ANALYTICAL INTERPRETATION OF HYDRODYNAMIC PRESSURE ON DAMS

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Abstract

Earthquakes pose a significant threat to dams and other reservoir structures. Beyond the ground shaking itself, the dynamic interaction between the earthquake ground motion and the water in the reservoir creates a phenomenon known as hydrodynamic pressure. This pressure can significantly exceed the static hydrostatic pressure that the dam is typically designed to withstand. Understanding and accurately predicting hydrodynamic pressure is crucial for ensuring the safety and stability of dams during seismic events. This paper provides an excursus on the literature present on this topic and delves into the formulation included in the Ministerial Decree of 1982 developing in detailed calculation steps and thus determining in closed form the resultant of the pressures and its point of application.

Keywords: Eathquakes, Hydrodynamic Pressure, Dams

1. Introduction

Dams are critical infrastructure, providing vital services like water storage, irrigation, flood control, and hydropower generation. However, earthquakes can subject dams to immense forces, potentially leading to catastrophic failure. One critical factor in dam safety during earthquakes is the hydrodynamic pressure exerted by the reservoir water. This pressure may be high enough to harm a dam, depending on how strong the ground excitation is. A look at the disastrous effects of a dam failure. Developing a suitable analytical technique to investigate the impact of an earthquake on a dam-reservoir system is imperative during an earthquake.

The numerous techniques for estimating the hydrodynamic forces that may develop on the dam face during earthquakes have been critically examined in the case of barrages. Based on widely divergent theories, these approaches apply to dams with slope upstream faces as well as vertical ones (Westergaard [4], von Karman, Bakhmeteff, Zanger [6], Housner [6], Kulmaci [7][8], Chopra. Westergaard (1933) was the first to derive an expression for the hydrodynamic pressure that an incompressible fluid in an endlessly long reservoir would exert on a rigid dam with a vertical upstream face. In his work, the "added mass" argument was put forth while ignoring the impact of surface waves.

Since Westergaard's groundbreaking discovery, numerous studies have been carried out to examine the seismic response of rigid dam-reservoir systems for both compressible and incompressible water. The hydrodynamic reaction of an incompressible fluid during an earthquake is not the same as that of a compressible fluid, as Chopra (1967) showed. Using an electrical analogue, Zanger (1953) and Zanger and Haefeli (1952) conducted experiments to establish the hydrodynamic pressures for a dam whose upstream face is not vertical. They came to the conclusion that, for a dam with the upstream face vertical for more than half of the overall height, the hydrodynamic pressure would be nearly equal to that of a completely vertical dam.

2. Hydrodynamic Pressure vs. Hydrostatic Pressure

Under normal conditions, water in a reservoir exerts hydrostatic pressure, which increases linearly with depth. This pressure is relatively constant over time. In contrast, hydrodynamic pressure arises due to the dynamic interaction between the earthquake ground motion and the water. As the dam vibrates due to the earthquake, the water within the reservoir also experiences movement. This movement creates additional pressure that can fluctuate rapidly and significantly exceed the static hydrostatic pressure.

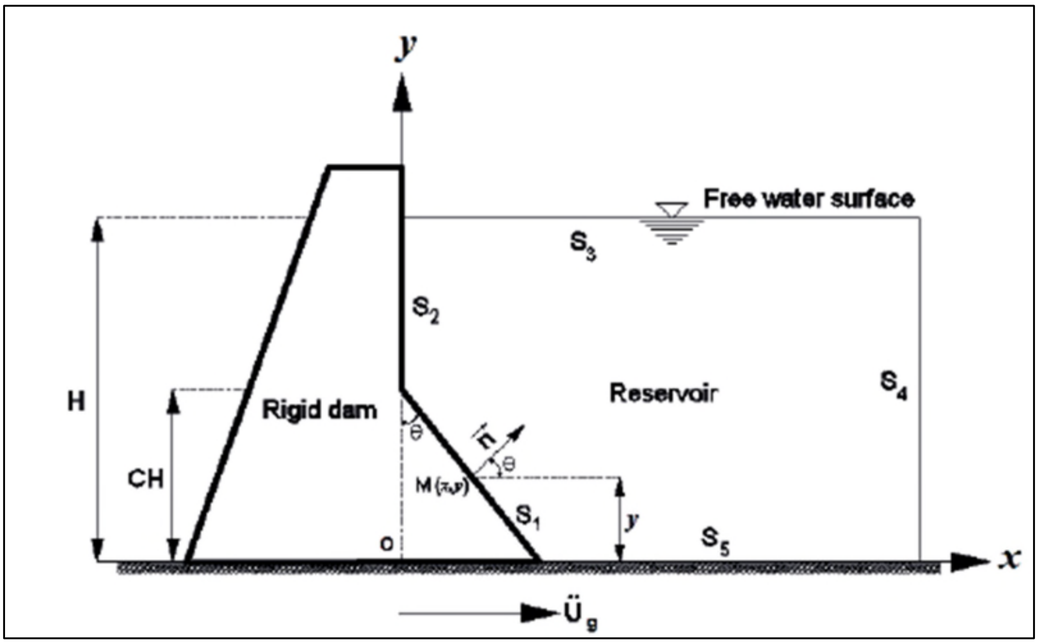

Figure 1 Rigid dam with partially inclined face and infinite length reservoir subjected to a horizontal ground motion [15]

2.1. Factors Affecting Hydrodynamic Pressure

The magnitude of hydrodynamic pressure depends on several factors, including:

- **Earthquake characteristics:** The intensity, duration, and frequency content of the earthquake ground motion play a crucial role. Stronger earthquakes and those with frequencies that resonate with the natural frequencies of the dam-reservoir system will induce higher hydrodynamic pressures.
- **Reservoir geometry:** The shape, depth, and bathymetry (underwater topography) of the reservoir influence the water's response to ground motion. Deeper reservoirs and steeper slopes can amplify hydrodynamic pressures.
- **Dam properties:** The dam's material properties, such as its stiffness and mass, affect its interaction with the reservoir water. More flexible dams are more likely to experience higher hydrodynamic pressures.

2.2. Consequences of Hydrodynamic Pressure

Excessive hydrodynamic pressure can have several detrimental effects on dams:

- **Increased structural loads:** The additional pressure acts on the dam face, adding significant forces that the dam needs to be designed to withstand.
- **Potential for cracking and damage:** High pressure can cause cracks and damage to the dam's concrete or embankment material, compromising its structural integrity.
- **Overtopping risk:** In extreme cases, hydrodynamic pressure can contribute to overtopping, where water rises above the dam's crest, potentially leading to catastrophic failure.

3. Engineering Analysis and Mitigation Strategies

To ensure dam safety during earthquakes, engineers employ sophisticated analytical methods to predict hydrodynamic pressure. These methods involve complex mathematical models that consider the dam-reservoir interaction and the dynamic behavior of the water under seismic loading.

Several mitigation strategies can be implemented to address hydrodynamic pressure:

- **Dam design considerations:** Dams can be designed to be more resistant to hydrodynamic pressure by incorporating features like thicker sections, buttress walls, and upstream slopes with a gentler inclination.
- **Upstream slope protection:** Protecting the dam's upstream slope with erosion-resistant materials can help dissipate the energy of water waves generated by the earthquake.
- **Early warning systems:** Real-time monitoring of dam behavior and reservoir water levels can provide valuable information for making informed decisions during an earthquake event.

4. Closed-Form Equations of Hydrodynamic Pressure of Resevoir Water According to Italian Ministerial Decree of 1982

In this paper, the impacts of boundary irregularity—which include compressibility, surface wave effects of the fluid, and dam flexibility—on seismic water pressures during earthquakes are investigated. A horizontal weight acceleration, ae, in either the x or y direction is responsible for the hydrodynamic pressure, which is the actual component of the complicated pressure.
This study operates under the following assumptions:

(l) Surface waves are present in the fluid, which is inviscid and compressible, and the flow is irrotational.
(2) Dams have vertical upstream faces.
(3) The reservoir's side boundary is stiff and vertical, but the reservoir bottom is flat.
(4) Density stratification is not present in the reservoir.
(5) The amplitude of the excitation is small.

According to the Italian Ministerial Decree of 1982 [1], the actions of water, the effects of which are to be added to those of inertia of the masonry mass, are assimilated to a continuous distribution of pressure normal to the upstream face of intensity:

$$p = C \gamma c y_o \quad (1)$$

where C is the coefficient of seismic intensity, g is the weight per unit volume of water, y_0 is the difference between the maximum height of the reservoir and the height of the most depressed point of the natural riverbed in correspondence with the upstream face of the structure, while c can be calculated using the function:

$$c = \frac{c_m}{2}\left[\frac{y}{y_0}\left(2-\frac{y}{y_0}\right)+\sqrt{\frac{y}{y_0}\left(2-\frac{y}{y_0}\right)}\right] \quad (2)$$

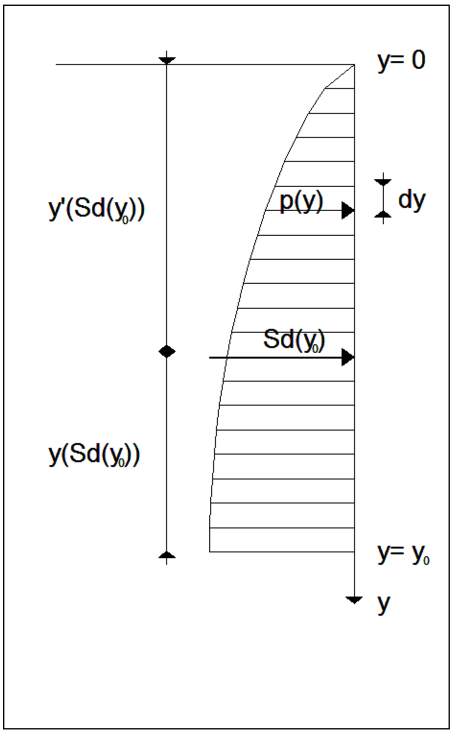

Figure 2 – Hydrodynamic pressure

where y is the difference between the maximum height of the reservoir and the height of the generic point of the face to which the pressure p indicated above is associated and c_m is a coefficient dependent on the angle of inclination of the face with respect to the vertical. Referring to Figure 2, the resultant of the pressures acting at different heights can be obtained both numerically (approximate solution) and in closed form (exact solution). To calculate this resultant in exact form means to add all the areas of prssure p_i dy between the dimension y = 0 and the generic dimension y, which consists of solving the integral (by substitution and twice by parts) :

$$S_d(y)=\int_0^y p(y)dy = \frac{c_m}{2}C\gamma\left[y^2-\frac{y^3}{3y_0}+\frac{y-y_0}{2}\sqrt{2yy_0-y^2}+y_0^2\cdot\arcsin\sqrt{\frac{y}{2y_0}}\right] \quad (3)$$

For $y=y_0$ we get the overall resultant of the pressures:

$$\boxed{S_d(y_0)=\int_0^{y_0} p(y)dy = \frac{c_m C\gamma}{24}(8+3\pi)\cdot y_0^2} \quad (4)$$

The point of application of the resultant Sd (y) with respect to the dimension y=0 can be evaluated as (5):

$$y'(S_d(y)) = \frac{\int_0^y y \cdot p(y)dy}{\int_0^y p(y)dy} = \frac{\frac{2}{3}y^3 - \frac{y^4}{4y_0} + \left(\frac{y^2}{3} - \frac{y_0 y}{6} - \frac{y_0^2}{2}\right)\sqrt{2yy_0 - y^2} + y_0^3 \cdot \arcsin\sqrt{\frac{y}{2y_0}}}{y^2 - \frac{y^3}{3y_0} + \frac{y - y_0}{2}\sqrt{2yy_0 - y^2} + y_0^2 \cdot \arcsin\sqrt{\frac{y}{2y_0}}}$$

The point of application of the resultant *Sd (y)* with respect to the dimension *y=y₀* is obtained by difference:

$$y(S_d(y)) = y_0 - y'(S_d(y)) \quad (6)$$

On the other hand, the point of application of the overall resultant *Sd (y₀)* , with respect to the dimension *y = 0* , is equal to:

$$y'(S_d(y_0)) = \frac{\int_0^{y_0} yp(y)dy}{\int_0^{y_0} p(y)dy} = \frac{1 + 3\pi}{8 + 3\pi} \cdot y_0 \quad (7)$$

In the same way as *Sd (y)* , the point of application of *Sd (y₀)* with respect to the dimension *y=y₀* is also obtained by difference:

$$\boxed{y(S_d(y_0)) = \frac{7}{8 + 3\pi} y_0 \cong 0{,}40 \cdot y_0} \quad (8)$$

5. Conclusion

Hydrodynamic pressure is a critical factor in dam safety during earthquakes. By understanding its causes, effects, and mitigation strategies, engineers can design and maintain dams that are more resilient to seismic events. Further research on advanced analytical methods and innovative dam design solutions is crucial for ensuring the safety and sustainability of dams in earthquake-prone regions.

There has been much research on the seismic behavior of dams, leading to the development of numerous analytical techniques. Regarding the seismic response of nonlinear elasto-plastic dams [and the dynamic properties of inhomogeneous visco-elastic dams significant advancements were achieved. Nevertheless, the majority of analyses simply take into account the hydrostatic portion of the reservoir pressures, ignoring the hydrodynamic pressures from the upstream reservoir. Westergaard was the first to study the hydrodynamic pressures on dams during earthquakes. He looked at a stiff dam with a vertical upstream face under harmonic stress. Liu suggested a sloped reservoir foundation, while Zangar and Chwang discussed dams with a sloped upstream face. Furthermore, Chopra examined the impacts of the compressibility of the reservoir water and demonstrated that the frequency of the applied loading determines the overall hydrodynamic force from a compressible reservoir on the dam.

Furthermore, a closed-form solution was created and the work of Chopra was expanded to examine the hydrodynamic pressures brought on by random acceleration loading.

6. References

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