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FUZZY STATISTICS-AIDED INFERENCE IN EXPERIMENTAL DESIGN

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1 INTRODUCTION

Conducting research based on active influence on the examined object or process requires distinguishing an explained quantity, measured quantitatively, the possible changes of which will be considered as influencing it through a group of quantities considered as explanatory quantities. This approach implicitly postulates the existence of a cause-and-effect relationship between the explanatory quantities and the explained quantity. In practice, especially industrial practice, explanatory quantities are often called controlled factors.

Knowledge of possible cause-and-effect relationships can be graded, from the most comfortable situation of the existence of appropriate binding equations and their exact solutions, through the existence of binding equations but without knowing the exact solutions, to the absence of such equations. While in the first case, experimental research serves to refine the results originally calculated for idealized models, in the second case, it is a necessary stage of identifying the parameters of the postulated model, and in the third case, it is a necessary stage of collecting data for which the simplest possible forecasting model will be constructed.

Carrying out experimental research is almost always an expensive and time-consuming endeavor. Available resources are limited, and cost minimization is essential. Design of experiments (DOE) is a methodology known since Fisher [1] for optimizing the design of multifactorial experimental studies, later extended by Yates to factorial designs [2], by Box and Wilson [3] to response surface designs, and by Scheffe to for mixtures [4]. Kiefer and Wolfowitz's formalization of the DOE concept [5] enabled the development of formal techniques, including optimal plans.

DOE has found wide application not only in experimental scientific research but also in industry, where it has been, among others, integrated into Six Sigma [6] or Red-X [7] procedures, allowing for the optimization of production processes in the steel industry [8], automotive [9, 10], machinery [11, 12], railway industry [13], as well as in the area of industrial materials engineering [14], armaments [16] and even phytochemistry [17].

3.1 Non-Classical Statistical Approaches

Despite all these successes, there were also quite strong limitations resulting from the postulation of simple forecasting models: factor models with low-order interactions [6] and polynomial RSM models of at most second order [17]. An additional limitation results from using stochastic approximation in the context of a small number of experimental samples. Probabilistic, as axiomatized by Kolmogorov, considers abstract probabilities, and their relationship to the real world is defined as limited compliance with large sample frequencies, which, in a sense, refers to the abandoned concepts of von Mises' empirical frequentist probabilism.

Unfortunately, in practice, experimental trials are rarely numerous. The real world, unlike virtual experiments, generates high or very high costs. For this reason, statistical assessments are burdened with large uncertainties, and in the case of individual samples, they lose their meaning altogether. Work on an alternative approach to individual events was conducted at one time by de Finetti [18], but after his death, they were not continued. Zadeh [19] proposed single fuzzy estimates as substitutes for subjective probability. His concept was consistently deepened when the fuzzy assessment of membership in a fuzzy set was introduced, thus creating type II fuzzy sets [20] and then inductively extending to type III sets, etc. Atanassov [21] developed Zadeh's concept to form two assessments, creating intuitionistic fuzzy sets. In turn, Pedrycz, wanting to avoid the need to specify membership too precisely, introduced shaded sets [22].

Buckley [23] developed a method of operationally combining classical statistics and the fuzzy approach, consistently developing subsequent operational elements of the modified formalism. Grzegorzewski [24] proposed a complete ontology and taxonomy describing hypothesis testing in the context of decision theory. Considering three components: data, hypotheses, and requirements, he identified eight different possibilities for combining fuzzy and non-fuzzy elements. One of them is traditional statistics, which have all three non-fuzzy components.

The development of fuzzy set operators (sum S and product T) led to an explosion of proposed operational variants of these operations, each satisfying the formal axioms. Pietraszek [26] proposed the interpretation of this ambiguity as resulting from the mutual correlation of fuzzy variables, at the same time providing an operational method of determining appropriate correlations and operators.

1.2 Electro-Spark Machining

In the era of globalization, companies are fighting for a competitive advantage both by introducing new products and by improving the quality, especially the reliability and durability of existing products. One way to improve durability and reliability is to improve the coatings of components that are exposed to corrosive agents or to heavy traffic loads, leading to excessive wear. In both cases, special protective coatings are used. In aggressive environments, paint coatings or metallic anti-corrosion layers are used. When exposed to excessive wear, surface layers with increased wear resistance are used. They are applied using various techniques, including beam techniques using a concentrated energy stream: electro-spark deposition (ESD), electro-spark alloying (ESA), pulse electrode surfacing (PES), and electro-discharge machining (EDM). During the process of applying coatings using electro-spark machining, the following physical phenomena occur: increase in electric field intensity as the electrodes get closer, electrical breakdown, gas ionization in the gap, formation of a plasma channel, light and thermal radiation and evaporation, short circuit of the electrodes, mechanical impact of the electrodes, erosion of the cathode and anode, material transfer, coating formation, diffusion and finally solidification. Electro-spark coatings are most often made of refractory metals (tungsten, molybdenum, chromium, titanium, zinc, tin, lead, cadmium), intermetallic compounds (including NiAl, Ni3Al, TiAl, Al3Ti, Ni3Ti), non-metals (graphite, oxides), carbides (tungsten, cobalt, molybdenum, silicon, boron) and borides (titanium or chromium). These coatings enable beneficial changes in the surface properties of the coated material: mechanical, electrical, thermal, and physical. The disadvantage, however, is the very high final roughness of the obtained coatings. This is compensated by the simplicity of the method and its low cost. Examples of use include the production of hard layers on the tips of turbine blades [26], increasing the mechanical properties of aluminum castings [27], or increasing resistance to biocorrosion [28].

Laser processing is used to improve the properties of electrodeposited coatings. Using a laser beam to smooth electrospark coatings reduces surface roughness and changes the shape of the unevenness profile. The porosity of the coating is reduced, and scratches, delaminations, and cracks on the coating surface are eliminated. For smoothing, low power densities and large laser beam diameters are recommended to melt the layer to a small depth. As a result of the laser modification of the surface layer, two zones appear: the remelting zone, with an increased concentration of the alloying element, and the heat-affected zone, with the same chemical composition as the substrate but a changed structure.

2 MATERIALS AND METHODS

2.1 Electrodes and laser

The electrodes used in the process of applying electric spark coatings were made from a mixture of elementary Co nanopowders and WC and Al2O3 ceramic nanopowders. The basic properties of nanopowders are presented in Table 1.

Table 1: Properties of nanopowders used to produce electrodes

Powder	Grade	Particle size	Producer
Co	Extrafine	0.4 um	Umicore (Belgium)
WC	Superfine	0.2 um	OMG (USA)
Al2O3	I317NH	0.15 um	Skyspring Nanomaterials Inc. (USA)

WC-Co-Al_2O_3 electrodes (84% WC, 6% Co, 10% Al_2O_3) in the shape of a cylinder with dimensions of 5x10 mm were manufactured using the pulsed plasma sintering (PPS) method. Sintering was carried out in a press furnace (Idea) at a temperature of 1100ºC and a pressure of 50 MPa. The coatings were applied to C45 carbon steel samples using the EIL-8A device (Portable EIL-8A electro-spark deposition facility, TRIZ, Ukraine). Laser modification of the surface layers was performed with a Nd:YAG laser (Baasel Lasertechnik 720 Nd:YAG) run in pulsed mode at a power level of 20 W, spot size of 0.7 mm, pulse duration of 0.4 ms, and pulse repetition frequency of 50 Hz. The T-01M pin-on-disc tribometer was employed to determine the dry friction behavior of the WC-Co-Al_2O_3 coatings. A ball-on-flat contact geometry was chosen to measure the friction force between a 100Cr6 grade steel ball, 6.3 mm in diameter, and the tested coating.

2.2 Design of Experiment

The observed (explained) quantity was the roughness parameter *Ra*, and the controlled quantities were laser power *P* and scanning speed *V*. The tests were carried out using a central composition plan (Box-Wilson) with values consistent with Table 2. For the classic DOE analysis, a full quadratic model for two factors with a second-order interaction (P V PP VV PV) was adopted.

Table 2: Design of experiment – Box-Wilson central composite, two factors

No	Power P (W)	Scanning speed V (mm/min)
1	17	220
2	23	220
3	17	285
4	23	285
5	16	250
6	25	250
7	20	208
8	20	300
9	20	250
10	20	250
11	20	250
12	20	250
13	20	250

3 CLASSIC RESULTS

After carrying out tribological tests, the following roughness coefficient values were obtained (Table 3).

Table 3: Obtained Ra roughness parameter

Treatment Number	Ra
1	10.16
2	8.27
3	7.86
4	8.55
5	6.39
6	9.42
7	5.64
8	4.77
9	7.96
10	6.03
11	6.89
12	7.21
13	8.13

After performing a classic regression analysis and eliminating statistically insignificant components, a model consisting only of a constant component and a quadratic component dependent on power was finally obtained – for coded values (Table 4).

Table 4: Coefficients of the reduced model (coded values)

Treatment Number	Coefficient	Std. dev	-95% CI	+95% Ci
const	6.89	0.30	6.05	7.72
PP factor	0.90	0.28	0.12	1.68

The reduced model expressed in uncoded quantities takes the form:

$$Ra = 5.90 + 3.848 \cdot 10^{-3} \cdot P^2 \tag{1}$$

Analyzing the variance of the reduced model made it possible to assess the significance of the lack of fit. The lack of fit turned out to be insignificant, which means that the model inaccuracies (residual values) are insignificant compared to the value of the pure error calculated from the repetitions (Table 5).

Table 5: ANOVA of the reduced coded model

Term	SS	df	MS	F	p
PP	7.53	1	7.53	10.40	0.032
Lack-of-fit	17.52	7	2.50	3.45	0.124
Pure error	2.90	4	0.723		
Total	27.95	12			

Calculating the predicted values using the obtained model allowed the determination of residual values (differences between measured and predicted values) – see Table 6.

Table 6: Measured, predicted, and residual values

No	Measured	Predicted	Residuals
1	10.16	7.02	3.14
2	8.27	7.94	0.33
3	7.86	7.02	0.84
4	8.55	7.94	0.61
5	6.39	6.89	-0.50
6	9.42	8.31	1.11
7	5.64	7.45	-1.81
8	4.77	7.45	-2.68
9	7.96	7.45	0.51
10	6.03	7.45	-1.42
11	6.89	7.45	-0.56
12	7.21	7.45	-0.24
13	8.13	7.45	0.68

According to the assumptions of the model, the distribution of residual values should be consistent with the normal distribution. The Shapiro-Wilk normality test was performed, and for the SW-W statistic value = 0.96, a critical level of p = 0.77 was obtained.

4 ANALYSIS

Variance analysis was chosen as an example of fuzzy interpretation. The null hypothesis in the analysis of variance is that the F statistic assumes a zero value, i.e., H_0: $F = 0$. The distribution of the F statistic is additionally parameterized by the degrees of freedom of the numerator, i.e., the tested source of variability, and the denominator, i.e., unexplained variability. In the case under consideration, the source of variability is the quadratic component related to the laser power with a degree of freedom $f_1 = 1$ and a statistic value $F = 10.40$. Unexplained variability has a degree of freedom $f_2 = 4$. Buckley's approach requires treating the statistical significance of β, i.e., the complement of the confidence interval (1 – α), as a membership value in the sense of fuzzy sets. Because the F distribution is one-sided, this means that the fuzzy description of the F statistic has the form:

$$m(x) = 1 - CDF(x, f1, f2) \qquad (2)$$

where CDF is the cumulative distribution function of the F distribution. If, additionally, limiting the support is applied only for values of m(x) greater than the assumed α significance level (alpha-cut), then the issue of accepting/rejecting the hypothesis comes down to determining whether the obtained empirical value of the F statistic falls within in support of such specific fuzzy zero value – see Fig.1.

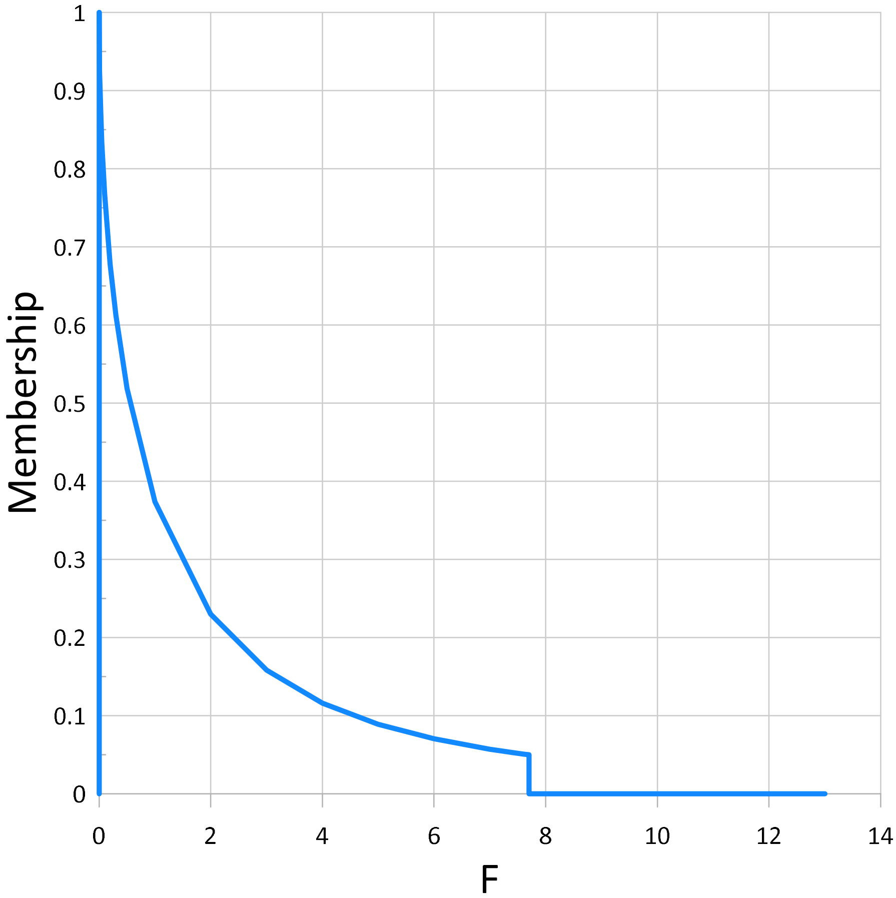

Figure 1: A fuzzy representation of the zero-value present in the null hypothesis of ANOVA i.e. H_0: F = 0 with $f_1 = 1$ and $f_2 = 4$

This way of representing the hypothesis allows you to express in a more transparent way than usual the fact of accepting or rejecting the null hypothesis: the calculated value of the statistic either falls within the range of the fuzzy zero support of the hypothesis and then the hypothesis is not rejected, or it does not fit, and then the hypothesis is rejected. In the actual sense, the state of inference is identical, but the transparency, also didactic, is much greater than in the situation when one has to explain quite complicated relationships between the α significance level and the critical level of the *p* test.

Additionally, the gradually decreasing membership of the fuzzy zero clearly shows the weakening position of the null hypothesis, which is finally rejected after reaching the rather arbitrarily selected alpha-cut position. In most cases, people who use statistics in practice do not realize that the acceptance/rejection of a hypothesis is not strictly one-and-done but is a process of gradual change of belief from one judgment to the opposite.

5 CONCLUSIONS

A designed experiment was carried out in accordance with the DOE methodology, consisting of laser processing of a special coating prepared using the ESD method. A forecasting model was identified that allows predicting the value of the surface roughness parameter Ra based on the knowledge of the laser power. The scanning speed turned out to be statistically insignificant and was not included in the set of model parameters. One of the elements of the diagnostic assessment of the obtained model was the ANOVA analysis of variance. The traditional way of interpreting the analysis is based on comparing the critical value of the p-test with an arbitrarily adopted significance level of alpha. The authors proposed using Buckley's approach to change how the null hypothesis is interpreted. The zero value appearing in the ANOVA null hypothesis is interpreted as a fuzzy number trimmed according to alpha-cut, and the obtained statistic value is compared. If the value of the statistic does not fall within the range of the fuzzy number support, then the null hypothesis is rejected.

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