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ON THE INSTABILITY OF ONE VERSUS TWO PROXIMATE MASSES MOVING ON AN INFINITE BEAM SUPPORTED BY VISCOELASTIC LAYERS

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Summary. This contribution provides a detailed analysis of the differences related to moving mass problems regarding the comparison of one and two moving masses. It has been proven that instability of one moving mass can only occur in the supercritical velocity range irrespectively of the type of the supporting structure. Additionally, damping in the supporting structure always shifts the onset of instability to higher velocity values. However, when two moving masses are under consideration, the situation can be completely different regarding the onset of instability and damping influence. For shorter distances between masses, the dynamic interaction between moving inertial objects can reverse the damping influence and instability can occur in the subcritical velocity range. Some preliminary results are presented in the previous author's works; however, more analyses are still required, specifically for the three-layer model.

1 INTRODUCTION

Moving load problems belong still to very active fields of research. Applications related to rail transport which is nowadays a preferential mode of transport due to its ability to reduce the CO2 footprint have recently received more attention. Increasing the speed of today's trains to meet demands on rail network capacity, as well as passenger expectations on reduced travel times without compromising their comfort, challenges the stability conditions. The instability of moving inertial objects as the anomalous Doppler effect is a physical phenomenon known for years. Several works of the author of this article show that the instability of one moving mass can only occur in the supercritical velocity range. The critical velocity in this context, also known as the critical velocity for resonance, corresponds to the critical velocity of a moving force which is equal to the lowest phase wave propagation velocity in the foundation. When the force moves at this velocity, then in the absence of damping the deflection tends to infinity. The possibility of instability of a moving vehicle was somehow overlooked because it was proven that a single moving mass or an oscillator can only achieve unstable behaviour in the supercritical velocity region, and because this region is a priori excluded from track design, the question of instability has not been found important. However, the instability of two moving masses does not respect such condition, [1-4].

This contribution concerns layered railway track models, which are very popular for their simplicity and computational efficiency. The results obtained by their use have an acceptable agreement with complex 3D models [5].

The present paper summarizes conditions under which instability of moving proximate masses occur in subcritical velocity range and what the influences of model parameters, within their allowable ranges which are detailed in [6], are. It is necessary to highlight that there are two important facts: first, the fact that the instability can occurs in subcritical velocity range is very dangerous, because the track design usually consider the limits in form of the critical velocity; second, the fact that the foundation damping can worsen the situation is also dangerous, because intuitively, one would increase the damping to stabilize the moving object.

2 PROBLEM DEFINITION

One of the models considered in the present paper is depicted in Figure 1. This is a three-layer model consisting of a beam and supporting layers. The layers contributing to the overall behavior by their inertia are the sleeper and ballast layer. Their mass is represented in Figure 1 as point mass, but for the sake of simplicity it is introduced into the governing equations in its distributed form. The model also has three viscoelastic layers representing from the top the railpads, ballast and foundation layers. Also, here the parameters are introduced in their distributed version. The shear stiffness is attributed to the ballast layer.

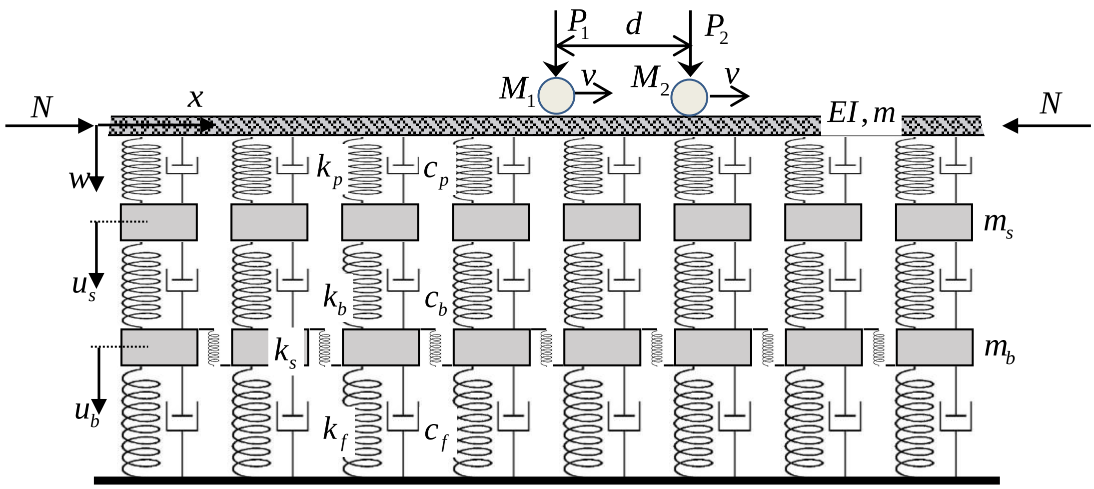


Figure 1: Thee-layer model traversed by two moving masses.

All results are presented using dimensionless parameters, which admissible ranges are detailed in [6]. Instability is analyzed with the help of so-called instability lines which are traced directly in the plane of velocity-moving mass ratios by identifying the real-valued induced frequencies, which in a damped case obligatorily alter by one the number of unstable induced frequencies, so that stable and unstable velocity intervals are clearly visible for any specific moving mass ratio. The usage of the D-decomposition method commonly implemented by other researchers is avoided.

In order to clearly state if the tested velocity is sub- or supercritical, the critical velocity must be uniquely determined. In a one-layer model, there is an analytical formula for one unique always valid value of the critical velocity. In the two-layer model, there are generally three resonances, two of which indicate critical velocities and one the so-called false critical velocity, [2-4]. Cases where there is only one resonance can be delimited by the range of dimensionless parameters of the model. If the shear resistance of the model is neglected and the action of the axial force is not considered, then the critical velocities can be determined analytically. In the three-layer model the situation is more complicated. There should be generally five resonances, three of which should indicate critical velocities and two the so-called false critical velocities. Cases with irregular behavior are generally difficult to predict, [6]. Sometimes it is possible to replace a critical velocity by a so-called pseudocritical velocity, however, sometimes there is no indication that the number of critical velocities could be completed.

3 ONE-LAYER MODEL

In a one-layer model, the situation is quite simple. This model does not have all layers as the one in Figure 1. In fact, there is only a foundation layer, so the model is just a beam on viscoelastic foundation. It can contain a shear layer, or equivalently the Pasternak modulus. Neglecting the shear stiffness and effect of the axial force, the critical velocity in dimensionless form equals unity. Then, there are almost no parameters to vary and analyze. It is only the foundation damping and the distance between masses. Conclusions are therefore simple to draw. In Figure 2 the case of one moving mass is plotted as a reference. It can be concluded that by decreasing the damping the instability curves are monotonically decreasing so that lower moving mass rations are affected by unstable behavior. No unstable case can be detected in the subcritical velocity range. Increased damping has positive influence, very low damping has a vertical asymptote that is approaching the critical velocity. This behavior was already reported by other researchers, [7], it is not a new finding.

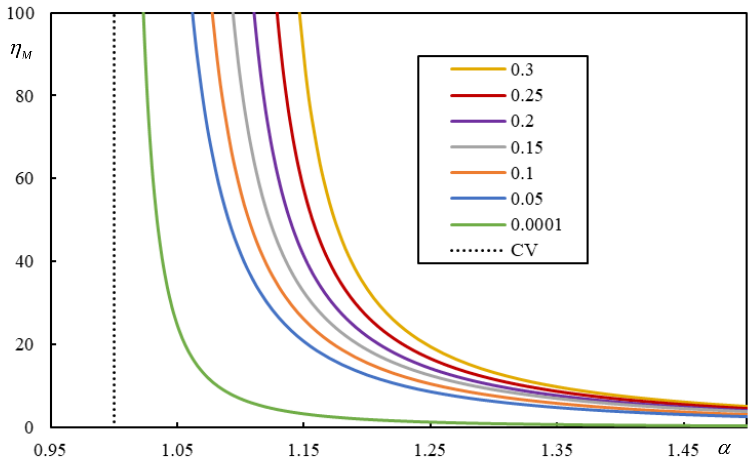
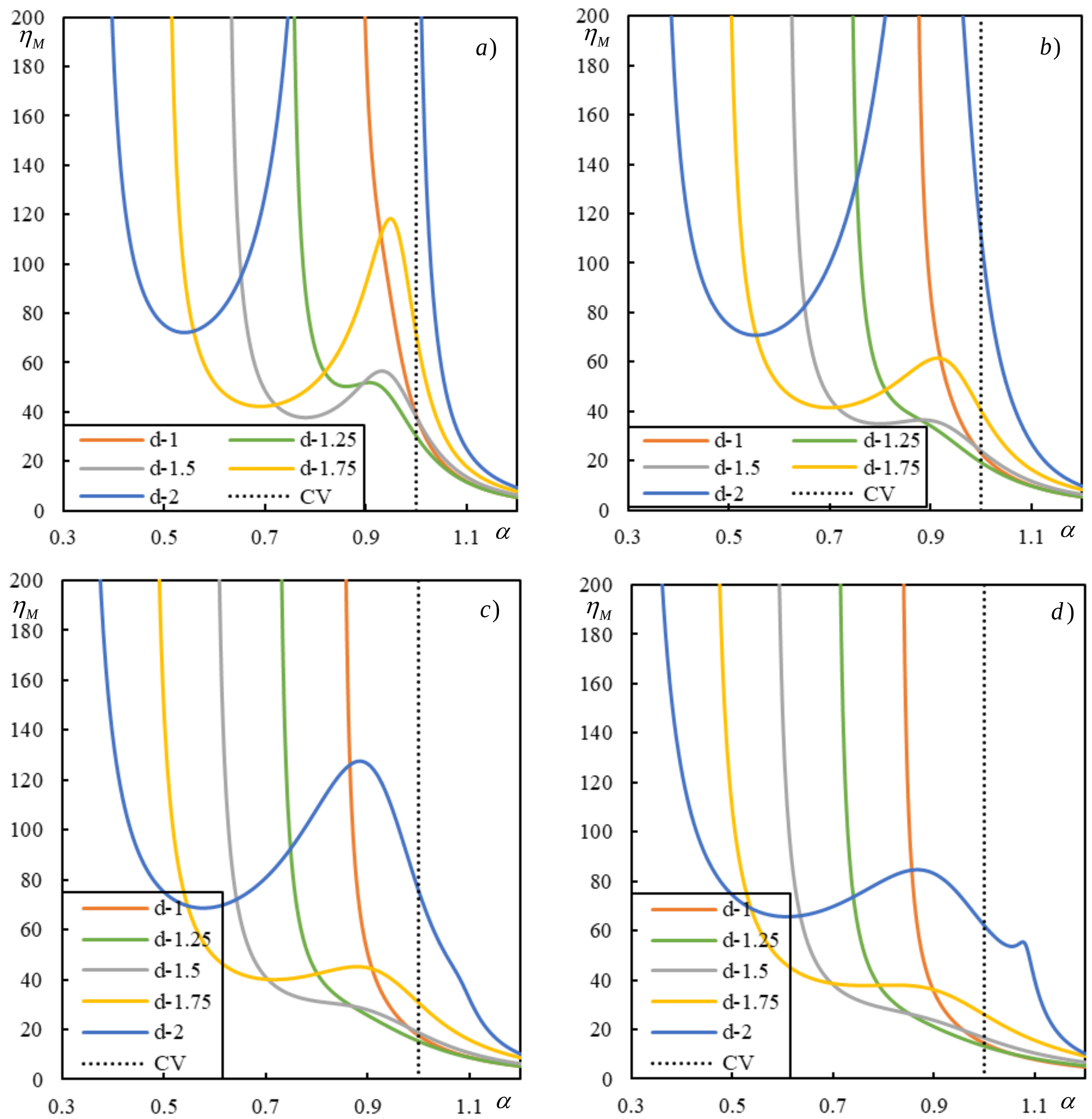


Figure 2: Instability lines for one moving mass traversing a one-layer model: different curves correspond to a different damping ratio η_f with values as indicated in the legend.

Nevertheless, when two moving masses are considered, then there are several distances that induce instability in subcritical velocity range. These situations are plotted in Figure 3. Different parts of Figure 3 correspond to different damping levels. It is seen that with increased damping the instability lines are getting lower, which is exactly the opposite effect as for one moving mass.

Graphs in Figure 3 are limited to the moving mass ratio $\eta_M = 200$ because higher values are not realistic. Velocity ratio α is extended slightly further into supercritical velocity range for better clarity. It is seen that until $\eta_f = 0.25$ the dimensionless distance $\tilde{d} = 2.25$ and higher does not induce any danger. Instability lines branches do exist, but η_M values are unrealistic. From higher distances these branches do not exist anymore.



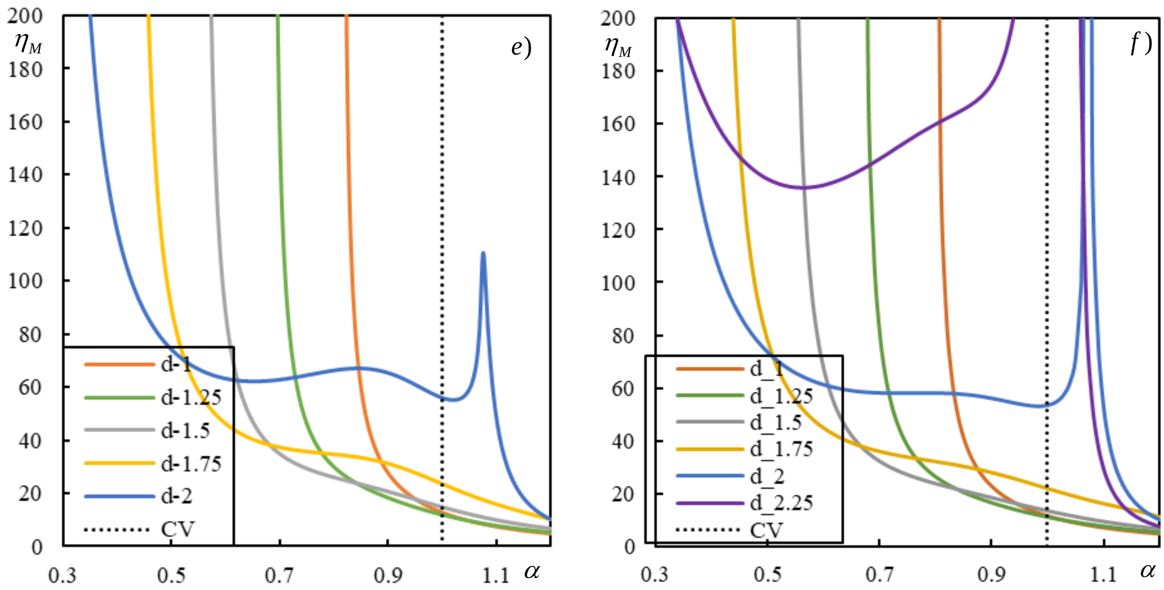
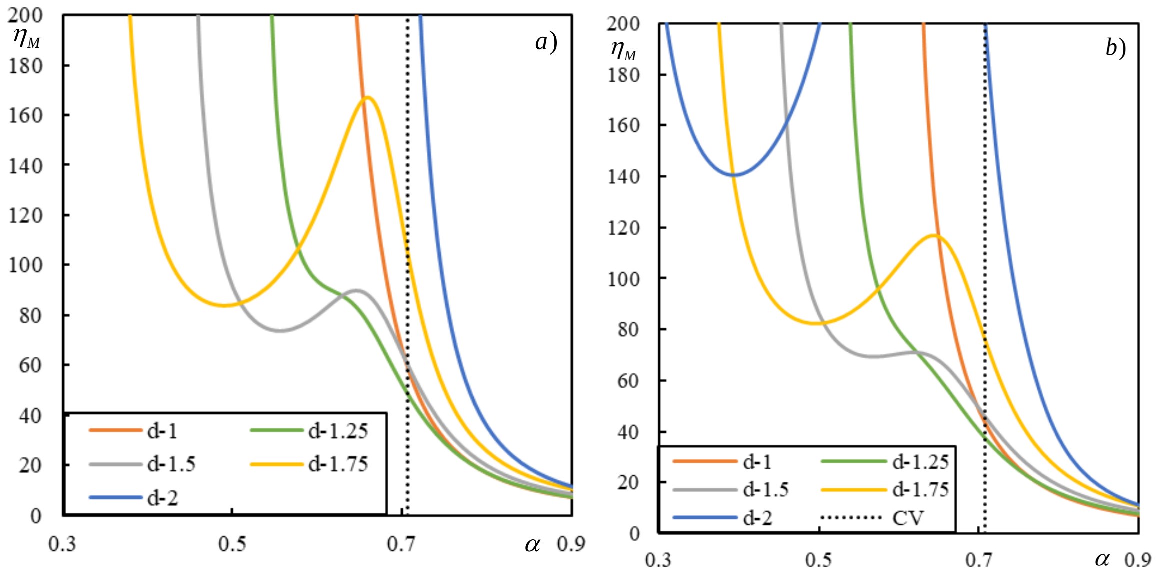


Figure 3: Instability lines for two moving masses traversing a one-layer model: different curves correspond to a different dimensionless distance $\tilde{d}$ with values as indicated in the legend. CV indicates the critical velocity; a)-f) are related to $\eta_f = 0.05:0.05:0.3$.

It can be concluded that when the masses are quite apart, then instability can occur only in the supercritical range as for one moving mass. However, the instability lines have generally more branches and sometimes their behavior is quite strange. On the other hand, it was demonstrated that the lowest distance does not correspond to the worst-case scenario.

4 TWO-LAYER MODEL



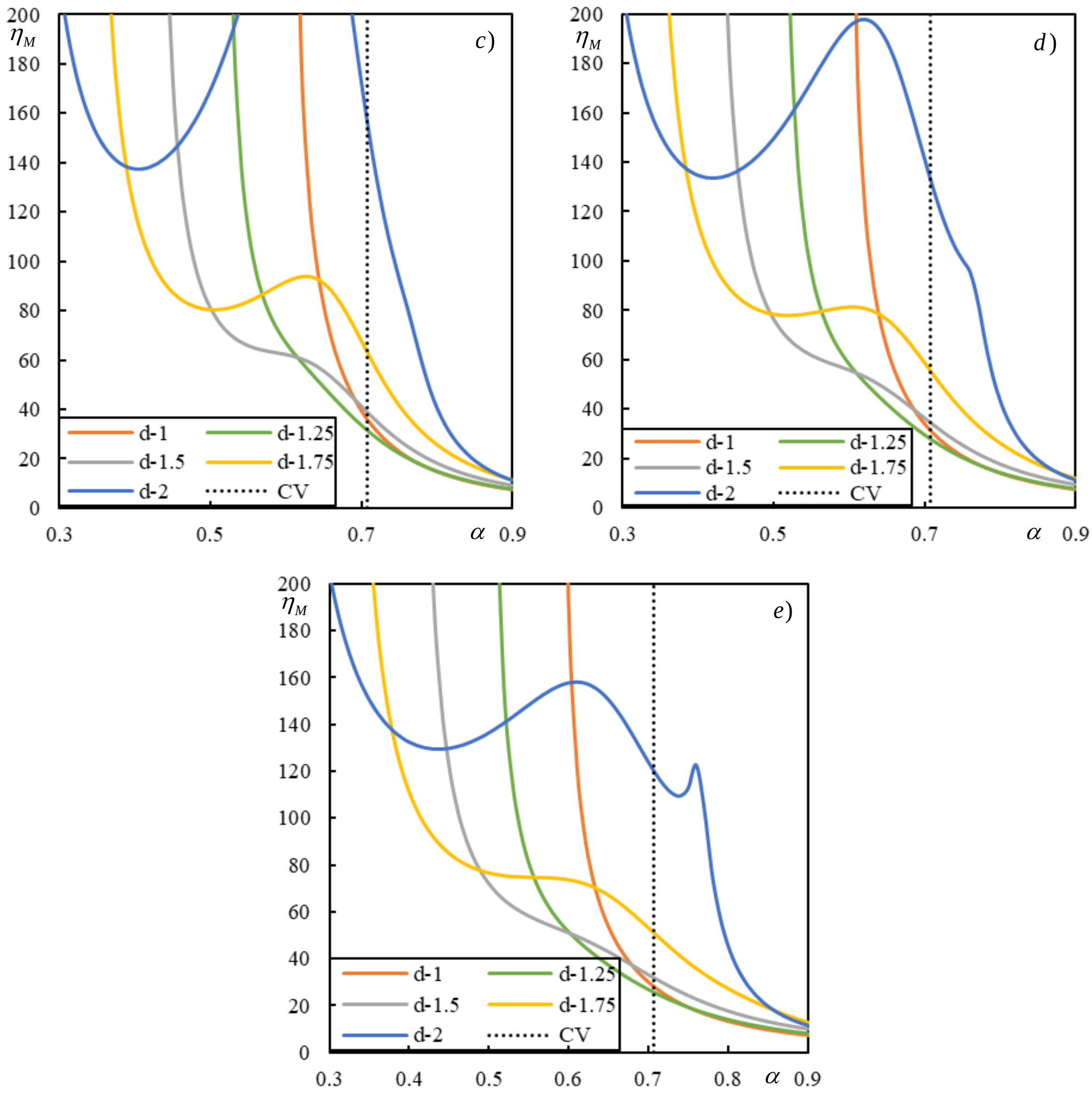


Figure 4: Instability lines for two moving masses traversing a two-layer model: different curves correspond to a different dimensionless distance $\tilde{d}$ with values as indicated in the legend. CV indicates the critical velocity; a)-e) are related to $\eta_f = 0.1:0.05:0.3$.

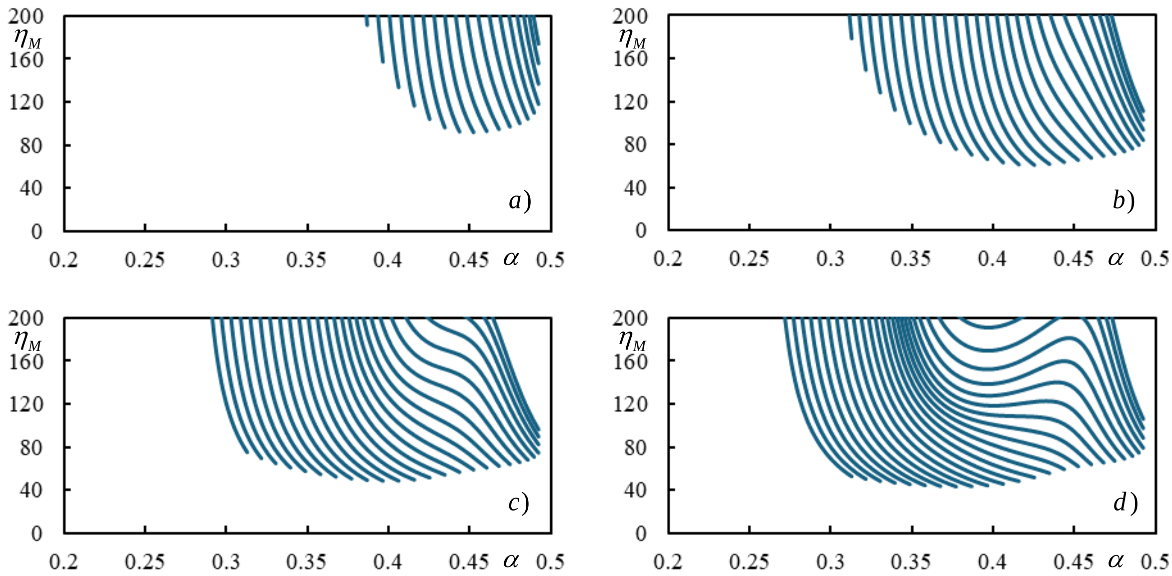
The general tendencies exemplified in the previous section are preserved on the two-layer model as well. Nevertheless, now there are more parameters to vary. It has been verified that the worst situation is related to the lowest sleeper layer mass ratio $\mu_s = 1$. Foundation damping ratio η_f has more influence than the rail pad layer damping ratio η_p and the rail pad layer stiffness ratio κ_p is not very important. This is exemplified in Figure 4.

Regarding the critical velocity, there should be three resonances, two identifying the critical velocity and one the false critical velocity. When the shear stiffness and axial force are neglected, critical velocities can be solved analytically. Then it is easy to identify the region with only one resonance where it is necessary to add the pseudocritical value. More details are given in [4]. It is of note that the region of irregular situations is located close to the boundary of admissible values, namely for low rail pad layer stiffness ratio κ_p.

5 THREE-LAYER MODEL

In the three-layer model, as indicated in [6], the question regarding the critical velocity itself is not sufficiently clarified, and this problem has not been analyzed by other researchers either. The expected five resonances, which should clearly indicate three critical velocities and two false critical values, occur in a limited number of regions of the entire range of admissible parameters. Moreover, it is not easy to analytically determine the separations between these regions. There are several cases where the role of the lowest critical velocity must be played by a pseudocritical value that can be determined by parametric analysis but also cases with three resonances where the pseudocritical velocity does not exist, and these cases are difficult to distinguish a priori. Parametric analysis does not always solve the problem of pseudocritical velocities because extreme displacements may not be very well pronounced and then the determination of the pseudocritical value can be ambiguous, as demonstrated in [6]. This usually happens further from the region of five resonances. Several examples of such behavior are shown in [6]. The way proposed in the present paper is connected to the fact that instability of one moving mass cannot occur in subcritical velocity range, therefore it is possible to implement negligible damping and search for the lowest vertical asymptote of the corresponding instability line.

Having settled the problem of the critical velocity, it is now possible to analyze the combination of parameters for which instability occurs in the subcritical velocity range. As for the two-layer model, mass ratios are kept at their lowest admissible values: $\mu_s = 1$, $\mu_b = 2$. An illustrative case is selected, and the corresponding graphs are plotted in Figure 5. Moving mass ratio is again limited by $\eta_M = 200$.



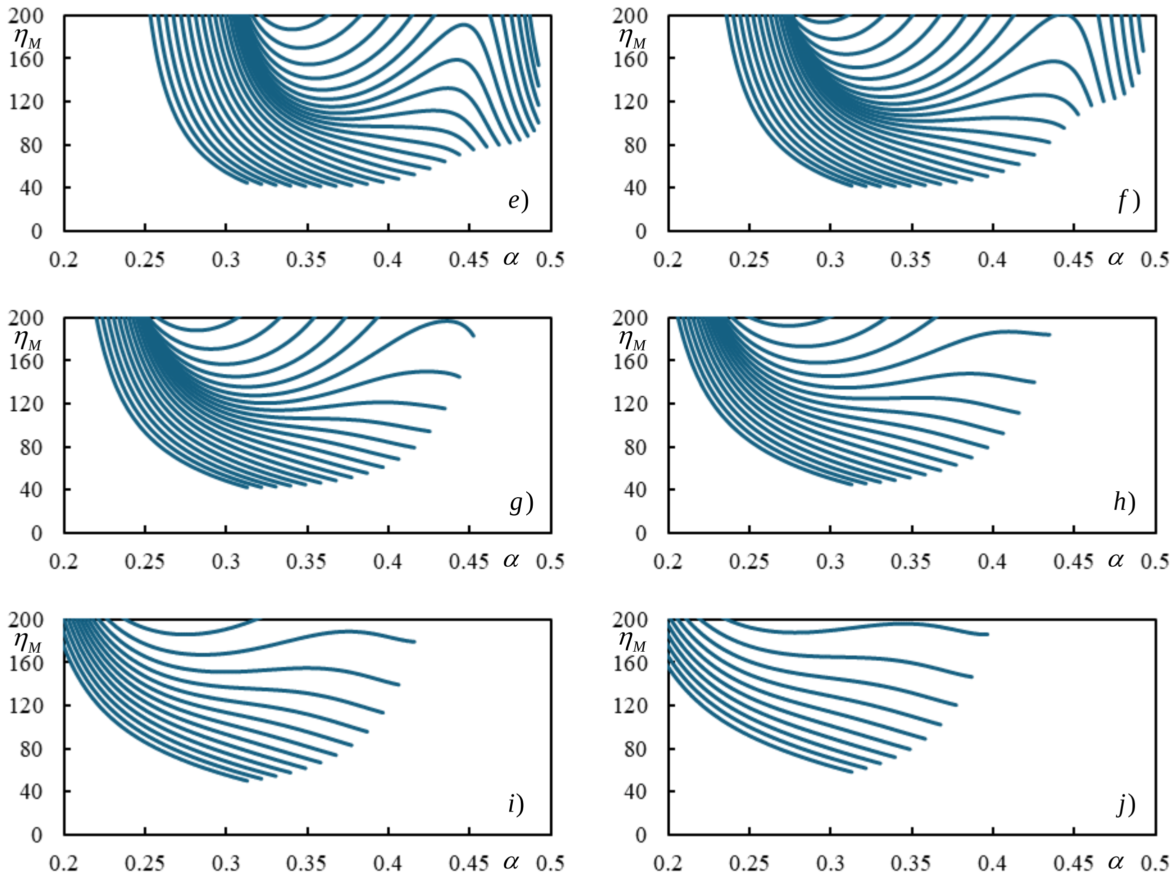


Figure 5: Instability lines two moving masses traversing a three-layer model: $\mu_s = 1$, $\mu_b = 2$, $\kappa_p = 3$, $\eta_p = \eta_b = \eta_f = 0.15$. Each curve corresponds to different κ_b starting from bottom with $\kappa_b = 0.04$ and following the logarithmic scale $\kappa_b = 0.04 \times 10^{(j-1)0.05}$; parts a)-j) correspond to $\tilde{d} = 1.25 : 0.25 : 3.5$.

For better clarity, instability lines are plotted only in the subcritical region, however, it is of note that these values are different for different combination of stiffness parameters κ_p and κ_b. The scale is maintained in all subfigures for better comparison. It can be concluded from Figure 5 that only for larger distances between the masses the instability lines occupy some region in the subcritical velocity range, while for shorter distances they are more vertical and thus reaching unrealistic values very close to the critical velocity. On the other hand, the number of curves is decreasing with increasing distance and thus only low values of κ_b are contributing. It is also possible to study the influence of other parameters and confirm the negative effect of increased damping, but these cases are omitted here due to the restrictions on paper length.

6 CONCLUSIONS

This contribution presents a detailed analysis of the combinations of parameters that induce instability of two moving proximate masses in the subcritical range of velocities when masses are traversing a layered model of the railway track. One- two- and three-layer models are considered. Several illustrative graphs of instability lines show the influence of parameters in the selected tested scenarios. Due to the large number of parameters, it is quite difficult to draw general conclusions on the three-layer model, because instability lines are not monotonic and can have several branches. Nevertheless, it was demonstrated that in order to instability occur in subcritical velocity range, all ratios, masses and stiffnesses, should be low. As far as the distance between masses, it should also be low, but here the relation is not monotonic, the lowest does not necessarily mean the worst. As far as damping is concerned, contrary to one moving mass, foundation damping always worsens the situation in the subcritical velocity range.

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