Reliability-based assessment of deep cement mixing column based on core strength

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ABSTRACT

In the quality assurance procedure of the deep cement mixing method, the statistical parameters of the unconfined compressive strength of core samples, core strength, are adopted to assess the quality of the cement-treated soil ground. Since the statistical parameters of the core strength are the sample statistical parameters, the statistical uncertainty emerges when estimating the population parameters. Moreover, the spatial correlation of the core strength should be considered on the evaluation of the overall strength of cement-treated soil ground. The paper presents a reliability-based assessment for the deep cement mixing soil column based on the core strength. The analysis method in which the statistical uncertainty included in the core strength and the spatial variability of the strength are considered simultaneously is adopted to calculate the overall failure probability of the cement-treated soil column. The statistical uncertainty is estimated using a Bayesian inference method and the random fields of the strength are generated with the statistical parameters involving the statistical uncertainty. The random finite element method with the generated random fields is performed to simulate the compression failure behaviour of a cement-treated soil column. The analysis result provides the cumulative distribution function of the overall strength of the cement-treated soil column. The reliability-based assessment is performed on the basis of the cumulative distribution function of the overall strength.

Keywords: deep cement mixing; quality assurance; statistical uncertainty; spatial variability.

1. Introduction

Deep cement mixing is one of the major ground improvement methods and has been widely adopted in practical projects. The strength of the cement-treated ground constructed by deep cement mixing method varies spatially due to in-situ soil variability, mixing effectiveness of cement, and other factors. Therefore, the quality assurance of the cement-treated ground is very important in this method. On the normal quality assurance procedure, core samples are retrieved from cement-treated soil columns and unconfined compression tests of the core samples are conducted to obtain the core strength. Then the mean and standard deviation of the core strength are considered when assuring the quality of the cement-treated soil columns. In the current procedure, the influence of the strength variability on the quality of the cement-treated soil columns is taken into account by considering the standard deviation of the core strength. However, the influence of the spatial variability of the strength on the overall strength of cement-treated soil columns is not considered in the current procedure. Moreover, since the core strength data provides the sample mean and standard deviation of the strength, the statistical uncertainty emerges when evaluating the population mean and standard deviation. Thus the statistical uncertainty should be considered in the evaluation of the statistical parameters of the strength.

This paper presents a reliability-based assessment for the deep cement mixing columns based on the core strength. In the proposed method, the spatial variability and the statistical uncertainty are simultaneously considered when assuring the quality of the cement-treated ground. The strengths of core samples retrieved from a lattice-shaped ground improvement by deep cement mixing method were used. The improved ground was constructed at a site located at Kobe city in Japan (Namikawa et al. 2007). The random finite element method (RFEM) analysis is adopted to calculate the overall failure behaviour of a cement treated soil column with the spatial variability of the strength. RFEM analysis has widely adopted to analyse ground with spatial variabilities of soil properties (e.g., Griffiths and Fenton 2001, Fenton and Griffiths 2008). The statistical uncertainty is estimated using a Bayesian inference method. In the Bayesian inference method, a Markov chain Monte Carlo (MCMC) method is adapted to generate the realization values from the joint probability distribution of the statistical parameters. The Bayesian inference method with MCMC has used to evaluate the uncertainty of soil materials (e.g., Zhang et al. 2009, Wang and Aladejare 2016, Ching et al. 2016, Ching and Wang 2016). The generated realization values involving the statistical uncertainty are used when generating the random fields for RFEM. The compression failure behaviour of the cement-treated soil column is simulated in the RFEM analysis. In the present study, the statistical uncertainty and spatial variability are simultaneously considered in the evaluation of the overall failure probability of the cement-treated soil column. The analysis result provides the cumulative distribution function of the overall strength of the column. The reliability-based assessment is performed on the basis of the cumulative distribution function of the overall strength.

2. Analysis method

Namikawa (2021, 2022) has proposed the analysis framework of RFEM with the statistical uncertainty. The analysis framework is illustrated in Figure 1. First, the realizations of the population mean μ_{quf}, variance σ_{quf}^2, and the autocorrelation distance θ_{quf}, of the strength are estimated using the Bayesian inference method. Second, the random fields of the strength are generated with the realizations of μ_{quf}, σ_{quf}^2, and θ_{quf}. Thus these parameters values vary with each random field. Finally, the RFEM analysis is performed for the random fields of the strength. Using this analysis framework, the statistical uncertainty and spatial variability can be simultaneously considered in the evaluation of the overall failure probability of the cement-treated soil column.

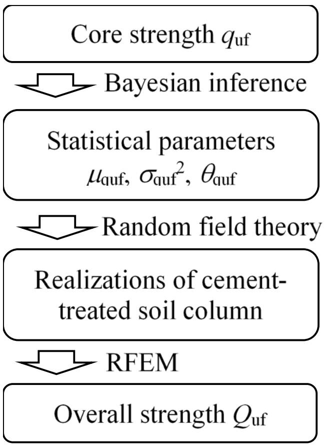

Figure 1. Analysis framework

2.1. Probability distribution of strength

Namikawa and Koseki (2013) examined the probability distribution of the core strength and concluded that the normal and lognormal distributions can be adopted for the probability distributions of the unconfined compressive strength q_{uf} of the deep cement mixing columns. In the present study, q_{uf} of the cement-treated soil is assumed to follow the multivariate lognormal distribution:

$$p\left(\mathbf{lnq_{uf}}\middle|\mu_{\mathrm{lnquf}},\sigma_{\mathrm{lnquf}}{}^2,\theta_{\mathrm{lnquf}}\right)=\frac{1}{\sqrt{(2\pi)^n\left(\sigma_{\mathrm{lnquf}}{}^2\right)^n|\mathbf{C}|}\prod_{i=1}^{n}q_{\mathrm{uf}}(\mathbf{r_i})}\exp\left\{-\frac{1}{2\sigma_{\mathrm{lnquf}}{}^2}\left(\mathbf{lnq_{uf}}-\boldsymbol{\mu}_{\mathbf{lnquf}}\right)^T\mathbf{C}^{-1}\left(\mathbf{lnq_{uf}}-\boldsymbol{\mu}_{\mathbf{lnquf}}\right)\right\} \tag{1}$$

$$\mathbf{lnq_{uf}}=\begin{bmatrix}\ln q_{\mathrm{uf}}(\mathbf{r_1})\\ \vdots\\ \ln q_{\mathrm{uf}}(\mathbf{r_n})\end{bmatrix},\quad \boldsymbol{\mu}_{\mathbf{lnquf}}=\begin{bmatrix}\mu_{\mathrm{lnquf}}\\ \vdots\\ \mu_{\mathrm{lnquf}}\end{bmatrix},$$

$$\mathbf{C}=\rho_{\mathrm{quf}}(\mathbf{d})=\exp\left(-\frac{[\mathbf{r_i}-\mathbf{r_j}]}{\theta_{\mathrm{lnquf}}}\right)$$

where n is the number of q_{uf} values, $\mathbf{r_i}$ is the space vector at the point i, μ_{lnquf} is the mean of ln q_{uf}, $\sigma_{\mathrm{lnquf}}^2$ is the variance of ln q_{uf}, and θ_{lnquf} is the autocorrelation distance of ln q_{uf}. In this equation, an exponential type autocorrelation function is assumed for the spatial variability of ln q_{uf}. μ_{lnquf} and $\sigma_{\mathrm{lnquf}}^2$ are expressed in terms of μ_{quf} and σ_{quf}^2:

$$\mu_{\mathrm{lnquf}}=\ln\mu_{\mathrm{quf}}-\frac{1}{2}\ln\left\{1+\frac{\sigma_{\mathrm{quf}}{}^2}{\mu_{\mathrm{quf}}{}^2}\right\} \tag{2}$$

$$\sigma_{\mathrm{lnquf}}{}^2=\ln\left\{1+\frac{\sigma_{\mathrm{quf}}{}^2}{\mu_{\mathrm{quf}}{}^2}\right\} \tag{3}$$

In the Bayesian inference approach, realizations of μ_{lnquf}, $\sigma_{\mathrm{lnquf}}^2$ and θ_{lnquf} are drawn from the posterior probability distribution. Thereafter, μ_{quf} and σ_{quf}^2 are calculated from the realizations of μ_{lnquf} and $\sigma_{\mathrm{lnquf}}^2$. It should be noted that the statistical uncertainty of θ_{lnquf} is examined as the parameter of the autocorrelation distance.

2.2. Bayesian inference of statistical parameters

2.2.1. Posterior probability distribution of statistical parameters

The population statistical parameters μ_{quf}, σ_{quf}^2, and θ_{quf} are expressed as the posterior probability distribution in the Bayesian inference. The posterior probability distribution is defined as a product of the prior distributions and the likelihood function of observed data (Gelman et al. 2014). The posterior distribution $p(\mu_{\mathrm{lnquf}}, \sigma_{\mathrm{lnquf}}^2, \theta_{\mathrm{lnquf}} \mid \mathbf{lnq_{uf}})$ after observing q_{uf} values is described as follows:

$$p\left(\mu_{\mathrm{lnquf}},\sigma_{\mathrm{lnquf}}{}^2,\theta_{\mathrm{lnquf}}\middle|\mathbf{lnq_{uf}}\right)\propto p\left(\mathbf{lnq_{uf}}\middle|\mu_{\mathrm{lnquf}},\sigma_{\mathrm{lnquf}}{}^2,\theta_{\mathrm{lnquf}}\right)p\left(\mu_{\mathrm{lnquf}}\right)p\left(\sigma_{\mathrm{lnquf}}{}^2\right)p\left(\theta_{\mathrm{lnquf}}\right) \tag{4}$$

where $p(\mathbf{lnq_{uf}} \mid \mu_{\mathrm{lnquf}}, \sigma_{\mathrm{lnquf}}^2, \theta_{\mathrm{lnquf}})$ is the likelihood function of the observed q_{uf} values, and $p(\mu_{\mathrm{lnquf}})$, $p(\sigma_{\mathrm{lnquf}}^2)$, $p(\theta_{\mathrm{lnquf}})$ are the prior probability distributions. The posterior distribution is expressed as a joint probability density function among μ_{lnquf}, $\sigma_{\mathrm{lnquf}}^2$, and θ_{lnquf}.

2.2.2. Markov chain Monte Carlo method

Markov chain Monte Carlo (MCMC) method is adopted to draw the realization values of μ_{lnquf}, $\sigma_{\mathrm{lnquf}}^2$, and θ_{lnquf} from $p(\mu_{\mathrm{lnquf}}, \sigma_{\mathrm{lnquf}}^2, \theta_{\mathrm{lnquf}} \mid \mathbf{lnq_{uf}})$ (Gamerman and Lopes 2006). The realizations can be sequentially sampled from the conditional distributions as follows:

$$p\left(\mu_{\mathrm{lnquf}}\middle|\sigma_{\mathrm{lnquf}}{}^2,\theta_{\mathrm{lnquf}},\mathbf{lnq_{uf}}\right)\propto p\left(\mathbf{lnq_{uf}}\middle|\mu_{\mathrm{lnquf}},\sigma_{\mathrm{lnquf}}{}^2,\theta_{\mathrm{lnquf}}\right)p\left(\mu_{\mathrm{lnquf}}\right) \tag{5}$$

$$p\left(\sigma_{\mathrm{lnquf}}{}^2\middle|\theta_{\mathrm{lnquf}},\mu_{\mathrm{lnquf}},\mathbf{lnq_{uf}}\right)\propto p\left(\mathbf{lnq_{uf}}\middle|\mu_{\mathrm{lnquf}},\sigma_{\mathrm{lnquf}}{}^2,\theta_{\mathrm{lnquf}}\right)p\left(\sigma_{\mathrm{lnquf}}{}^2\right) \tag{6}$$

$$p\left(\theta_{\mathrm{lnquf}}\middle|\mu_{\mathrm{lnquf}},\sigma_{\mathrm{lnquf}}{}^2,\mathbf{lnq_{uf}}\right)\propto p\left(\mathbf{lnq_{uf}}\middle|\mu_{\mathrm{lnquf}},\sigma_{\mathrm{lnquf}}{}^2,\theta_{\mathrm{lnquf}}\right)p\left(\theta_{\mathrm{lnquf}}\right) \tag{7}$$

$p(\mathbf{lnq_{uf}} \mid \mu_{\mathrm{lnquf}}, \sigma_{\mathrm{lnquf}}^2, \theta_{\mathrm{lnquf}})$ is calculated with the previous realization values of μ_{lnquf}, $\sigma_{\mathrm{lnquf}}^2$, and θ_{lnquf}. It is assumed that $p(\mu_{\mathrm{lnquf}})$ follows a normal distribution and $p(\sigma_{\mathrm{lnquf}}^2)$ follows an inverse gamma distribution. Then these prior distributions become natural conjugate distributions. The Gibbs sampling can be adopted to draw the realizations of $\mu_{\ln quf}$ and $\sigma_{\ln quf}^2$. It is assumed that $\theta_{\ln quf}$ follows a truncated normal distribution. Since this prior distribution is not a natural conjugate distribution, the Metropolis-Hastings algorithm is adopted to draw the realizations of $\theta_{\ln quf}$. The drawn realization values of $\mu_{\ln quf}$ and $\sigma_{\ln quf}^2$ can be transformed to μ_{quf} and σ_{quf}^2 using the following relationships.

$$\mu_{\text{quf}} = \exp\left\{\mu_{\text{lnquf}} + \frac{\sigma_{\text{lnquf}}{}^2}{2}\right\} \tag{8}$$

$$\sigma_{\text{quf}}{}^2 = \exp\{2\mu_{\text{lnquf}} + 2\sigma_{\text{lnquf}}{}^2\} - \exp\{2\mu_{\text{lnquf}} + \sigma_{\text{lnquf}}{}^2\} \tag{9}$$

The Bayesian inference and MCMC methods adopted in this study have been described in detail in another publication (Namikawa 2019).

2.3. Random field of strength

The covariance matrix decomposition method (Fenton and Griffiths 2008) is adopted to generate the realizations of the random field of q_{uf}. In this method, a production of a lower triangle of the correlation matrix **C** in Eq.(1) and a standard normal random variable vector yields a standard normal random field in the presence of the spatial autocorrelation with $\theta_{\ln quf}$. Then the random field is generated from the standard normal random field with $\mu_{\ln quf}$ and $\sigma_{\ln quf}^2$.

In a normal RFEM, the random fields are generated with constant values of the statistical parameters. In the preset study, the random fields are generated with the $\mu_{\ln quf}$, $\sigma_{\ln quf}^2$, and $\theta_{\ln quf}$ values calculated using the Bayesian inference method. Thus the statistical parameter values vary in each realization of the random field of q_{uf}. The RFEM analysis that simultaneously accounts for the statistical uncertainty and the spatial variability is possible with the random fields generated by the method in the present study.

2.4. Random finite element method

A three-dimensional FEM analysis was conducted to calculate the unconfined compressive strength Q_{uf} of the full-scale cement-treated soil column. The FEM software DIANA was used in the FEM analysis. A full-scale cement-treated soil column of 1 m in diameter and 2 m in height is modelled as shown in Figure 2. A mesh consists of eight-node isoparametric elements. Most of the elements are cubic with a side length of 100 mm. The boundary conditions are smooth at the top and bottom of the model. A uniform displacement is applied at the top surface in the vertical direction during the loading process. 200 realizations of the random field of q_{uf} were analyzed in the RFEM analysis.

An elasto-plastic model proposed by Namikawa and Mihira (2007) was used to describe the mechanical behaviour of cement-treated soils. This model can describe the compressive and tensile failure behaviour of cement-treated soils appropriately. The material parameter values of the elasto-plastic model are listed in Table 1. These values for q_{uf} = 3 MPa were determined based on the laboratory test results. (Namikawa and Koseki 2006: Namikawa and Mihira 2007). Namikawa and Mihira (2007) performed the triaxial compression and tension tests of cement-treated soils and provided the elastic modulus E, the internal friction angle ϕ, cohesion c, and tensile strength T_f of cement-treated soils. Namikawa and Koseki (2006) performed the plane strain compression and bending tests of cement-treated soils and provided Poisson's ratio ν, dilatancy characteristic, localization size t_{s0}, and fracture energy G_f of cement-treated soils.

The q_{uf} values for each element were calculated from the assigned random variables generated as the random fields. The material parameter values shown in Table 1 was determined from the q_{uf} values assigned at each element. E, c, T_f, and G_f were assumed to be stochastic parameters and vary with q_{uf} proportionally. The other parameters, ϕ, ν, hardening parameter α, and e_y, softening parameter e_r, dilatancy coefficient D_c, t_{s0}, and characteristic length l_c, were assumed to be constant. The determination of the material parameters has been described in detail in other publications (Namikawa and Koseki 2013; Namikawa 2021).

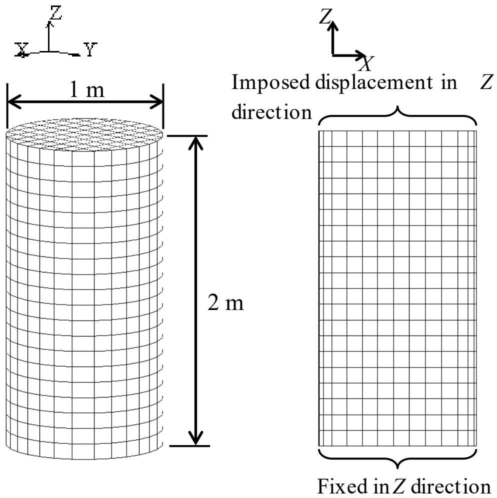

Figure 2. Mesh of a full-scale column

Table 1. Material parameters for cement-treated soil with q_{uf} = 3 MPa

Parameter	Stochastic or deterministic	Value
Elastic modulus E	Stochastic	5280 MPa
Poisson's ratio ν	Deterministic	0.167
Friction angle ϕ	Deterministic	30 degree
Cohesion c	Stochastic	0.866 MPa
Tensile strength T_f	Stochastic	0.672 MPa
Fracture energy G_f	Stochastic	15.9 N/m
Hardening parameter α	Deterministic	1.05
Hardening parameter e_y	Deterministic	0.0002
Softening parameter e_r	Deterministic	0.4
Dilatancy coefficient D_c	Deterministic	-0.4

3. Analysis results

3.1. Core strength data

The strengths of core samples retrieved from a lattice-shaped ground improvement by deep cement mixing method were used. The improved ground was constructed at a site located at Kobe city in Japan (Namikawa et al. 2007). Namikawa and Koseki (2013) have reported the core strength data in this project. The core strength data is shown in Figure 3. The quality of the cement-treated clay columns was assured using the core strength data shown in Figure 3.

The statistical parameters calculated from the core strength are shown in Table 2. Three analyses were conducted using the core strength data. q_{uf} of the core samples retrieved from the single column was used in the C-1 and C-2 cases. The two columns data was used in the C-12 case. The sample size n is around 20 in the single column case, and that is 40 in the two columns case. The autocorrelation distance of ln q_{uf}, $s\theta_{lnquf}$, in the vertical direction was calculated by the maximum likelihood method. Table 2 shows the sample mean $s\mu_{quf}$, variance $s\sigma^2_{quf}$, and autocorrelation distance $s\theta_{lnquf}$. The statistical uncertainty emerges when evaluating the population statistical parameters from the core strength data.

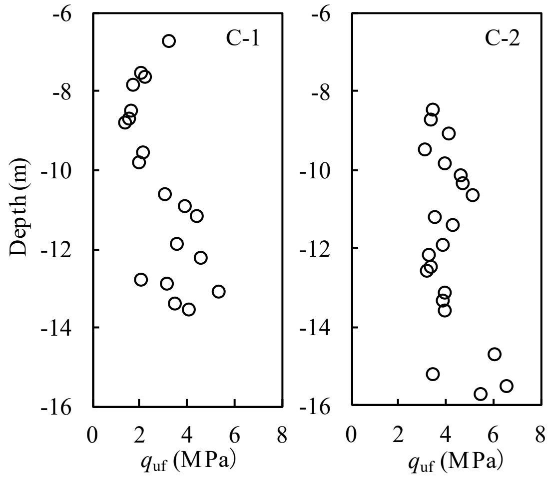

Figure 3. Core strength data.

Table 2. Sample statistical parameters of core strength data

Case	n	$s\mu_{quf}$	$s\sigma_{quf}^2$	$s\theta_{lnquf}$
C-1	19	2.92 MPa	1.37	0.6 m
C-2	21	4.20 MPa	0.910	0.3 m
C-12	40	3.59 MPa	1.52	0.9 m

3.2. Population statistical parameters

The population statistical parameters were evaluated using the Bayesian inference with the MCMC method. 11000 realizations of μ_{quf}, σ_{quf}^2, and θ_{lnquf} were drawn in each case. The realizations of the population mean μ_{quf} of the strength drawn by the MCMC method are shown in Figure 4 for the C-1 case. The variability of the realizations indicates the degree of the statistical uncertainty emerging in the evaluation of the population mean. Figure 4 shows that the the statistical uncertainty cannot be ignored when evaluating the population statistical parameter from the core strength data with the sample size n = 19.

200 realizations of μ_{quf} were randomly selected from the MCMC analysis results. Then the initial 1000 realizations were discarded to avoid the influence of the starting values. The μ_{quf}, σ_{quf}^2, and θ_{lnquf} values selected for generating the random field of q_{uf} are shown in Figure 5 for the C-1 case.

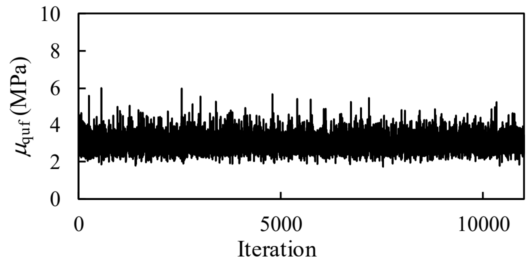

Figure 4. Realizations of population mean μ_{quf} of strength in C-1 case.

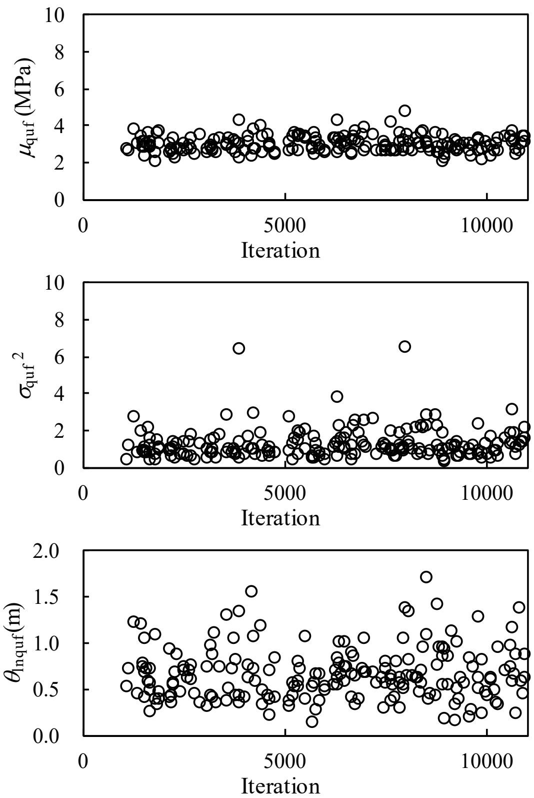

Figure 5. Population mean μ_{quf}, variance σ_{quf}^2, and autocorrelation distance θ_{lnquf} used in generating random field of strength in C-1 case.

3.3. Random finite element analysis result

The RFEM analysis was performed to simulate the unconfined compression behaviour of the cement-treated soil column with the spatial variability. The 200 realizations involving the statistical uncertainty and the spatial variability were analysed in the C-1, C-2, and C-12 cases. The analysis in which the statistical uncertainty is not taken account into was also performed in all the cases. An example of the strength distribution of the realization is shown in Figure 6. Although the same random variables are used for the cases with and without the statistical uncertainty, the generated random field with the statistical uncertainty differs from that without the statistical uncertainty.

The histogram of the overall strength Q_{uf} calculated by the RFEM analysis is shown for the C-1 case. It can be seen that the variability of Q_{uf} for the case with the statistical uncertainty is larger than that without the statistical uncertainty.

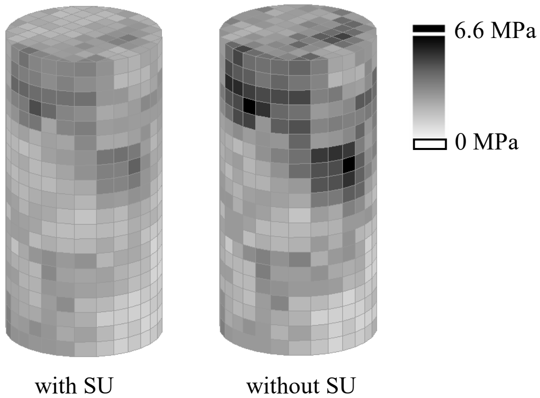

Figure 6. Example of random field of strength in a full-scale column with and without statistical uncertainty (SU) in C-1 case.

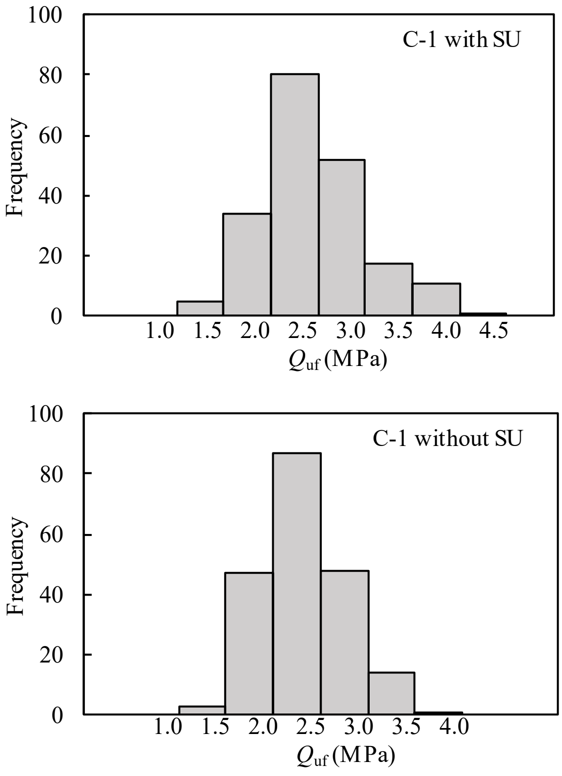

Figure 7. Histogram of overall strength Q_{uf} in C-1 cases with and without statistical uncertainty (SU).

Table 3. Statistical parameters of overall strength Q_{uf}

Case	Mean $s\mu_{Quf}$	Coefficient of variation sV_{Quf}
C-1 with SU	2.44 MPa	0.230
C-1 without SU	2.29 MPa	0.202
C-2 with SU	3.79 MPa	0.195
C-2 without SU	3.75 MPa	0.0864
C-12 with SU	3.01 MPa	0.284
C-12 without SU	2.93 MPa	0.212

Note: SU = statistical uncertainty

The mean $s\mu_{Quf}$ and coefficient of variation sV_{Quf} of the calculated Q_{uf} are summarized in Table 3. There is little difference in $s\mu_{Quf}$ between the analysis results with and without the statistical uncertainty. Table 3 shows that sV_{Quf} in the cases with the statistical uncertainty is larger than that without the statistical uncertainty. Thus the statistical uncertainty affects the variability of the overall strength calculated by the RFEM analysis significantly.

4. Reliability-based assessment

The reliability-based assessment based on the failure probability of the full-scale cement-treated soil column was conducted using the RFEM analysis results. In this project, the specific design strength of the cement-treated soil was 1.18MPa and the improved ground was adopted to prevent the large deformation of soil deposits. The quality assurance of the cement-treated soil column was performed against seismic events. Normally the safety factor for seismic events is 2 in the design and quality assurance of the deep cement-mixing method (CDIT 2002). Thus the strength of the cement-treated column should be more than 0.590 MPa in this project.

The empirical cumulative distribution function of Q_{uf} obtained from the RFEM analysis results is shown in Figure 8. The smallest values of Q_{uf} is larger than 0.590 MPa in all the cases. Since the number of the realizations of the cement-treated soil column is 200, the RFEM analysis results indicate that the failure probability of the cement-treated soil column is less than 0.5%. The failure probability for the seismic events is normally set to be less than 1.0%. The reliability-based assessment indicates that the constructed cement-treated soil columns satisfies the quality specified in the design procedure.

The characteristic value X_{quf} of the strength can be determined from the empirical cumulative distribution function of Q_{uf}. Eurocode 7 (CEN 2004) has proposed that the characteristic value should be derived so that the calculated probability of a worse value governing the occurrence of the limit state under consideration is not greater than 5%. According to Eurocode 7, the characteristic value X_{quf} of the strength is defined as the 5% fractile value of Q_{uf}. The characteristic value is plotted in Figure 8. X_{quf} is 1.91 MPa for the C-12 case. The statistical uncertainty and spatial variability are properly taken into account in the estimation of this value. However, the characteristic value should be estimated in the design procedure. The cement-treated soil columns are normally constructed after the design procedure. Further study is required to determine the characteristic value of the cement-treated soil strength without the core strength data in the design procedure.

5. Conclusions

The present study demonstrated the reliability-based assessment of the cement-treated soil columns by the deep mixing method. The reliability-based assessment was performed based on the RFEM analysis results. In the present study, the statistical uncertainty which emerges in the population statistical parameters was evaluated from the core strength data and the strength

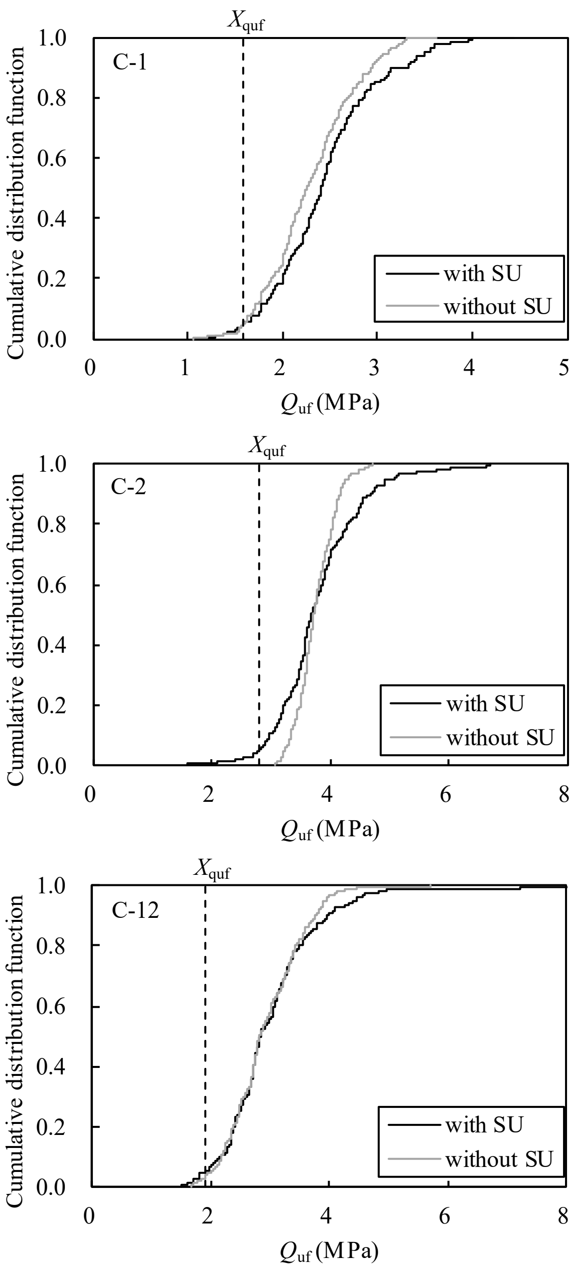

Figure 8. Empirical cumulative distribution function of overall strength Q_{uf}.

random field was generated with the statistical parameters involving the statistical uncertainty. The statistical uncertainty was estimated using a Bayesian inference method. In the Bayesian inference method, a Markov chain Monte Carlo (MCMC) method was adapted to generate the realization values from the joint probability distribution of the statistical parameters. The generated realization values were used when generating the random fields for RFEM. The compression failure behaviour of the cement-treated soil column was simulated in the RFEM analysis.

The RFEM analysis results showed that the statistical uncertainty involved in the sample statistical parameters affect significantly the variability of the overall strength Q_{uf} of the cement-treated soil column. The cumulative distribution function of Q_{uf} provided from the RFEM analysis results indicated that the failure probability is less than 0.5% for the design strength in the project. Moreover, the cumulative distribution function was used to determine the characteristic value of the strength of the cement-treated soil column.

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