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Quasi-region-specific model uncertainties of simplified liquefaction triggering analysis

Jiun-Shiang Wang[1#], *Jianye* Ching[2], and *Yu-Jhang* Tu[1]

[1] *National Taiwan University, No. 1, Sec. 4, Roosevelt Rd., Daan Dist., Taipei City 106319, Taiwan (R.O.C.)*
[#]*Corresponding author: d11521014@ntu.edu.tw*

ABSTRACT

In earthquake-prone regions, assessing soil liquefaction potential is indispensable for contemporary seismic design. Various procedures for liquefaction triggering analysis have emerged over the past decades. However, most of them are derived from generic liquefaction databases, such that the model uncertainties in liquefaction potential assessments applied to a specific region of concern remain unknown, which poses a challenge for engineers to evaluate the liquefaction risks of target sites. This study aims to propose a hierarchical Bayesian model (HBM) to learn the inter-region characteristics of model uncertainties of the traditional simplified liquefaction potential evaluation methods based on a database containing global case histories of liquefaction categorized into several regions where those triggering events occurred. The learning outcomes can yield the model uncertainty of the target region, and the liquefaction probability at the target site under a given ground motion condition. For an illustration of the proposed model, a case history of liquefaction from a specific region is adopted to construct a quasi-region-specific model uncertainty and evaluate the liquefaction probability in the target soil. The illustration shows that the constructed quasi-region-specific model uncertainty with liquefaction histories in the target region can improve liquefaction occurrence prediction in comparison with the prediction without any histories, which is believed to benefit the engineering practice.

Keywords: soil liquefaction; model uncertainty; quasi-region-specificity; hierarchical Bayesian model.

1. Introduction

Assessing soil liquefaction potential is a cornerstone for contemporary seismic design. Diverse approaches for assessing soil liquefaction potential have been developed over the past decades (e.g., Seed and Idriss 1971; Dobry and Ladd 1980; Olsen 1997; Youd et al. 2001; Boulanger and Idriss 2016; Cetin et al. 2018; Hwang et al. 2021). Their features and applicability are also comprehensively compared and discussed (e.g., National Research Council 2016; Cetin and Bilge 2022). Among them, the stress-based simplified procedures that exploit the measurements of in situ tests, such as the blows of standard penetration tests (SPT-N), the cone resistance of piezocone penetration tests (CPT-q_c), and the shear wave velocity (V_S) for liquefaction potential evaluations, are the most prevalent in practice and accepted by seismic design codes of various countries (e.g., Architectural Institute of Japan (AIJ) 2001; American Association of State Highway and Transportation Officials (AASHTO) 2014; Japan Road Association (JRA) 2017; Ministry of Interior (MOI) of Taiwan 2022).

Although simplified procedures are prevailing in practice due to their simplicity and convenience, since they do not perform perfectly for predicting the occurrence of liquefaction, their model uncertainties become a further focus of studies in geotechnical engineering. These studies have developed the probabilistic models for these simplified procedures to quantify their prediction bias and model uncertainties (e.g., Juang et al. 2003; Cetin et al. 2004; Moss et al. 2006; Kayen et al. 2013; Boulanger and Idriss 2016; Hwang et al. 2021).

However, most of the simplified procedures and their probabilistic versions were driven based on generic liquefaction potential databases, such that their quantified model uncertainties are also generic. Namely, the model uncertainties of simplified procedures for liquefaction potential assessments applied to a specific region of concern remain unknown, which poses a challenge for engineers to evaluate the liquefaction risks of target sites.

Therefore, this study aims to propose a hierarchical Bayesian model (HBM) for two objectives. One is to learn the inter-region characteristics of model uncertainties of the traditional simplified liquefaction potential evaluation procedures. The other one is to utilize the learning outcomes to acquire the quasi-region specific model for predicting liquefaction occurrence. This model can yield the quasi-region-specific model uncertainty that incorporates the information contained in a generic database and the target region data. Accounting for quasi-region-specific model uncertainty, this model yields the liquefaction probability of the target site adapted to the concerned region under a given ground motion condition.

Developing the aforementioned HBM requires two items: a database containing global case histories of liquefaction categorized into several regions where those triggering events occurred, and a hierarchical model that can accommodate the inter-region characteristics of liquefaction triggering and an indicator to determine whether the soil liquefies. The former is introduced in

Sec. 2, and the latter is elaborated on in Sec. 3, respectively.

2. Soil liquefaction potential database

A soil liquefaction potential database is compiled in this paper. Most of the data in this database were collected from 20 studies published in scientific journals or investigation reports. The others came from Next Generation database of Liquefaction (NGL) (Ulmer et al., 2023).

This liquefaction potential database contains 2759 site-investigation records from 60 regions where historical earthquake-induced liquefaction hazards occurred, such as Niigata in Japan (Ishihara and Koga, 1981), Imperial Valley in USA (Bennett et al., 1984), Kocaeli in Turkey (PEER, 2000), Tangshan City in China (Cai et al., 2012), Canterbury in New Zealand (Green et al., 2014), and Chang-Hwa County in Taiwan (Hwang et al., 2021). Those site-investigation data cover 6 parameters that are adopted in common for liquefaction risk evaluation, including the ratio of overburden stress (σ_v/σ_v'), clean-sand normalized SPT-blows ($(N_1)_{60}$), normalized cone tip resistance (q_{t1N}), soil behavior index (I_c), normalized shear wave velocity (V_{S1}), and fines content (FC). The distribution of the dataset is shown in Fig. 1 via histograms.

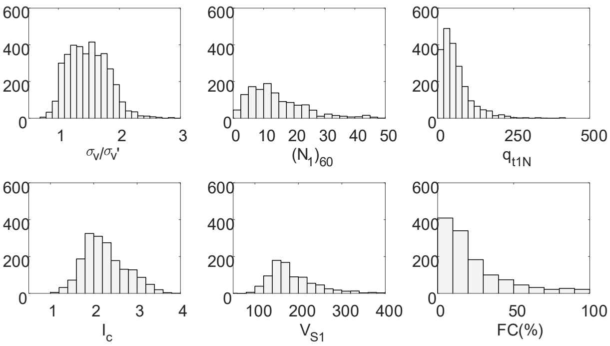

Figure 1. Histograms of soil data in liquefaction potential database.

Along with these investigation data, the corresponding liquefaction histories (i.e., the soil liquefied or not) given the event conditions (i.e., moment magnitude and peak ground acceleration), are also included in this database. This database is utilized to develop an HBM for region-specific liquefaction risk evaluation.

3. Quasi-region-specific model uncertainty of simplified liquefaction triggering analysis

3.1. Hierarchical Bayesian model

To capture the statistical uniqueness of each data group in a generic soil/rock database, a hierarchical Bayesian model (HBM) was formulated by Ching et al. (2021a). HBM can characterize the inter-group and intra-group correlation of each group of soil/rock property data. The structure of the HBM shown in Fig. 2 is a two-level hierarchical tree. The top level contains hyper-parameters (μ_0, C_0, Σ_0, υ_0) which are modeled to characterize the inter-group correlation. The second level comprises the statistical parameters of each group (the mean vector, μ_i, and the covariance matrix, C_i) in the database, which is modeled the intra-group uniqueness. The bottom level is houses the soil/rock data in which are modeled as Gaussian distributed.

$$f(x_{ij}|\mu_i, C_i) = |C_i|^{-\frac{1}{2}}(2\pi)^{-\frac{n}{2}} \exp\left[-\frac{1}{2}(x_{ij} - \mu_i)^T C_i^{-1}(x_{ij} - \mu_i)\right] \quad (1)$$

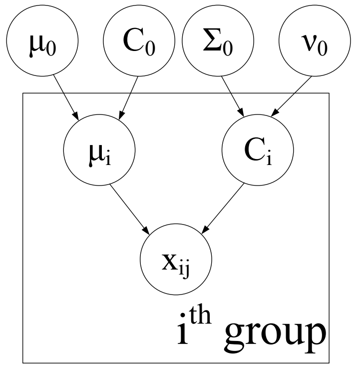

Figure 2. Model structure of HBM (modified from Ching et al., 2021a).

The procedure for performing HBM can be divided into two steps. The first step is referred to as the "learning step", and the second one is the "inference step". The former yields the hyper-parameters characterizing inter-group correlations based on a generic database, and the latter comes out the quasi-group-specific distribution of the target group inferred based on the sparse data from the target group and the hyper-parameters driven at the "learning step".

The effectiveness of constructing quasi-site-specific models along with the computational efficiency of HBM have been demonstrated via several real cases in previous studies (e.g., Ching et al., 2021a; Ching et al., 2021b; Ching et al., 2022). Its closed-form probability structure is attributed to that computational efficiency, such that it become a desirable candidate for the issue of this paper concerns. However, HBM cannot accommodate categorical parameters (e.g., liquefied soils or non-liquefied soils), such that it cannot learn the data group uniqueness of liquefaction triggering in our liquefaction potential database for driving a liquefaction triggering analysis model. As it transpired, HBM should be modified for liquefaction triggering analysis.

3.2. HBM for liquefaction triggering analysis

This paper proposes a modified HBM considering a "group" as a "region". This modified HBM is called as "GR-Liq-HBM". Its model structure is shown in Fig. 3. The main modifications can be enumerated below:

1. A model factor (denoted by m) modelled log-normally distributed is introduced into GR-Liq-HBM. It is defined as follows:

$$m = \frac{FS_a}{FS_n} \quad (2)$$

where FS_a is the actual value of anti-liquefaction factor of safety (FS) which indicates the soil actually liquefies or not: FS_a < 1 indicates the soil liquefies whereas $FS_a \geq 1$ points out the soil does not liquefy. FS_n is the nominal *FS* computed via simplified procedures of liquefaction triggering analysis (e.g., Youd et al., 2001; Boulanger and Idriss, 2016).

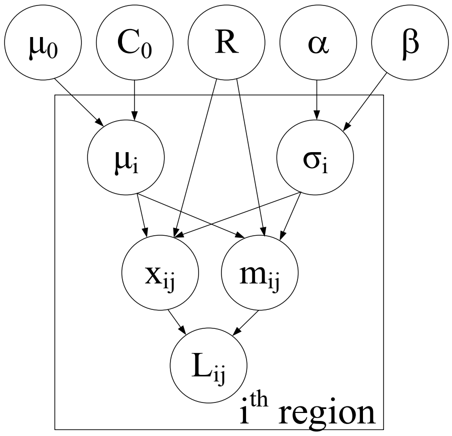

Figure 3. Model structure of GR-Liq-HBM.

2. An indicator (L) is also introduced in GR-Liq-HBM. L can be derived from the nominal *FS* incorporated with the model factor (m). L = 1 as the soil liquefies, whereas L = 0 as the soil does not liquefy.
3. The region-specific variance of soil properties (x_{ij}) and the model factor (m_{ij}) are modelled separately in GR-Liq-HBM. It is worth noting that the values of model factors cannot be observed in reality, whereas the soil properties can be acquired via site investigation, such that the uncertainties of soil properties (x_{ij}) and the model factor (m_{ij}) can be significantly different, which cannot be considered by the original HBM. This is because the original HBM models the variance of soil properties (x_{ij}) and the model factor (m_{ij}) in one covariance matrix which is governed by one parameter υ_0.
4. Due to the independence of modelling the variance of soil properties (x_{ij}) and the model factor (m_{ij}), the region-specific covariance matrix should be modified as follows:

$$C_i = S_i R S_i = \begin{bmatrix} \sigma_{i,1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{i,m} \end{bmatrix} \begin{bmatrix} 1 & \cdots & \rho_{1m} \\ \vdots & \ddots & \vdots \\ \rho_{m1} & \cdots & 1 \end{bmatrix} \begin{bmatrix} \sigma_{i,1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{i,m} \end{bmatrix} \quad (3)$$

where S_i is a diagonal matrix composed by the i^{th} region-specific standard deviation of each uncertain variables of x_{ij} and m_{ij}; R is the correlation matrix of x_{ij} and m_{ij}. Their conditional probability structures are elaborated on below.

A common choice of the prior distribution for the Bayesian updating for the variance of a Gaussian distributed variable is inverse-gamma distribution (Gelman et al. 2014). Thus, this paper modelled the prior distribution of the region-specific variance of x_{ij} and m_{ij} are inverse-gamma distributed:

$$\sigma_{i,k}^2 \sim \mathrm{IG}(\alpha_k, \beta_k) \quad (4)$$

where $\sigma^2_{i,k}$ is the k^{th} uncertain variable in the i^{th} region of soil data; IG(.) is the inverse-gamma PDF; α_k and β_k are the shape parameters and the scale parameters of inverse-gamma distributed variances respectively.

α_k and β_k are hyper-parameters but not group-specific parameters. They also need prior settings for Bayesian inference:

- prior for α_k: For the inference of the shape parameter of an inverse-gamma distribution, there is not a conjugate prior distribution. Besides, the shape parameter cannot be negative due to the properties of the inverse-gamma distribution. Thus, this study adopts a uniform distribution, [-10^4, 10^4], as the prior of each ln(α_k). That is, ln(α_k) ~ U([-10^4, 10^4]).
- prior for β_k: For the inference of the scale parameter of an inverse-gamma distribution, the conjugate prior is the gamma distribution (Gelman et al. 2014). Thus, this study adopts the gamma distribution as the prior for inferencing βk. For fair non-informativity of prior, the parameters for this prior distribution are set as $\alpha_\beta = 0.5$, $\beta_\beta = 10^4$. Namely, ln(β_k) ~ Gamma(0.5,10^4).

On the other hand, the correlation matrix of x_{ij} and m_{ij}, R, is modelled as an inter-region parameter in this paper. That is, R denotes a correlation matrix universally applicable across all regions in the database.

For inferencing R, it is also necessary to select a probabilistic distribution as a prior, while this selection is not quite intuitive. It should be noted that R is a special matrix: it should be symmetric, its diagonal entries should be one, and its off-diagonal elements should be bounded within [-1, 1]. Fortunately, have proposed a distribution to model the uncertainty of correlation matrix, which is so-called LKJ distribution (Lewandowski et al. 2009):

$$\mathrm{R} \sim \mathrm{LKJ}(\eta) \quad (5)$$

where η is a positive scalar parameter which tunes the strength of the correlations. If η = 1, the density is uniform over all correlation entries. For non-informativity of a prior, this study adopts η = 1 as the parameter of LKJ distribution to infer the correlation matrix, R.

3.3. Quasi-region-specific model uncertainty

Based on the model settings mentioned above, the quasi-region-specific model uncertainty of a simplified liquefaction triggering analysis procedure applied to the region of concerns can be obtained by two steps. This two-step procedure is shown in Fig. 4.

The first step (Step 1) is to learn the inter-region characteristics from the soil liquefaction potential database. In this paper, the learning outcomes of GR-Liq-HBM are the posterior PDF of the hyper-parameters f(μ_0, C_0, α, β, R|D) along with its Gibbs samples, where D is the liquefaction potential database. For the consistency of Gibbs sampling, the burn-in period is set as 1000, the interval of sampling is 10, and the total iterations is 20,000, such that the number of total Gibbs samples of the hyper-parameters comes out to be 2,000. These samples stand for the "prior" information for the next step.

The second step (Step 2) aims to update the learning outcomes from the Step 1 to infer the quasi-region-specific distribution of the model factor conditioning three information: the hyper-parameters learning from the global liquefaction potential database, the investigation data from the target site, and the liquefaction histories of the target region. The first two information is required for this current step, while the third one is optional but beneficial to improvement for prediction of liquefaction occurrence, which is demonstrated in Sec. 4.

The first piece of required information for the Step 2 can be obtained from the Step 1 by this study, such that the project engineers (i.e., the users) do not put their efforts into acquiring it. On the other hand, the site-investigation data of the site where the project focus on, the require information of the Step2, should be investigated by project engineers (i.e., the users).

Step 2 yields a posterior PDF of the quasi-region-specific model uncertainty and its Gibbs samples as well:

$$f[\mu_m, \sigma_m^2 | \Theta, D, D_R, D_s] \propto f(\Theta|D) f(\mu_m, \sigma_m^2|\Theta) f(D_R, D_s|\mu_m, \sigma_m^2) \quad (6)$$

where $\Theta = \{\mu_0, C_0, \alpha, \beta, R\}$; D_R denotes the data of the liquefaction histories of the target region; D_s denotes the investigation data of the target site; μ_m and σ^2_m are the mean value and the variance of the quasi-region-specific model factors, respectively.

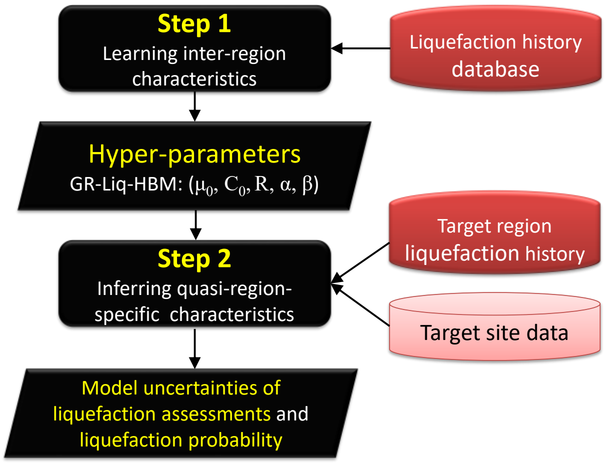

Figure 4. Two-step procedure of GR-Liq-HBM for inferring quasi-region-specific model uncertainty.

4. Application to a real case

This section approaches to demonstrate how the proposed model performed in inferring the quasi-region-specific model uncertainty of simplified procedures and to yield the liquefaction probability of the target site.

A site located in North Kaiapoi, Canterbury, New Zealand is indicated to be a liquefied site given the shock of the 2011/2/22 Canterbury earthquake (Shen et al. 2016). For demonstration and validation, let us consider this liquefaction information is unknown, and a construction project aims to evaluate the liquefaction potential of this site. For this evaluation, the project engineers need at least two pieces of information: the site-investigation data, and the ground motion condition of concern.

For the former, the investigation report shows that the critical liquefiable layer of that site is located at 3.5-4.5m in depth, with a median depth of 4.0m. The report also presents CPT data of that stratum: the average cone resistance $q_c \approx 5.55$MPa, the average sleeve friction $f_s \approx 0.028$MPa, and the soil behavior index $I_c \approx 1.83$ (Shen et al. 2016). For the latter, on the other hand, let us consider the scenario the 2011/2/22 Canterbury earthquake occurs. In accordance with the motion record (Shen et al. 2016), the peak ground acceleration ≈ 0.18g, and the moment magnitude ≈ 6.3.

Based on the above information, the nominal factor of safety (FS_n) can be evaluated via a CPT-based simplified procedure (Juang et al. 2003) ≈ 1.45, which says that the stratum of concerns does not liquefy. Note that in fact, it was liquefying given the shock of the 2011/2/22 Canterbury earthquake, but in this illustration this liquefaction information is set unknown, such that the model uncertainty of the liquefaction potential assessment should be further inferred.

4.1. Inference of quasi-region-specific model uncertainty

A project engineer can derive the quasi-region-specific model uncertainty of the concerned region by performing the Step 2 of the proposed model, if the Step 1 is finished by this study. To infer the concerned model uncertainty, the project engineer can perform the Step 2 in either two options: conditioning liquefaction history data of other sites in this region, or conditioning no liquefaction information. The former information can be found in the liquefaction potential database compiled in this paper.

Those two conditioned inference results of model uncertainty via the proposed model along with the Gibbs sampler are shown in Fig. 5: where Fig. 5(a) shows the inference of statistical parameters of model uncertainty of the target region, and Fig. 5(b) shows the inference of model uncertainty of the liquefaction potential evaluation of the target stratum.

In accordance with the inference results in Fig. 5(a), the posterior mean of the quasi-region-specific model factor conditioned on the regional liquefaction histories is evidently lower not only than the one conditioned on no regional liquefaction histories, but also than zero, which points out that the bias of the liquefaction potential evaluation in this target region exists. These inference results yield the posterior samples of the model factor of the target stratum, which is shown in Fig. 5(b).

4.2. Liquefaction occurrence prediction

The posterior samples of the model factor of the target stratum can yield the liquefaction probability of the layer:

$$P_L \approx \frac{1}{N}\sum_{i=1}^{N} \mathbb{1}(m_i \times FS_n < 1) \quad (7)$$

where P_L stands for the liquefaction probability; N is the number of total posterior samples of model factor; m_i is the inferred model factor via the proposed model; $\mathbb{1}(.)$ is the indicator function.

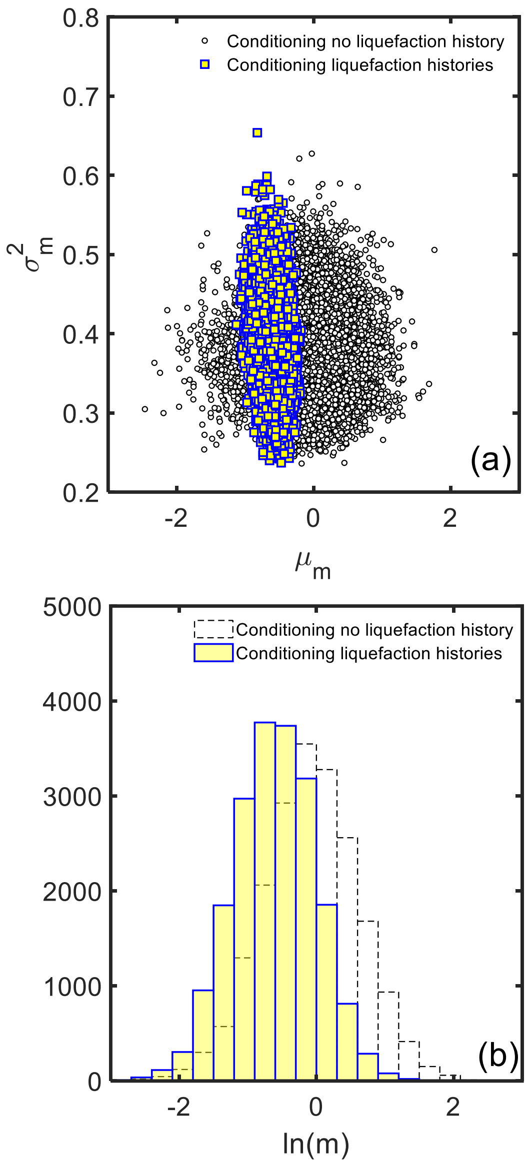

Figure 5. Quasi-region-specific model uncertainty

The liquefaction probability is adopted to predict the occurrence of liquefaction: $P_L \geq 0.5$ indicates that the target stratum liquefies, while $P_L < 0.5$ indicates that the target stratum does not liquefy.

The figures of liquefaction probability estimated via traditional model (Juang et al. 2003) and GR-Liq-HBM are enumerated in Table 1. Among these predictions, only the one by the GR-Liq-HBM with regional liquefaction history for the target stratum is correct, while the others by the traditional model (Juang et al. 2003) and the GR-Liq-HBM without regional liquefaction history are misleading.

For further comparison, this paper derives a "generic" probabilistic model for liquefaction triggering analysis based on the liquefaction potential database compiled in this study. This generic probabilistic model contains a generic model factor characterized by the improved transitional Markov chain Monte Carlo (TMCMC) approach (Ching and Wang 2016). The posterior generic model factor also can yield the liquefaction probability of the target stratum using Eq. (7). The estimation is shown in Table 1. However, its prediction of liquefaction occurrence is also misleading.

Table 1. The liquefaction probability in the target stratum via four models.

Model	P_L
Generic probabilistic model (Juang et al. 2003)	0.016
Generic probabilistic model (Derived based on our database)	0.292
GR-Liq-HBM (Without regional liquefaction history)	0.326
GR-Liq-HBM (With regional liquefaction history)	0.642

5. Conclusions

It is believed that taking the liquefaction histories of vicinity into account when determining the liquefaction potential of the target site is quite intuitive for engineers in practice. However, engineers rely on their empiricism to evaluate the liquefaction potential of sites they concern due to the absence of the approaches that incorporate regional liquefaction histories and the target site investigation data. As the era of digital transformation approaches, data-centric measures should be gradually developed to address this issue.

This paper aims to propose a hierarchical Bayesian model, GR-Liq-HBM, for data-centric liquefaction triggering analysis. The model characterizes the inter-region and the intra-region correlations between the soil properties and the model uncertainties of the simplified procedures for liquefaction triggering analysis. The characterizing outcomes can assist engineers yielding the model uncertainty of the target region adaptively, and the liquefaction probability of the target site under a given ground motion condition. Also, the model can accommodate the regional liquefaction history data and the target site investigation data to infer the quasi-region-specific model uncertainty and the liquefaction probability of the target site.

A real case in New Zealand is adopted to illustrate how to perform GR-Liq-HBM on a target site to acquire quasi-region-specific model uncertainty and the liquefaction probability of the target site. The illustration shows that GR-Liq-HBM, along with the regional liquefaction history data, yields an improved prediction of liquefaction occurrence, which is believed to benefit the site characterization work in practice.

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