conditions can be significantly reduced by robust optimization methods. [2, 3]

Most manufacturing processes are single-stage operation meaning that the transformation of raw material to the final product occurs in a single step. The output of the process depends on many variables some of which are easy to control while others are difficult, expensive or energy-consuming to control. The later are called noise variables in robust optimization and can be

described using a probability distribution. Robust optimization aims to find the optimal values for controllable variables (Design variables) while considering the statistics of noise variables to achieve the required performance and has been applied in many disciplines [4, 5, 6]

Additive manufacturing is rapidly evolving, enabling the production of highly intricate and functional parts. It significantly reduces material waste compared to traditional subtractive methods and reduces the lead times for low-batch production. Various methods of additive manufacturing encounter specific challenges, particularly when it comes to predicting part performance. Changes in material properties or process conditions can have a significant impact, making it difficult to ensure consistent outcomes.

In this study, an additive manufacturing process, laser powder bed fusion (LPBF), is used to demonstrate how robust optimization and tailored scatter can contribute to sustainable process optimization, minimizing the impacts on unwanted input. Section 2 introduces efficient formulations of robust optimization using Robustimizer and the concept of tailored scatter. Section 3 introduces the additive manufacturing model used in this work. The results are presented and discussed in Section 4 .

2 Robust optimization and tailored scatter

While numerical simulations and analytical models provide insight about LPBF process, the major challenge that is still open is how to obtain the model parameters. Statistical methods provide a reasonable approach by assigning distributions to uncertain variables instead of assuming fixed values. This approach is also practical because, in reality, measuring the exact value of a parameter is tedious and susceptible to measurement and environmental errors.

A surrogate model-based robust optimization procedure consists of several building blocks. First a design of experiments (DOE) is generated in the combined design and noise space. Then the responses of the process model are evaluated for the discrete DOE points. In the next step, a surrogate model is constructed, which is an approximate representation of the process. The search for the robust optimum design parameter consists of uncertainty quantification and the repetitive evaluation of objective function value and constraints, which subsequently leads to the optimal design variables. Since the reliance on a surrogate model might lead to loss of accuracy, iterative improvement of the surrogate model is usually applied [7].

In the context of surrogate modeling, exploration and exploitation are two key concepts that relate to how the surrogate model is utilized. Exploration refers to the process of gathering information about the input output relationship across the entire input space. Exploitation, on the other hand, involves utilizing the surrogate model to make predictions or decisions that maximize the desired outcome within the known regions of interest. Striking a balance between exploration and exploitation is crucial in surrogate modeling to ensure that the model is both accurate and efficient in finding the robust optimum. New DOE points are added to the initial DOE to improve the surrogate model of the process. A new infill point is selected in the combined design and noise space and added to the initial DOE. This procedure can be repeated until the updated surrogate model does not lead to further improvement of the predicted optimum design variables. For each building block of robust optimization, various methods exist and a variety of functions and toolboxes can be combined to perform robust optimization [8, 9]. In this study, Robustimizer software is used offering various possibilities for all these steps in one easy-to-use graphical interface.

Table 1: Model parameters for Ti-6Al-4V

Name	Symbol	Value
Laser absorptivity (-)	A	0.36
Thermal conductivity (W/(mK))	k	7.4
Boiling temperature (K)	T_b	3560

Table 2: Ranges of design variables

Name	Symbol	Upper bound	Lower bound
Scanning speed (m/s)	v	0.5	1.5
Laser power (w)	P	100	200

3 Additive manufacturing via laser powder bed fusion

LPBF is one of the promising additive manufacturing methods in which the powder is selectively melted layer-by-layer using a laser beam. Manufacturing challenges arise while producing the components. Specifically in LPBF, various types of defects can be introduced such as balling, keyhole-induced pores, lack of fusion, and cracks if the processing parameters are not optimized. Many recent studies have focused on modeling and simulation of LPBF using different levels of complexity [10]. Most studies focus on predicting the melt pool size which is an important metric for print quality and defect formation [11, 12]. Existing models and simulations include many parameters which might not be easy to determine. Therefore, robust optimization opens a new horizon in investigating these processes with a statistical approach.

In this work an analytical model of melt pool depth is chosen to demonstrate how uncertainties in process modeling can be taken into account during process optimization. The keyhole mode melting [13, 14] is selected in which instabilities can cause keyhole-induced pore formation among other issues. In this case, the melt pool depth can be described by:

$$D = \frac{AP}{2\pi k T_b} \ln \frac{a + \alpha/v}{a} \quad (1)$$

where A is laser absorptivity (-), P is laser power (W), k is the thermal conductivity (W/(m.K)), T_b is the boiling temperature (K), a is laser beam size (m), α is the thermal diffusivity (m²/s), and v is the scanning speed (m/s).

The model parameters are grouped into three categories. Some model parameters are assumed to be fixed. The nominal values for these parameters are listed in table 1. Laser power and scanning speed are chosen to be the design variables which can be adjusted to achieve the desired performance within their lower and upper bounds (Table 2). Moreover, thermal diffusivity and laser beam size are assumed to be noise variables meaning that their exact values are unknown. These can be assumed as probabilistic inputs to the model with given means and standard deviations as shown in Table 3. Laser beam size can be considered a source of process variability, influenced by factors such as optical alignment, thermal effects, lens quality, and environmental conditions. In addition, thermal diffusivity can be considered as a source of material uncertainty due to inconsistencies in material composition, and complexity of thermal response of the material under process. The aim is to identify the design variables necessary to achieve a consistent melt pool depth, considering uncertainties.

Table 3: Statistics of noise variables

Name	Symbol	Mean	Standard deviation
Thermal diffusivity (m ² /s)	α	8e-6	0.5e-6
Laser beam size (m)	a	40e-6	0.5e-6

4 Results and discussion

To achieve a constant melt pool depth in the presence of noise, the following objective is implemented in the robust optimization framework:

$$\underset{\mathbf{x}}{\text{minimize}} \quad (\mu_{\text{D}}(\mathbf{x}) - D_t)^2 + 3(\sigma_{\text{D}}(\mathbf{x}))^2 \quad (2)$$

where μ_{D} defines the mean depth of the melt pool, σ_{D} is the standard deviation, and D_t is the target depth which is set to $100\mu\text{m}$ in this work. An initial DOE of 20 points in a 4D space of design-noise variables is created via combination of full factorial design and Latin hypercube and the results are evaluated using Equation (1). A surrogate model based on Gaussian processes which is an approximate of this function is created in Robustimizer and is used to search for robust optimum. This approach is used to compare the accuracy of the results and it must be noted that in general the relationship between input and output is not explicitly known. A modified infill criterion based on [7] is used to search for the robust optimum by adding new points to the initial DOE. Figure 1 represents the initial DOE with unfilled circles and the added points in the combined design-noise space with filled circles. The darker and larger the filled circle, the later it was added to the DOE. 15 points are added sequentially until the expected improvement value converged to a small number (< 0.001). Figure 2 shows the search for the robust optimum via exploration and exploitation. The darker and larger the filled circle, the later the optimum was found in the search process. After adding 15 points adaptively, the optimum was found at $x = [121.81, 0.50]$.

The ground truth objective function in the design space using Equation (1) is represented by contour lines in Figure 2. The reference robust optimization solution is at $x = [122.77, 0.50]$, which means the surrogate-based approach could predict the optimum with only 0.8% deviation.

To achieve a target melt pool depth without considering uncertainties, process optimization in a deterministic manner leads to $x = [198.68, 0.86]$. This highlights the importance of considering noise in the optimization process, as ignoring it can lead to suboptimal settings. In contrast, a robust approach can lead to significantly different process settings. It is interesting to see that higher stability against noise is achieved with less laser power and scanning speed.

Inverse robust optimization also known as tailored scatter, is a recently introduced method [15] that explores the possibilities of designing uncertainty for a given robust performance. This approach identifies the largest possible noise variations that still ensure a tightened robust performance. The formulation applied in this work is as follows:

$$\begin{aligned} \underset{\mathbf{x}, \sigma_z}{\text{min}} \quad & \frac{1}{\sum_i w_i \bar{\sigma}_i} \\ \text{s.t.} \quad & \frac{(\mu_{\text{D}}(\mathbf{x}, \sigma_z) - D_t)^2 + 3(\sigma_{\text{D}}(\mathbf{x}, \sigma_z))^2}{(\mu_{\text{D}}(\mathbf{x}, \sigma_n) - D_t)^2 + 3(\sigma_{\text{D}}(\mathbf{x}, \sigma_n))^2} - C_{\text{tol}} \leq 0 \end{aligned} \quad (3)$$

where w_i are the weights corresponding to the cost of tightly controlling the noise variable i , $\bar{\sigma}_i = \frac{\sigma_{z_i}}{\sigma_{n_i}}$, σ_{z_i} is the optimization variable representing the standard deviation for noise variable

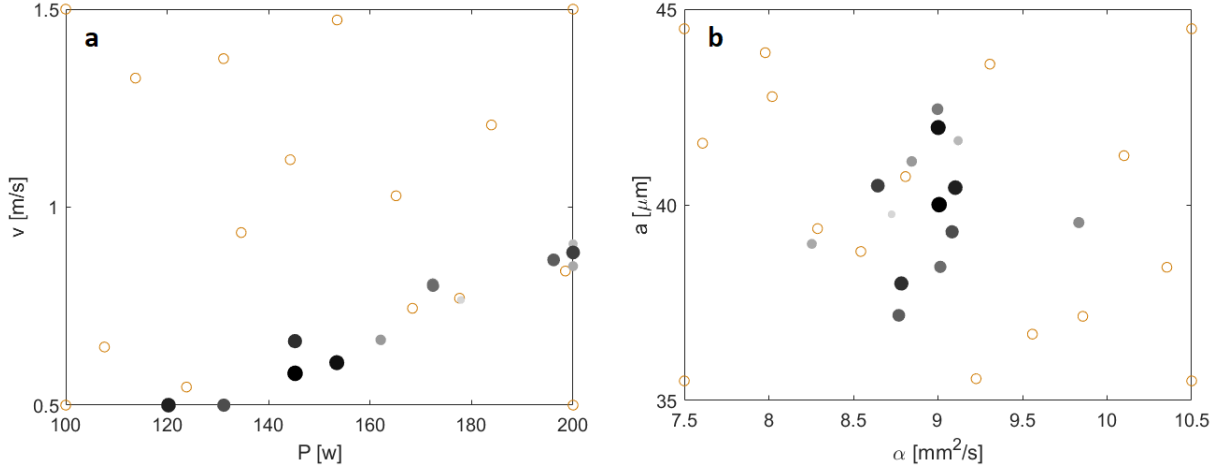


Figure 1: Initial DOE shown with unfilled circles and the additional points added through exploration and exploitation shown with filled circles in the (a) design space and (b) noise space. (The darker and larger the filled circle, the later it was added to the DOE)

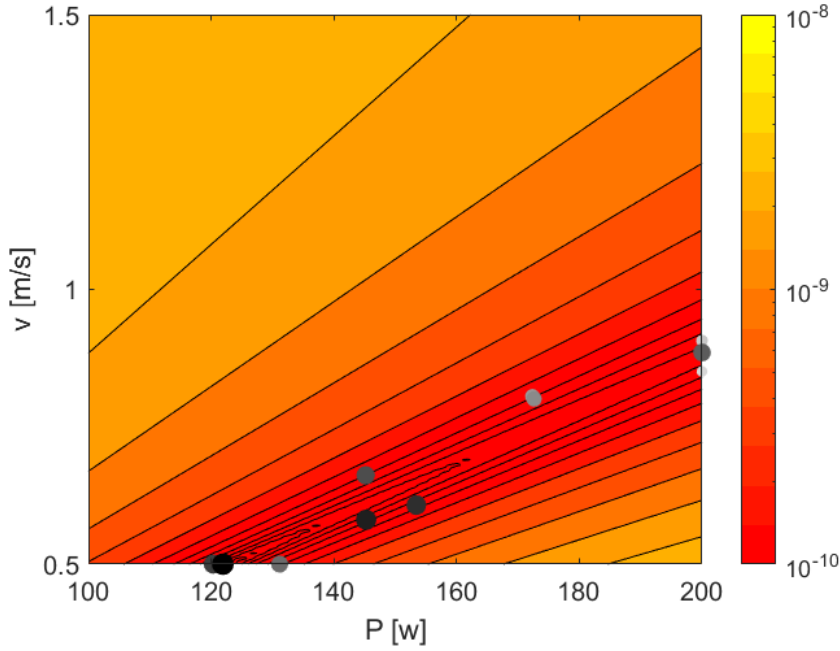


Figure 2: Search for the robust optimum via exploration and exploitation. (The darker and larger the filled circle, the later the optimum was found in the search process)

i , σ_{n_i} is the nominal standard deviation for noise variable i , and C_{tol} represents the tightening ratio of the objective value to the nominal objective value.

Figure 3 shows the tolerable variations for two C_{tol} values with various control cost weights. This figure indicates how to achieve a tighter tolerance on the melt-pool depth by tightening the control on two noise variables. In general, variations in laser beam size have a smaller

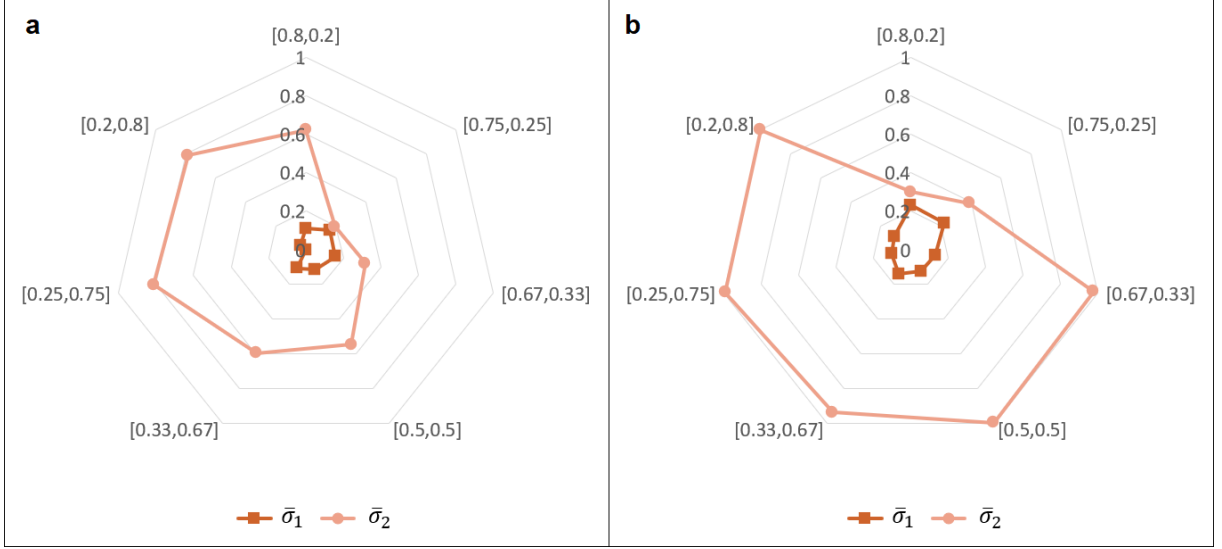


Figure 3: Tolerable scatter for (a) $C_{\text{tol}} = 0.25$ and (b) $C_{\text{tol}} = 0.5$ for various control cost weights

influence on overall performance. However, if the cost of controlling the thermal diffusivity is around three times that of the laser beam size, the variation in both noise variables needs to be controlled upto 20% of the nominal variation, to achieve a melt pool depth variation of 25% of the original depth variation. These results provide insight for decision-making in controlling the LPBF process from a statistical point of view.

5 CONCLUSIONS

This study explored the application of robust optimization to the laser powder bed fusion process, addressing key challenges related to manufacturing quality. A surrogate model-based robust optimization procedure was effectively applied to minimize variations and achieve a consistent melt pool depth, which is crucial for enhancing print quality and reducing defects. By employing the tailored scatter technique, the potential for controlled reduction of uncertainties and improvement of performance was demonstrated. The approach facilitated the identification of the largest possible noise variations that ensured required performance. This work contributed to a deeper understanding of how statistical approaches can be employed to achieve a more robust process for reducing manufacturing-related variations, paving the way for more efficient, reliable and sustainable production methods.

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