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Control-Relevant Identification of the Unstable Inertial Systems

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ABSTRACT

Nowadays, with the rise of computing power, control-relevant identification methods have gained attention in various industrial applications, incorporating the requirements for control design into the process of system identification. Mathematical identification of stable linear dynamical systems is a widely studied problem in the literature, and it is prevalently performed in open-loop structures that may lead to high-order models suitable for control system design with imposed control objectives. However, in the case of unstable systems identification can become a challenge task, and usually is performed using closed-loop identification techniques. This paper presents the control-relevant identification approach for two kinds of unstable processes. The contribution focuses on establishing a well-fitted identified model by using a strategy that involves collecting data from the closed-loop system's operation with a proportional controller when the system achieves an underdamped step response. In addition, a proportional-integral-derivative (PID) controller for each process was synthesized using maximal stability degree method. Concerning identification, the simulation results were compared with those of the genetic algorithm and offered better model estimation than the genetic algorithm. On the other hand, it is also demonstrated that the designed control algorithm offered a high degree of stability to the system and is more reliable in stabilizing the behavior of the unstable system than the genetic algorithm and the parametric optimization method.

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1 Introduction

Stable and unstable systems are core concepts in control theory, system dynamics, and engineering. These concepts play a fundamental role in various areas, including control systems, process engineering, aerospace and robotics, electrical engineering, biological systems [1,2]. System identification and control algorithm design are critical processes in system stability analysis and maintains of the desired system performance and robustness. Over the last few decades have been developed many approaches for system stability analysis and stability ensuring. These approaches depend on such factors as whether the system is linear or nonlinear, operates continuously or discretely, exhibits time-varying or time-invariant behavior, etc. [3].

System identification and control algorithm design for unstable systems are significantly more complicated and challenging tasks compared to stable systems [4–7]. Unstable systems can be found in various industries where dynamic processes, complex interactions, and sensitive conditions exist in chemical industries [8], power systems [1], automotive industry [9], and manufacturing industries [10]. For so many kinds of systems, it often requires the use of advanced control theories, numerical simulations, and experimental validation to achieve effective control strategies. Unstable systems often exhibit more complex and nonlinear dynamics and are highly sensitive to initial conditions and small disturbances due to the sensitive nature of unstable systems, control algorithms must be highly robust to uncertainties, noise, and parameter variations and the open loop identification methods are not applicable on unstable systems [10–12]. From these considerations, the identification of unstable systems presents a complex challenge due to the necessity of performing the procedure of identification within a closed loop [13–15].

Closed-loop identification poses a significant challenge due to the inherent correlation between unmeasurable noise and the input. This correlation arises because, in cases where the feedback controller is not consistently zero, a relationship between the input and the noise is established. Consequently, identification methods effective in open-loop scenarios often are not directly suitable when applied to closed-loop data due to this inherent input-noise correlation [16,17].

Over the last decades have been proposed different methods and algorithms for model estimation of the unstable systems in the closed-loop. These methods address the complexities associated with unstable systems and aim to improve the accuracy of model identification. These methods propose solutions in models estimation of the unstable systems as approximation models with inertia first, or second order with time delay and positive zeros [11], or discrete-time models as auto regressive model with exogenous inputs (ARX), auto regressive moving average model with exogenous inputs (ARMAX) [15–17], modified output error model [4], Box-Jenkins [18], q-Markov Covariance equivalent realization (QMC), etc. [19,20]. These models employ coefficient fitting through techniques such as least squares, as well as identification methods like subspace state space system identification (N4SID), multivariable output-error state space identification (MOESP), and canonical variate analysis (CVA) [11]. Another group of optimization techniques, which are so effective in system estimation is meta-heuristic algorithms, which are inspired by natural processes, such as evolution, swarm behavior, and other biological phenomena. They are particularly effective for solving problems where the solution space is large, multi-dimensional, and nonlinear [21–23]. In the context of identification, which involves estimating model parameters from data, meta-heuristic algorithms can be applied to find the best set of parameters that fit the given data and model structure.

One of the main goals of model estimation is control algorithm design. In the case of unstable systems, the control algorithm design often involves making trade-offs between stabilizing the system and achieving the desired performance of the system. Utilizing feedback control through the proportional-integral-derivative (PID) control algorithm offers the means to stabilize inherently unstable systems [24–26]. This approach involves adjusting control inputs based on the system's error, integrating accumulated errors, and considering the rate of change of errors to effectively counteract instability and establish stable behavior.

Incorporating the requirements for control algorithm design into the process of system identification offers the potential to streamline the modeling process and enhance the model's applicability in application of control system design and system stability ensuring the unstable systems [16]. This approach, known as control-relevant identification, lies at the core of the concept as discussed by Hjalmarsson in 2005 and van den Hof and Callafon in 2003 and it empowers engineers to harness and control unstable systems, enhancing safety and efficiency across a diverse array of industries and applications [27–29].

In this paper, the given system is unstable, and for implementing the identification algorithm, it is assumed that experimental input/output data are generated under a stabilizing feedback system with a proportional (P) controller. In the literature, this type of identification is widely used in connection with control-relevant identification, where the goal is model estimation for control algorithm design. The control algorithm design is proposed to be realized using the maximum stability degree iterative (MSDI) method.

This paper proposes an approach for control-relevant identification of unstable inertial systems, where the main contributions of research are as follows:

1. The closed-loop identification algorithm that allows for the estimation of a mathematical model, which approximates the unstable system with a second-order inertial transfer function with one positive pole, with or without a time delay.
2. The methodology for synthesizing the PID control algorithm for an unstable control system that is based on the maximum stability degree iterative method. This approach achieves two goals: stabilizing the system by providing a high degree of stability and optimizing system performance through the iterative selection of tuning parameters.

The rest of this paper is organized as follows: Section 2 presents algorithms for closed-loop identification, Section 3 covers algorithms for tuning the PID controller, and Section 4 introduces the algorithm for control-relevant identification. Simulation results are provided in Section 5, and conclusions are presented in Section 6.

2 Closed-Loop Identification of the Unstable Systems

2.1 Identification of the Unstable Inertial Model

To tackle the issue of system identification for an unstable system within a closed loop, a solution is proposed that involves approximating the system's behavior with a second-order inertial transfer function with one positive pole:

$$H(s) = \frac{k}{(T_1 s - 1)(T_2 s + 1)} = \frac{k}{a_0 s^2 + a_1 s - a_2} = \frac{B(s)}{A(s)}, \tag{1}$$

where T_1, T_2 are time constants; k is a transfer coefficient of the system; $a_0 = T_1 T_2, a_1 = T_1 - T_2, a_2 = 1$.

To implement the closed-loop identification, the automatic control system with a P controller (Fig. 1) is considered.

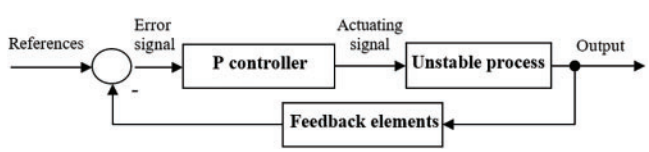

Figure 1: Block scheme of the automatic control system

The transfer function of the P controller is as follows:

$$H_P(\mathrm{s}) = k_p, \tag{2}$$

where k_p–proportional tuning parameter of P controller.

It is assumed that the system can be stabilized by the P controller only if $T_1 > T_2$ [30]. Based on this, the closed-loop identification algorithm proposed in this paper is developed under this condition.

The transfer coefficient of the system is determined according to the expression [30]:

$$k = \lim_{t\to\infty} \frac{\Delta y}{\Delta u} = \lim_{t\to\infty} \frac{y_{st} - y_{initial}}{u - u_{initial}}, \tag{3}$$

where y_{st} is the steady-state output value, $y_{initial}$ is the initial value of the output response, u–input signal, $u_{initial}$ is the initial value of the input signal.

In the transfer function (1), the unknown parameters a_0, a_1, and a_2 are proposed to be calculated based on the following parameters: natural frequency–ω_n, proportional tuning parameter–k_p, and transfer coefficient–k:

$$a_0, a_1, a_2 = f(\omega_n, k_p, k). \tag{4}$$

The tuning parameter $k_p > 0$ is varied until the system achieves the step response shown in Fig. 2. Next, based on the underdamped step response of the closed-loop system, the oscillation period is calculated as follows:

$$T_0 = t_2 - t_1. \tag{5}$$

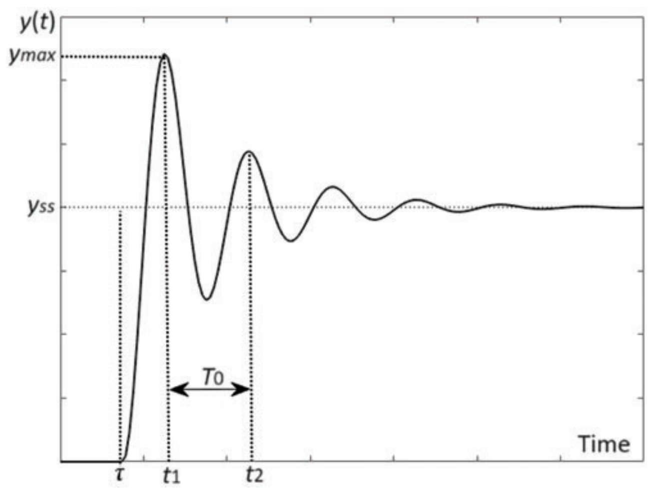

Figure 2: Underdamped step response of the closed-loop system

The damping value is calculated as follows:

$$\xi = \frac{1}{\sqrt{1 + (2\pi / log\, d)^2}}, \tag{6}$$

where d is decay ratio.

According to Eqs. (5) and (6), the natural frequency is calculated according to the expression [30]:

$$\omega_n = \frac{2\pi}{T_0\sqrt{1 - \zeta^2}}. \tag{7}$$

The characteristic equation of the closed-loop system with P controller is:

$$A(s) = \left(a_0 s^2 + a_1 s - a_2\right) + k \cdot k_p. \tag{8}$$

Next, the following substitution of the Laplace transform is proposed:

$$s = j\omega_n. \tag{9}$$

The characteristic Eq. (8) will become:

$$A(j\omega) = \left(a_0 (j\omega_n)^2 + a_1 (j\omega_n) - a_2\right) + k \cdot k_p = -a_0\omega_n{}^2 - a_2 + k_p k + a_1\omega_n j = P(\omega) + jQ(\omega)$$

Next, the real part is set equal to zero, denoted as $P(\omega) = 0$. Based on this, the following expressions are obtained:

$$\begin{cases} a_0 = \dfrac{k_p k - a_2}{\omega_n^2}; \\ a_2 = 1. \end{cases} \tag{10}$$

The coefficient a_1 remains unknown and for its calculation, the maximum stability degree criterion is used [31]. Under this criterion, the maximum shift of the dominant poles of the closed-loop system towards the imaginary axis in the left half of the complex plane is denoted by η_{max}, which analytically corresponds to the following expression:

$$\eta_{max} = \frac{a_1}{n a_0}, \tag{11}$$

where n is the order of the polynomial $A(s)$.

According to [31]:

$$\eta_{max} = \omega_n \cdot \xi. \tag{12}$$

By equating expressions (11) with (12), the expression for calculating the value of the a_1 coefficient is obtained as follows:

$$a_1 = \frac{2(k_p k - a_2)\xi}{\omega_n}. \tag{13}$$

Thus, using expressions (3), (10), and (13), the coefficients k, a_1, and a_2 from the transfer function (1) can be determined.

2.2 *Identification of the Unstable Inertial Model with Time Delay*

It is considered that the unstable system is approximated with a second-order inertial transfer function with one positive pole and a time delay as follows:

$$H(s) = \frac{ke^{-\tau s}}{(T_1 s - 1)(T_2 s + 1)} = \frac{ke^{-\tau s}}{a_0 s^2 + a_1 s - a_2} = \frac{B(s)}{A(s)}, \tag{14}$$

where τ is a time delay.

The system can be stabilized by a P controller when $T_1 > T_2$ and $T_1 > \tau$ [30], where the transfer coefficient of the closed-loop system is calculated according to Eq. (3). The unknown parameters of the transfer function (14) are proposed to be estimated based on the following parameters:

$$a_0, a_1, a_2 = f(\tau, \omega_n, k_p, k), \tag{15}$$

where τ–time delay, ω_n–natural frequency, k_p–proportional tuning parameter, k–transfer coefficient.

Based on the underdamped step response of the closed-loop system (Fig. 2), achieved by varying the proportional tuning parameter k_p, the period of oscillations is calculated as follows:

$$T_0 = t_2 - t_1 - \tau. \tag{16}$$

The damping value is calculated using Eq. (6), and the natural frequency ω_n is determined using Eq. (7).

The closed-loop characteristic equation is as follows:

$$A(s) = \left(a_0 s^2 + a_1 s - a_2\right) + k \cdot k_p \cdot e^{-\tau s}. \tag{17}$$

Based on substitution (9) and Euler's formula, the characteristic Eq. (17) becomes:

$$A(j\omega) = \frac{1}{k}\left(a_0 (j\omega_n)^2 + a_1 (j\omega_n) - a_2\right) + k_{cr} e^{-\tau\omega n j} = \left(-a_0 \omega_n{}^2 - a_2 + k_{cr} k \cos \tau\omega_n\right) + \\ j\left(a_1 \omega_n - k_{cr} k \sin \tau\omega\right) = P(\omega) + jQ(\omega). \tag{18}$$

Next, the real part is set to zero, $P(\omega) = 0$, and based on this condition, the expression for calculating the a_0 coefficient is obtained as follows:

$$a_0 = \frac{k k_{cr} \cos \tau\omega_n - a_2}{\omega_n^2}. \tag{19}$$

The maximum stability degree of the system with time delay is:

$$\eta_{max} = \frac{a_1}{n\tau a_0}, \tag{20}$$

where n is the order of the characteristic equation, which is 2 for the characteristic Eq. (17), and τ is a time delay.

Equating Eq. (20) with Eq. (12) yields:

$$\frac{a_1}{n\tau a_0} = \omega_n \cdot \xi. \tag{21}$$

From expression (21) and based on expression (19), the relationship for calculating the unknown parameter a_1 is as follows:

$$a_1 = \frac{2\zeta\tau\left(k k_{cr} \cos \tau\omega_n - a_2\right)}{\omega_n}. \tag{22}$$

The coefficient a_2 is assumed to be equal to 1.

According to Eqs. (19) and (22), and based on the step response of the closed-loop system, the mathematical model that approximates the unstable system with time delay can be estimated.

3 Synthesis the PID Control Algorithm by the Maximum Stability Degree Method with Iterations

3.1 Maximum Stability Degree Method with Iterations

The MSDI method is an effective analytical approach for optimizing the parameters of P, PI, and PID controllers [32].

It is assumed that the transfer function of the control object is described as follows:

$$H(s) = \frac{k}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n} = \frac{B(s)}{A(s)}. \tag{23}$$

It is necessary to synthesize the control algorithm for the system described by the transfer function (23), where the control law is described by the following transfer function:

$$H_R(s) = \sum_{l=0}^{N} k_l s^{l-1}, \tag{24}$$

where k_l represents the tuning parameters, N represents the number of tuning parameters.

The characteristic equation of the closed-loop system with respective control law (24) is as follows:

$$F(s, k_l) = \frac{A(s)}{B(s)} + \sum_{l=0}^{N} k_l s^{(l-1)}. \tag{25}$$

The problem of tuning using this method consists of ensuring the maximum degree of stability for the designed system with the control law given by (24). The stability degree of the system is denoted by J, and in the characteristic Eq. (25), the substitution $s = -J$ is introduced.

$$F(-J, k_l) = \frac{A(-J)}{B(-J)} + \sum_{l=0}^{N} k_l (-J)^{l-1}. \tag{26}$$

By performing differentiation operations on Eq. (26), algebraic equations are obtained from which the tuned controller parameters can be calculated:

$$k_l = f(a_i, k, J), i = \overline{1, n}. \tag{27}$$

According to Eq. (27), the tuning parameters of the respective control law depend on the known parameters of the control object $k, a_0, a_1, \ldots, a_n$ as well as the unknown value of stability degree. In the MSDI method, by varying the stability degree within the range $J > 0$, different sets of tuning parameters for the corresponding controller can be obtained. These parameter sets result in varying performance characteristics for the automatic control system, such as settling time–t_s, rise time–t_r, and overshoot–σ.

3.2 The PID Control Algorithm

For the unstable control system described by transfer function (1), it is proposed to use a PID control algorithm, which allows for shifting the positive pole to the left half of the complex plane, thereby ensuring system stability. The PID control algorithm is described by the following transfer function:

$$H_R(\mathrm{s}) = k_p + \frac{k_i}{s} + k_d s, \tag{28}$$

where k_p–proportional tuning parameter, k_i–integral tuning parameter, k_d–derivative tuning parameter.

In accordance with the MSDI method, the following analytical expressions are obtained:

$$k_p = \frac{1}{k}\left(-3a_0J^2 + 2a_1J + a_2\right) + 2k_dJ, \tag{29}$$

$$k_i = \frac{1}{k}\left(a_0J^3 - a_1J^2 - a_2J\right) - k_dJ^2 + k_pJ, \tag{30}$$

$$k_d = \frac{1}{k}\left(3a_0J - a_1\right). \tag{31}$$

Next, it was proposed to synthesize a PID controller, described by the transfer function (28), for the object model described by the transfer function (14).

In accordance with the MSDI method, the analytical expressions for calculating the tuning parameters are obtained as follows:

$$k_p = \frac{e^{-\tau J}}{k}\left(\tau a_0J^3 - (\tau a_1 + 3a_0)J^2 + (2a_1 - \tau a_2)J + +a_2\right) + 2k_dJ, \tag{32}$$

$$k_i = \frac{e^{-\tau J}}{k}\left(a_0J^3 - a_1J^2 - a_2J\right) - k_dJ^2 + k_pJ, \tag{33}$$

$$k_d = \frac{e^{-\tau J}}{2k}\left(\tau^2a_0J^3 - \left(\tau^2a_1 + 6\tau a_0\right)J^2 + (4a_1\tau - \tau^2a_2 + 6a_0)J - (2a_1 - \tau a_2)\right). \tag{34}$$

4 Algorithm for Control-Relevant Identification

Based on the proposed approach for closed-loop identification, an algorithm for control-relevant identification is presented, consisting of the following steps:

1. Control system design with P controller.
2. Achievement of the underdamped step response by varying the proportional tuning parameter.
3. From the obtained step-response determination of period of oscillations–T_0, time delay–τ, damping ratio–ξ.
4. Determination the value of natural frequency according to Eq. (7).
5. Choosing the object model for approximating the unstable process and calculating the parameters of the transfer function according to Table 1.
6. Tuning the PID controller in conformity with expressions (29)–(31), or (32)–(34) for the case of time delay.
7. Verification of the system performance and variation of the maximum stability degree value to achieve the desired performance.

Table 1: Expressions for estimating the parameters of the transfer functions

No.	Transfer function	Expressions for calculation of the parameters
1	$H(s) = \dfrac{k}{a_0 s^2 + a_1 s - a_2}$	$\begin{cases} a_0 = \dfrac{k_{cr}k - a_2}{\omega_n^2}; \\ a_1 = \dfrac{2(k_{cr}k - a_2)\xi}{\omega_n}; \\ a_2 = 1. \end{cases}$
2	$H(s) = \dfrac{ke^{-\tau s}}{a_0 s^2 + a_1 s - a_2}$	$\begin{cases} a_0 = \dfrac{kk_{cr}\cos\tau\omega_n - a_2}{\omega_n^2}; \\ a_1 = \dfrac{2\zeta\tau(kk_{cr}\cos\tau\omega_n - a_2)}{\omega_n}; \\ a_2 = 1. \end{cases}$

5 Study-Case and Computer Simulation

5.1 Control Relevant-Identification of the Unstable Inertial System

It is assumed that the unstable system is described by the following transfer function:

$$H(s) = \frac{1}{(6s-1)(2s+1)} = \frac{1}{12s^2+4s-1} = \frac{B(s)}{A(s)}. \tag{35}$$

The unstable system described by Eq. (35) was simulated in MATLAB with P controller. The proportional tuning parameter k_p was varied until an underdamped transient response of the closed-loop system was achieved for $k_p = 5$. The resulting underdamped step response is shown in Fig. 3, from which there are obtained: a period of oscillation $T_0 = 11.40$ s, and the damping ratio $\xi = 0.29$. The natural frequency was determined using Eq. (7), yielding $\omega_n = 0.57$.

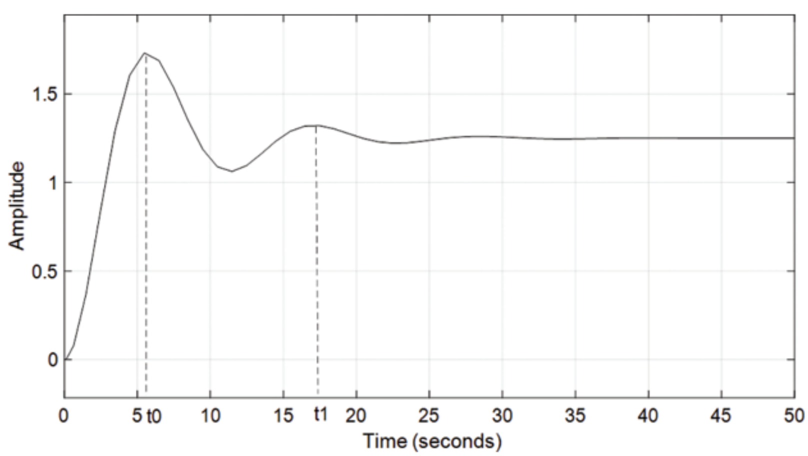

Figure 3: Underdamped step response of the closed-loop system

Using the calculated values of natural frequency, period of oscillation, and proportional transfer coefficient, the model parameters are determined based on the expressions from Table 1:

$a_0 = 12.01;\ a_1 = 4.15;\ a_2 = 1.$

In this way, the identified transfer function is as follows:

$$H\,(s) = \frac{1}{12.01s^2 + 4.15s - 1} = \frac{B(s)}{A(s)}. \tag{36}$$

For comparison of the obtained results, it was proposed to use a genetic algorithm for the experimental identification of the system in open-loop conditions, which was implemented in the MATLAB software package. The genetic algorithm is a powerful optimization technique inspired by the process of natural selection. It works by generating a population of potential solutions, each represented by a set of parameters (often encoded as a 'chromosome'). These solutions undergo selection, crossover (recombination), and mutation to create new generations of solutions. The algorithm iteratively refines the population by selecting the best-performing solutions according to a fitness function, which evaluates how well each solution matches the desired system behavior. This approach is especially valuable for unstable systems where traditional identification methods may be less effective due to the system's inherent instability and sensitivity to perturbations [33].

In Table 2, there are presented the obtained results of system identification and goodness of fit for estimated models.

Table 2: Comparison of the system identification methods

Identification method	No. of iter.	a_0	a_1	a_2	k	Fit, %
Closed-loop identification		12.01	4.15	1	1	96.06
Genetic algorithm	58	4.15	5.37	1	1	57.25
	80	9.37	4.46	1	1	87.57
	82	3.36	5.52	1	1	52.4

From Table 2, it can be observed that the best fitting of the identified model was obtained for the case of using closed-loop identification with a goodness of fit 96.06%.

Next, the PID control algorithm was tuned for the identified object model (36) using the MSDI method. Based on the analytical expressions for calculating the tuning parameters (29)–(31), the dependencies $k_p = f\,(J)$, $k_i = f\,(J)$, $k_d = f\,(J)$ were obtained, which are presented in Fig. 4.

According to these dependencies, four sets of tuning parameters were selected and are presented in Table 3. The results were compared with those obtained using the genetic algorithm (GA) and parametric optimization (PO) from MATLAB. The results of tuning the PID controller and the system performance for each case (t_r–rise time, s; t_s–settling time, s; σ–overshoot, %) are given in Table 3.

Next, a computer simulation of the automatic control system with the PID controller tuned by the MSDI method, genetic algorithm, and parametric optimization was performed. The obtained transient responses are presented in Fig. 5, where the curve numbering corresponds to the numbering from Table 2.

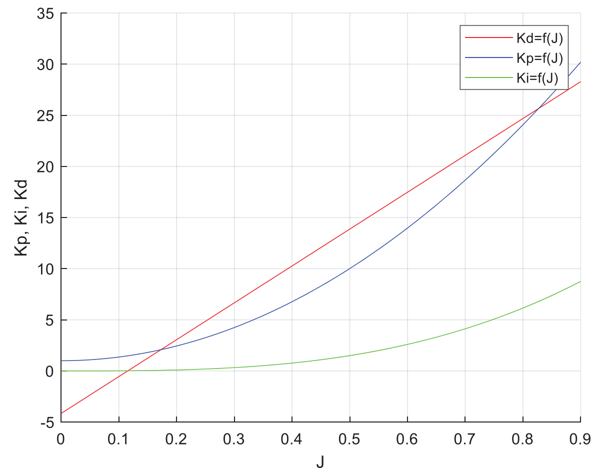

Figure 4: Dependences $k_p = f(J)$, $k_i = f(J)$, $k_d = f(J)$

Table 3: Tuning parameters and system performance

No.	Method	J	k_p	k_i	k_d	t_s, s	t_r, s	σ, %
1	MSDI	0.50	8.01	1.50	13.80	11.30	1.25	24.20
2	MSDI	0.80	22.06	6.15	24.68	5.75	0.67	18.90
3	MSDI	1.30	59.91	26.30	42.70	2.87	0.37	18.60
4	MSDI	1.90	129.10	82.40	64.30	1.79	0.23	19
5	PO		12.58	0.98	18.0	20.90	0.87	15.50
6	GA		37.56	4.51	41.80	7.17	0.45	10.20

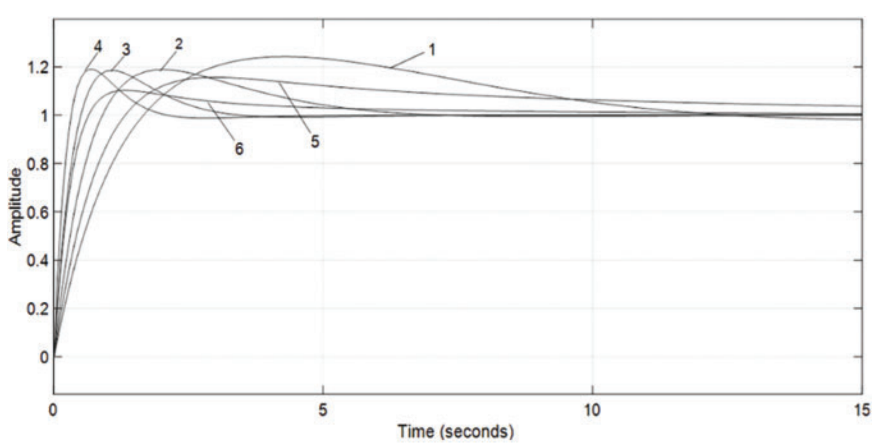

Figure 5: Transient responses of the automatic control system

From the transient responses of the automatic control system, it was observed that the best performance was achieved with the PID controller tuned using the MSDI method (Curve 4, Fig. 5). Fig. 6 presents the pole-zero map of the closed-loop system with PID controllers tuned by the MSDI method, genetic algorithm, and parametric optimization. Fig. 7 shows the Bode diagram.

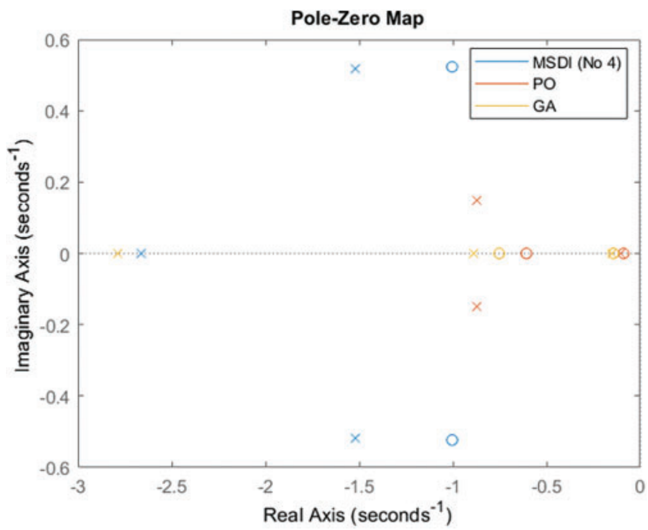

Figure 6: Pole-zero map

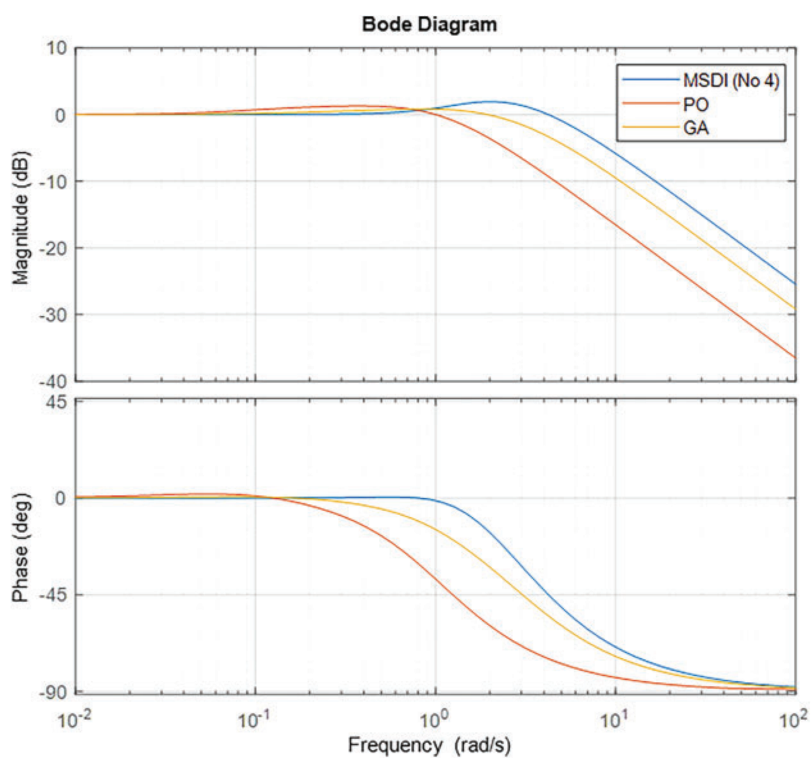

Figure 7: Bode diagram

From Fig. 6, it was observed that the automatic control system with PID controller tuned by the MSDI has the highest stability degree ($\eta = -1.51$) in comparison with the genetic algorithm ($\eta = -0.87$) and parametric optimization ($\eta = -0.15$).

5.2 *Control Relevant-Identification of the Unstable Inertial System with Time Delay*

The unstable process is assumed to be described by the following transfer function:

$$H(s) = \frac{2e^{-3s}}{(10s-1)(2s+1)} = \frac{2e^{-3s}}{20s^2+8s-1} = \frac{B(s)}{A(s)}. \tag{37}$$

This system was simulated with P control algorithm and by varying the k_p tuning parameter, an underdamped transient response of the closed-loop system was achieved for $k_p = 0.8$, as shown in Fig. 8.

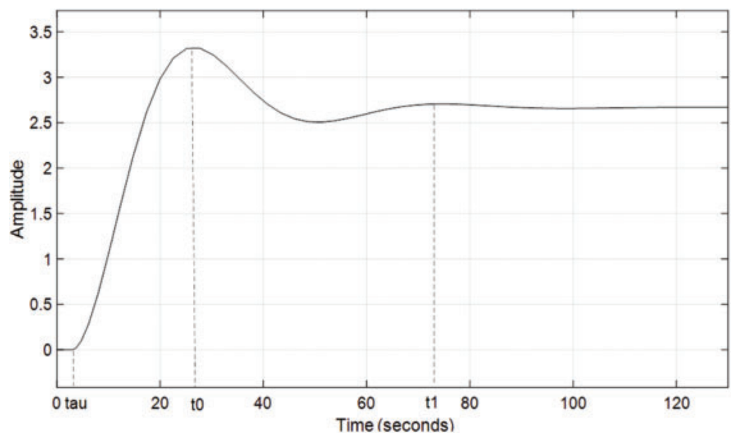

Figure 8: Underdamped step response of the closed-loop system with time delay

From Fig. 8, the following values were calculated: $T_0 = 45.47$ s and $\xi = 0.40$. Next, the natural frequency was determined to be $\omega_n = 0.15$, the time delay $\tau = 3$ and the transfer coefficient $k = 2$.

According to Table 2, and expressions (19) and (22), the transfer function that approximates the dynamics of the unstable system with time delay is as follows:

$$H(s) = \frac{2e^{-3s}}{19.13s^2+9.69s-1}. \tag{38}$$

For comparison of the identification method, a genetic algorithm was used, which provides the capability to estimate the mathematical model of the open-loop system based on the step response. The results of the system identification are presented in Table 4.

Table 4: Comparison of the system identification methods

Identification method	No. of iter.	a_0	a_1	a_2	k	τ, s	Fit, %
Closed-loop identification		19.13	9.69	1	2	3	88.70

(Continued)

Table 4 (continued)

Identification method	No. of iter.	a_0	a_1	a_2	k	τ, s	Fit, %
Genetic algorithm	50	6.47	8.53	1	2	3	31.90
	165	11.63	7.75	1	2	3	54.90
	200	8.78	8.18	1	2	3	42.50

The PID controller was tuned for the identified transfer function (38) using the MSDI method. Based on the analytical expressions (32)–(34), the dependences $k_p = f(J)$, $k_i = f(J)$ and $k_d = f(J)$ were obtained and are presented in Fig. 9.

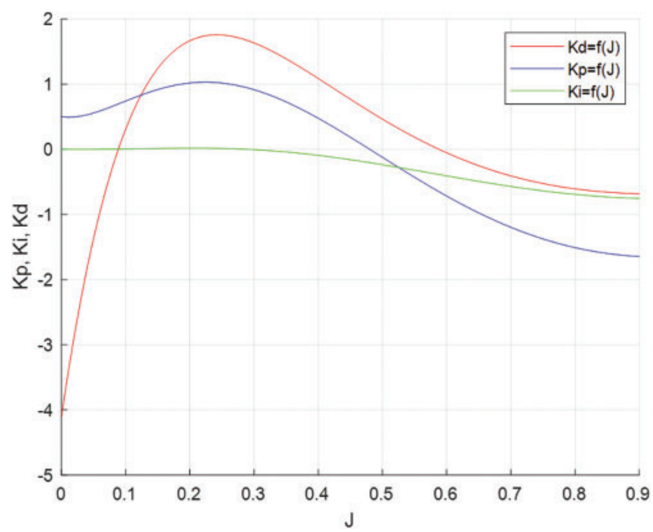

Figure 9: Dependences $k_p = f(J)$, $k_i = f(J)$, $k_d = f(J)$

Based on these dependencies, four sets of tuning parameters were chosen and are listed in Table 5. The PID controller tuning results were then compared with those obtained using genetic algorithm and parametric optimization and the obtained transient responses are presented in Fig. 10, where the numbering of curves corresponds with numbering from Table 5.

Table 5: Tuning parameters and system performance

No	Method	J	k_p	k_i	k_d	t_s, s	t_r, s	σ, %
1	MSDI	0.22	1.02	0.017	1.73	95.70	3.30	134
2	MSDI	0.18	0.98	0.016	1.54	93	3.50	142
3	MSDI	0.11	0.77	0.006	0.54	148	4.60	212
4	MSDI	0.27	0.98	0.007	1.72	235	3.40	127
5	PA		0.98	0.014	1.57	110	3.50	139
6	GA		1.60	0.086	5.46	54.80	1.50	129

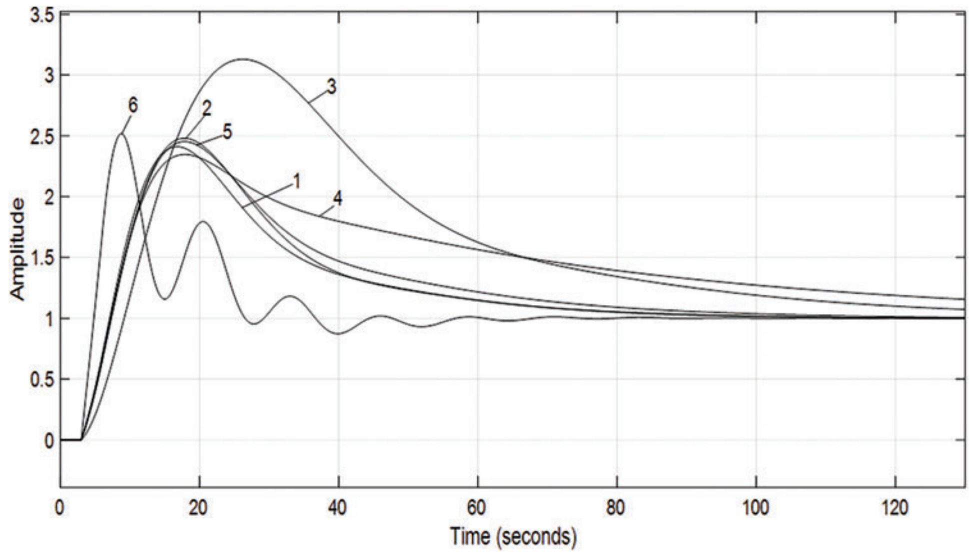

Figure 10: Transient responses of the automatic control system

From Fig. 10 and the system performance analysis, it can be concluded that the PID controller tuned using the MSDI method, genetic algorithm, and parametric optimization all ensure system stability, though with an overshoot greater than 100%. Fig. 11 presents the Bode diagram.

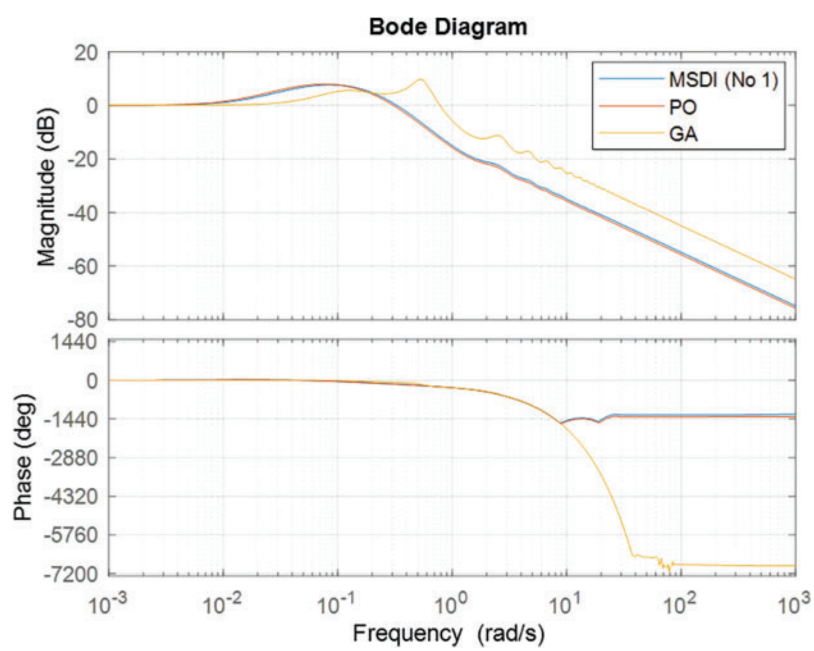

Figure 11: Bode diagram

6 Conclusions

Control-relevant identification of unstable systems is crucial in various industries and applications. This field aims to address the challenges associated with system identification in closed-loop and synthesis of the control algorithm, which is inherently difficult to realize due to the system's unstable nature and the presence of delays in their response.

This paper proposed the methodology for control-relevant identification of the inertial unstable systems with, or without time delay. The procedure for system identification is performed in a closed-loop and offers a goodness of fit of around 90%, which was verified by computer simulation and compared with a genetic algorithm that offered a goodness of fit of around 70%. The automatic control system with a PID controller tuned using the MSDI method has a stability degree that is 1.73 times higher than the system with the controller tuned using the genetic algorithm, and 10 times higher than with the PID controller tuned by the parametric optimization method.

Analyzing the performance of the automatic control system tuned by the MSDI method, it can be concluded that this method provides a high degree of stability. However, in the presence of time delays, the system's performance deteriorates, with the resulting transient responses exhibiting overshoot greater than 100%.

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