STUDY OF THE DYNAMIC BEHAVIOR OF CELLULAR STRUCTURES FOR THE ABSORPTION OF MECHANICAL VIBRATIONS

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Key Words: Local Ressonators, Cellular Structures, Elastic Band Gaps, Vibration, FEM.

INTRODUCTION

Vibration is a prevalent issue in structural engineering, encompassing a wide array of problems that, if left unaddressed, can lead to severe consequences. These consequences can vary from causing discomfort for pedestrians traversing a perceptibly moving bridge to inducing premature fatigue in aeronautical structural components, ultimately resulting in catastrophic failures and loss of human life. Various sources can induce vibration in structural components, such as misalignment of rotating systems, seismic excitations, road loads on vehicles, and aerodynamic loads.

To address these phenomena, in addition to appropriate structural design, various mechanisms, which can operate actively or passively, are used to attenuate oscillatory effects and minimize their impact. Active systems use electronic controllers to generate a response via actuators, reducing the signal transmissibility level based on specific oscillatory signals. Passive systems, on the other hand, mainly rely on viscoelastic polymeric materials, utilizing the reduction of the natural frequency associated with their use and a characteristic phenomenon of these materials for energy dissipation, hysteresis [1].

Hysteresis is a phenomenon where mechanical deformation energy is dissipated in the form of heat. In other words, part of the energy that would be transmitted to the structure is dissipated, thereby increasing the system's damping.

Active systems are extremely efficient in their purpose, as they can isolate vibrations across a wide frequency spectrum and can be applied to structures of different magnitudes, from small and lightweight systems using piezoelectric actuators to large structures using hydraulic actuators, such as in active stabilization systems for reducing vibrations caused by seismic activities in buildings. However, they tend to be quite costly and imply an additional layer of systems, which, if not properly designed, can reduce the structure's reliability [2].

As mentioned earlier, passive systems have low costs and perform well when correctly sized; however, they are not free from side effects. Cushions, for example, significantly increase the structure's flexibility, which can cause considerable deflection when subjected to static loads. Moreover, the significant increase in damping causes the opposite of the desired effect at higher frequencies, as it increases the signal transmissibility over the system above a certain frequency, as shown in Figure 1.

An efficient and underexplored phenomenon in practical applications is elastic band gaps. These are regions in the frequency vibration spectrum where the propagation of elastic waves in periodic media is attenuated. This can be done by using, for example, local resonators, which will be the focus of exploration in the present work.

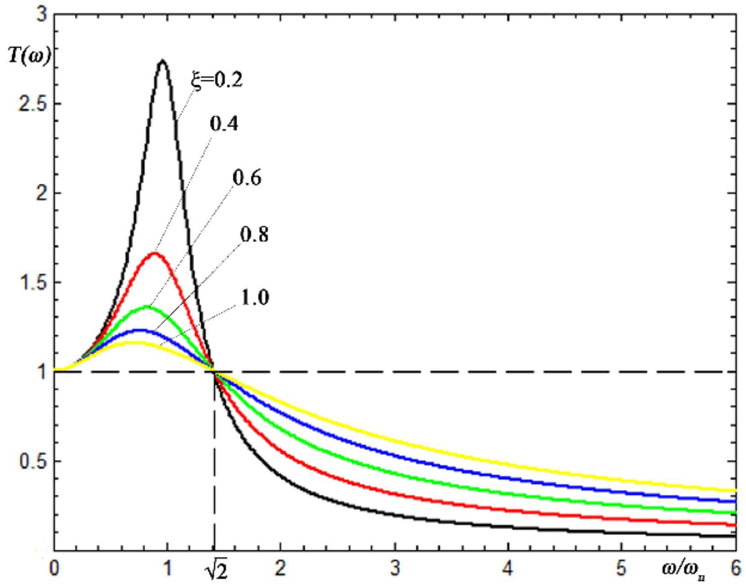

Figure 1 – Transmissibility for diferent values of damping [3].

Elastic Waves

The forces acting on a given body can be basically divided into two types: static forces and dynamic forces. In theory, all forces have a temporal variation in their application; however, when the rate of application is sufficiently low, such a load can be considered as static, a hypothesis widely used in structural engineering. Thus, dynamic load is characterized as a force applied in a very short period of time or at high frequencies, in which the rate of variation of its magnitude is considerably high [4].

As shown in Figure 2, with each increment of force during the application of a static load, represented by the forces F1 and F2, the entire system can be considered in static equilibrium, meaning that the internal forces are transmitted throughout the body immediately. This is evidenced in Figures (b) and (c), where, starting from an initial state (a), with the application of forces F1 and F2, there is no variation in the transverse deformation between sections A and B.

For dynamic loads, however, there is no instant transmission of internal deformations within the material from the point of force application. As shown in Figure 2, from the application of a dynamic force, represented by the arrow to the left of the figure, the propagation of deformations varies along the same component, which is evidenced by the difference in dimensions, and consequently, deformations, between sections A and B.

The propagation of waves within a material is then defined as the transmission of movement within it from an external excitation. Such movement will generate internal stresses and deformations in the material, and these parameters are considered to define a wave as elastic.

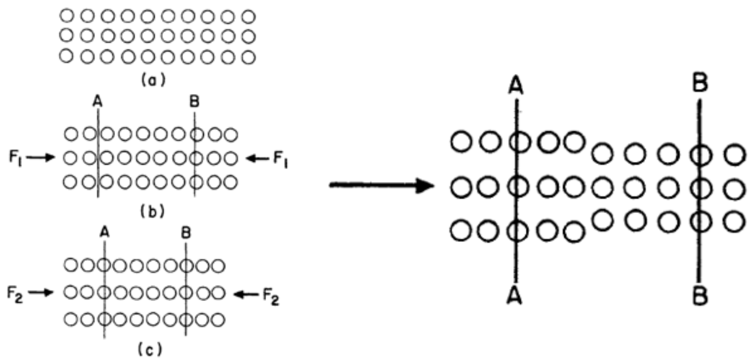

Figure 2 – Body under static (left) and dynamic (right) load [4].

Elastic waves are deformation waves within a solid that generate only stresses within the material's elastic regime; that is, after the passage of the wave, the body returns to its original state without permanent deformations.

Elastic Band Gap

The elastic band gaps are frequency ranges in which elastic waves has its propagation in a solid medium reduced or completelly forbiden [2]. This phenomenon is mechanically equivalent to the forbidden bands of electromagnetic waves and can be observed in photonic crystals, which are used, for example, in electrical insulators and extremely low-threshold lasers [5].

Where there is a variation in the wave propagation velocity, part of the incident wave will be reflected and will carry part of its energy with it. The propagation velocity of elastic waves in solid media depends basically on two factors, the material density and its stiffness. Therefore, whether through metamaterials, which are formed by two or more other materials, or cellular structures, the elastic band gaps can be formed through the "Bragg Scattering" phenomenon [2].

Bragg scattering occurs when the reflected elastic waves, due to local variations in density and/or stiffness, interact constructively with each other, causing all the energy present in the incident waves, within a certain frequency range, to be reflected and unable to propagate in the medium in question. The repetitive structures that use this phenomenon to produce forbidden bands are called phononic structures [2]. In practice, for a finite periodic structure, waves are not completely prevented from passing through, which occurs only in ideal infinite periodic structures [6].

A simple and didactic example of this phenomenon can be observed in the phononic structure shown in Figure 3, developed and studied by Policarpo [7]. It is a bar composed of a variation of steel and cork along its main axis, which, from harmonic excitations in the same direction, produces the frequency response function shown in Figure 4. As shown, such an arrangement produces forbidden bands starting from 2000 Hz.

Figure 3 – Experimental setup [7].

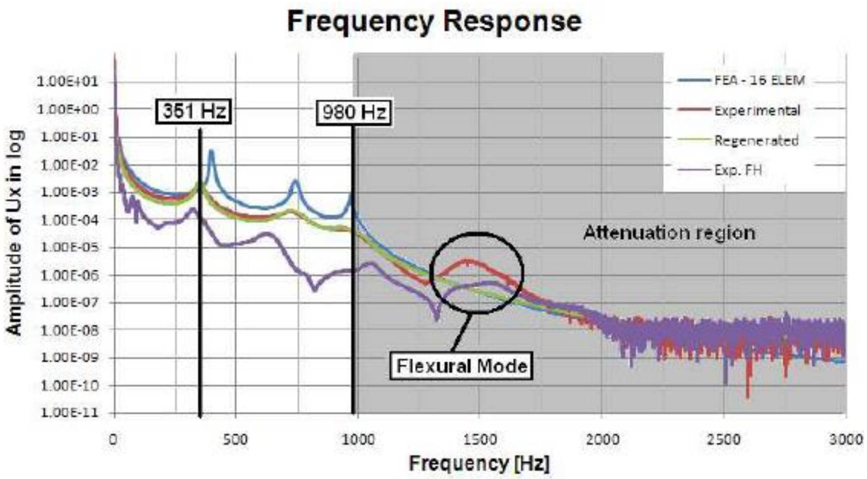

Figure 4 – Frequency response spectrum [7].

As in the previous example, most practical applications of this phenomenon revolve around vibration attenuation mechanisms. Other practical cases involving the same phenomenon, however, using bi- and three-dimensional structures can be observed in Liu et al. [5] and Wang et al. [8].

The length of the path traveled by the waves between constructive interferences, when originating from the same point, along a cellular structure, must be a multiple of the wavelength in which the forbidden passage phenomenon is desired. The path length mentioned above, in a cellular structure, is given by the length of a unit cell, meaning that such a cell must have a length equal to or multiple times greater than the desired wavelength.

This means that, to attenuate vibrations at low frequencies, usually relevant in structural engineering, very large cellular structures are required, which makes it impractical to use this phenomenon in small and medium-sized structures [2]. For example, to filter longitudinal elastic waves traveling through a steel cellular structure, with a frequency of 500 Hz, it is necessary for the structure to have cells with a characteristic length of approximately 1.85m, something completely unrealistic for most real-world applications.

Given these limitations, Liu et al. [9] was the first to propose the use of localized resonators, which allowed even cellular structures that do not have the necessary characteristic length for the desired frequency range to act as vibration attenuators at low frequencies. Since then, there has been significant advancement in the field of phononic structures, with periodic structures that utilize the local resonator mechanism being called elastic metamaterials

OBJECTIVE

The present study aims to evaluate and demonstrate the influence of localized resonators in cellular structures on the reduction and attenuation of elastic wave propagation.

METHODOLOGY

The analysis of virtual experiments were conducted using the finite element method in a commercial software, SIMCENTER®. It utilizes the NASTRAN libraries for solving static and dynamic problems, which are widely used in academia and various branches of the industry, especially in aerospace.

Arrangement of Numerical Simulations

As shown in Figure 5, a two-dimensional cantilever beam composed of unit cells was selected in a 4x10 arrangement, where resonators were inserted into some of the tested structures.

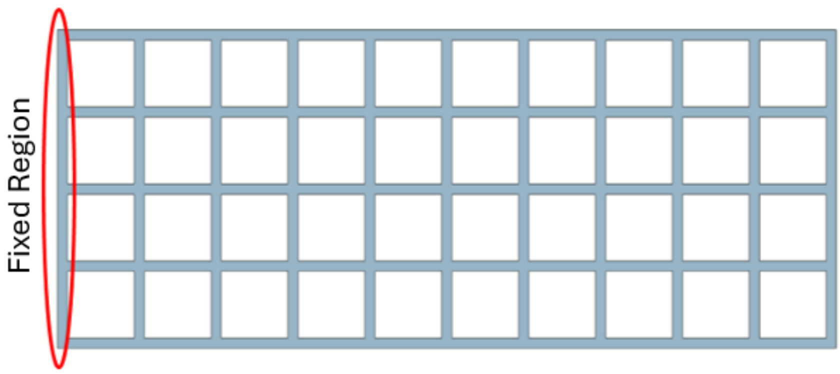

Figure 5 - Two dimentional beam composed of cellular structures.

As proposed by Lee [2], the resonator used consists of a small beam with a mass at the end. This arrangement is highly valuable because, in addition to being straightforward, it allows the natural frequency of the resonator, and consequently its effective frequency range, to be customized according to the necessity and through simple analytical formulation. Masses were added to the ends of all resonators to configure their natural frequency according to the natural frequencies defined for the study. For this purpose, CONM2 type elements were used.

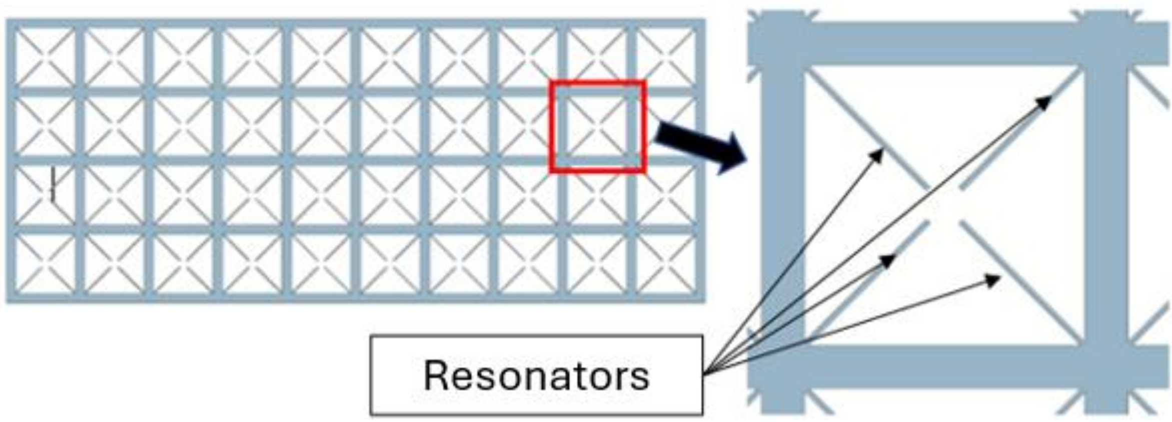

Figure 6 – Bidimentional beam with the local resonators in the cells.

The dimensions, in mm, of the analyzed cells are presented below:

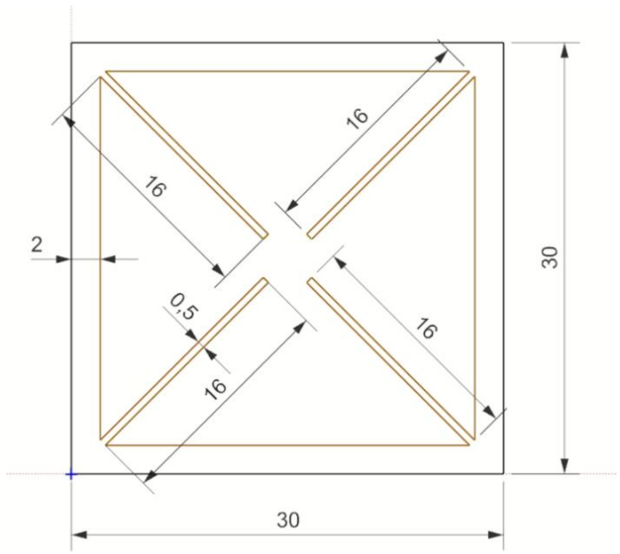

Figure 7 – Dimensions of the cell and ressonators.

After the formation of the cells, the thicknesses of the four ends of the resulting beam were increased by 2 mm so that all thicknesses of the geometry were the same.

Both the baseline cellular structure and the resonators added later were considered to be of the same material, in this case, steel. Its properties are presented in Figure 18.

```
Material properties:
Referenced library material : physicalmateriallibrary.xml
Library Version : 5.0
Material Type:   Isotropic
Label:   1
    Alternate Name                        :
    Category                              : METAL
    Sub-Category                          : Carbon Steel
    Material Property Dependency          : Constant
    Mass Density (RHO)                    : 7.872e-06 kg/mm³

     ======== Mechanical
    Young's Modulus (E)                   : 200000000 kPa
                                          : Major Poisson's Ratio
    Poisson's Ratio (NU)                  : 0.25
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Figure 8 – Mechanical properties of steel.

The structure was fixed at one of its ends, where it was also vertically excited in the frequency response analysis.

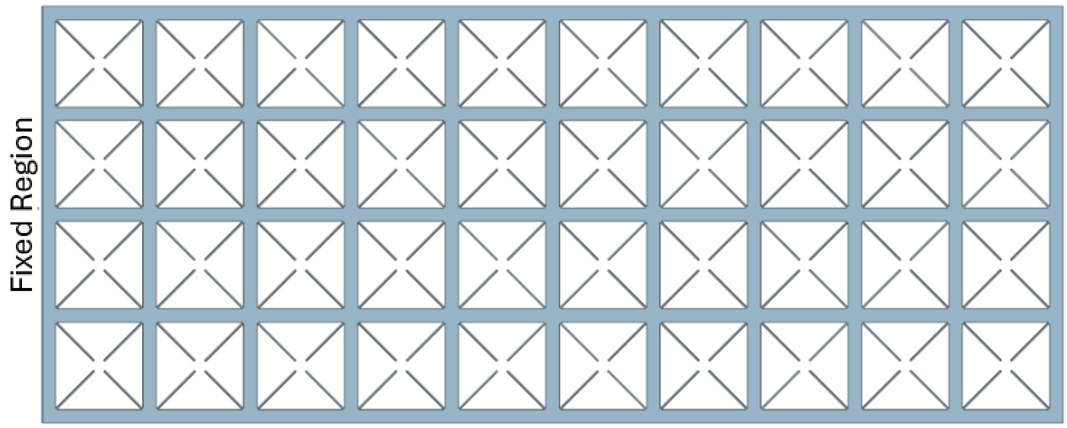

Figure 9 – Complete beam with resonators.

FEA Analysis Parameters

In the FEA modeling, the following parameters and configurations were used:

1. Second-order two-dimensional elements of type CQUAD8
2. At least 2 elements along the thicknesses for the correct physical representation of deformations along the structure.
3. In the frequency response analysis (FRF), a vertical displacement of 1 mm at the fixing point was input into the system, varying the frequency range from 1 Hz to 450 Hz, depending on the analysis performed.

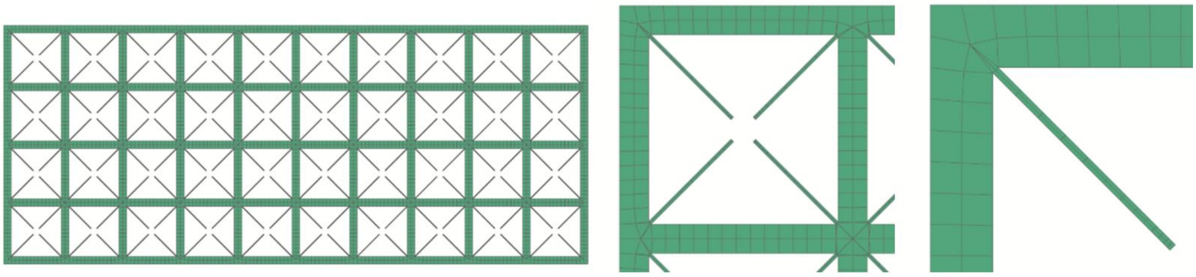

Figure 10 – Mesh used for the FEA analysis.

The output was measured at the central part of the beam, at the end opposite to the fixed end, as shown below:

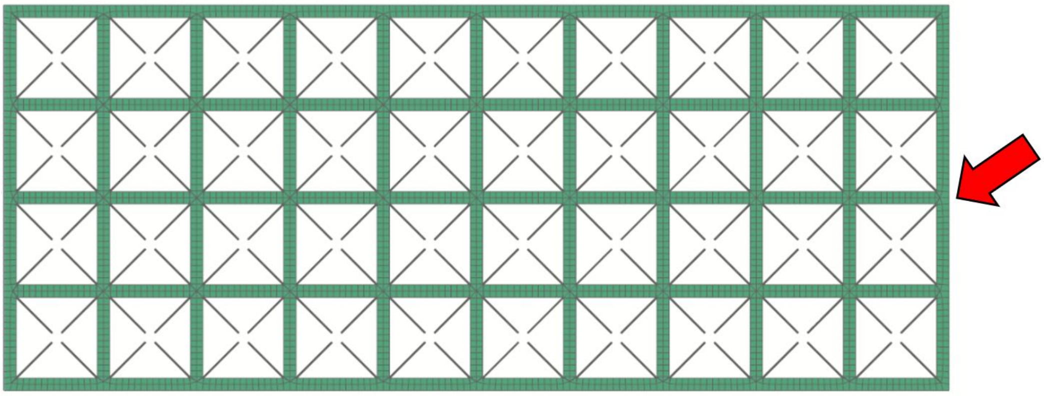

Figure 11 – Point on the structure where the response was analysed.

The Experiments

To evaluate the effects of the resonators on the system's transmissibility, one case containing only the cellular structure was analyzed, and three cases where resonators with natural frequencies of 120 Hz, 125 Hz, and 130 Hz were added. Resonators with different natural frequencies were evaluated to understand the influence of frequency variation on the system's damping.

RESULTS

As shown in the graphs below, the insertion of localized resonators inside the cells significantly reduced the transmissibility in the spectrum region near the natural frequency defined for each resonator. The X-axis represents the excitation frequencies, and the vertical axis represents the transmissibility, as the displacement input in the system is unitary along the analyzed frequency spectrum.

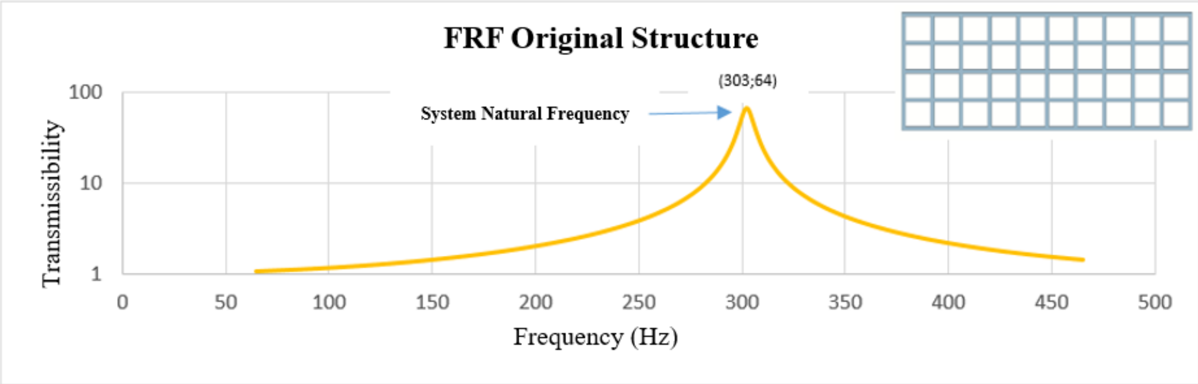

Figure 12 – Structure FRF without resonators.

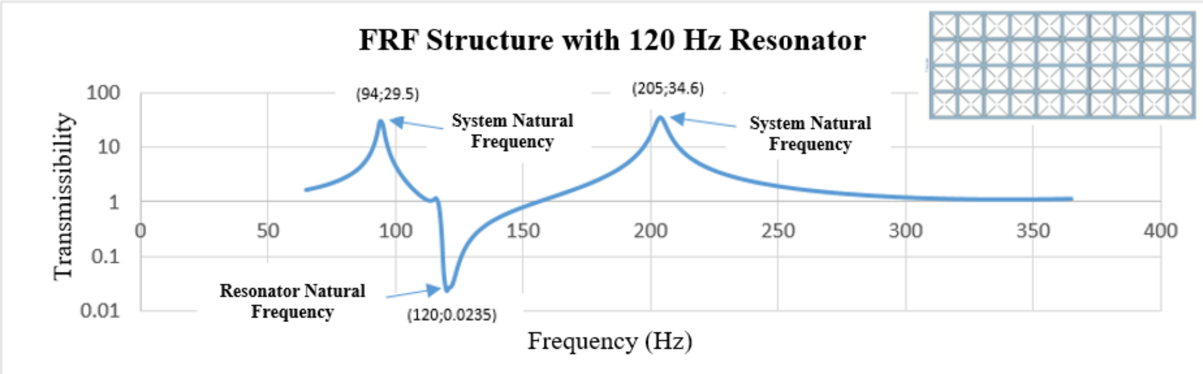

Figure 13 – Structure FRF with 120 Hz resonators.

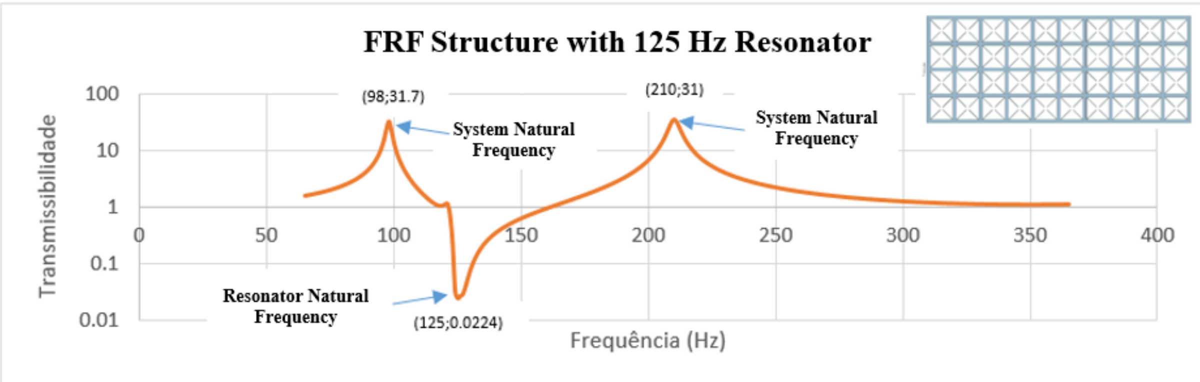

Figure 14 – Structure FRF with 125 Hz resonators.

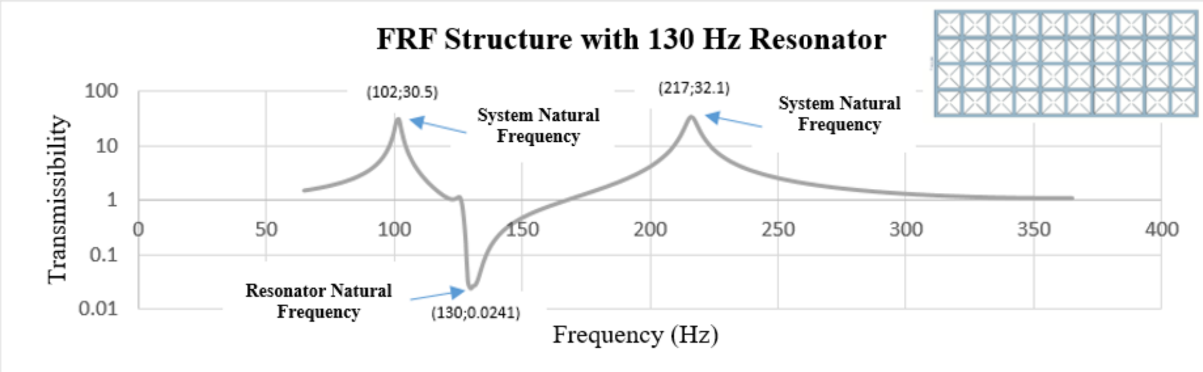

Figure 15 - Structure FRF with 130 Hz resonators.

In Figure 16, the previous graphs are unified into one, showing that there is no significant variation in the attenuation level depending on the natural frequency of the resonator used.

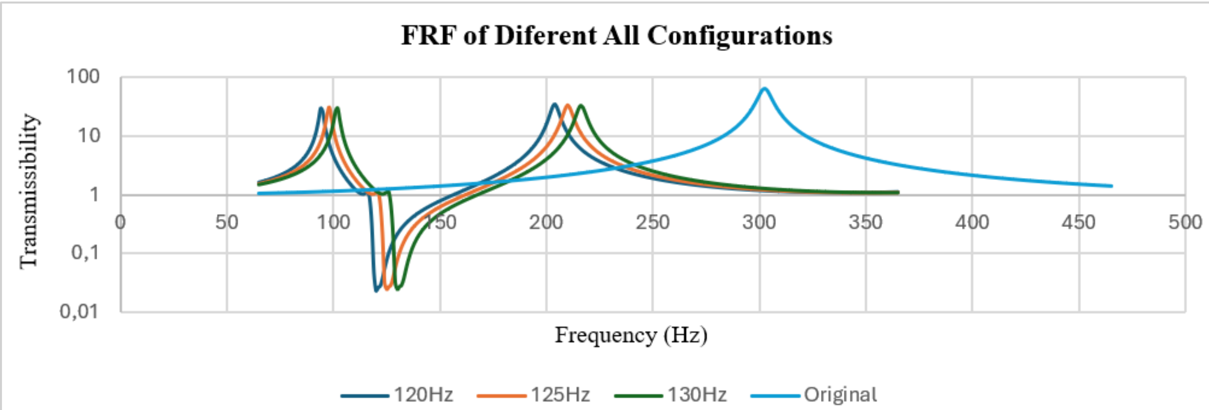

Figure 16 – Comparison of FRF for diferent resonators.

Due to the insertion of masses at the ends of the resonators, in addition to the desired damping, changes in the natural frequencies of the complete system are observed. This change occurred in the natural frequency of the original structure, which is reduced to 205 Hz in one of the cases. However, in the same case, a natural frequency of 94 Hz is added to the structure. In all cases where resonators were tested, the way the structure deforms did not vary, only the frequencies where the response peaks occurred. Therefore, below are the structural responses of the geometry at the response peaks and valleys, along with the related natural modes.

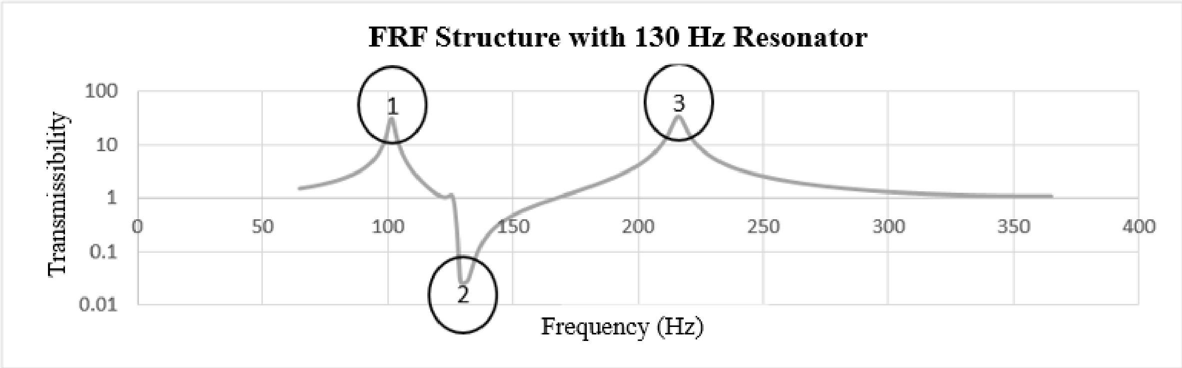

Figure 17 – FRF of the structure with 130 Hz resonators.

Respectively, natural mode and response shape associated with point 1:

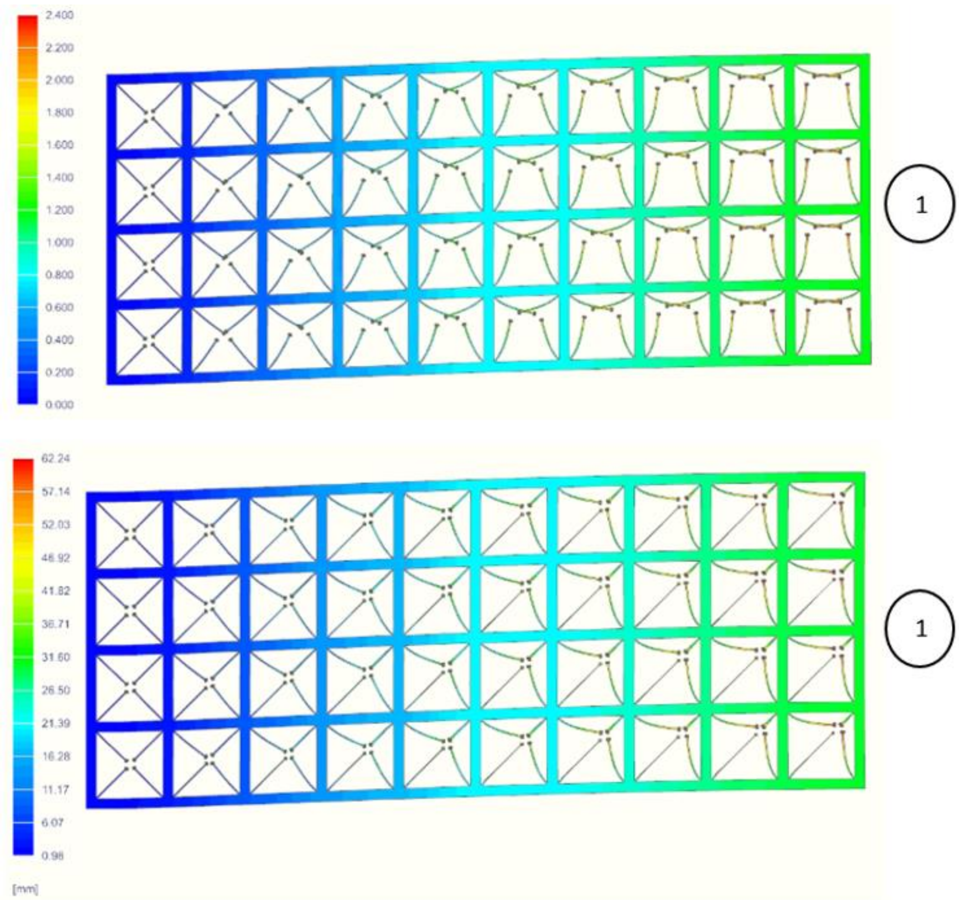

Figure 18 – Natural mode (up) and response shape (down) associated with point 1 in figure 17.

Respectively, natural mode and response shape associated with point 2:

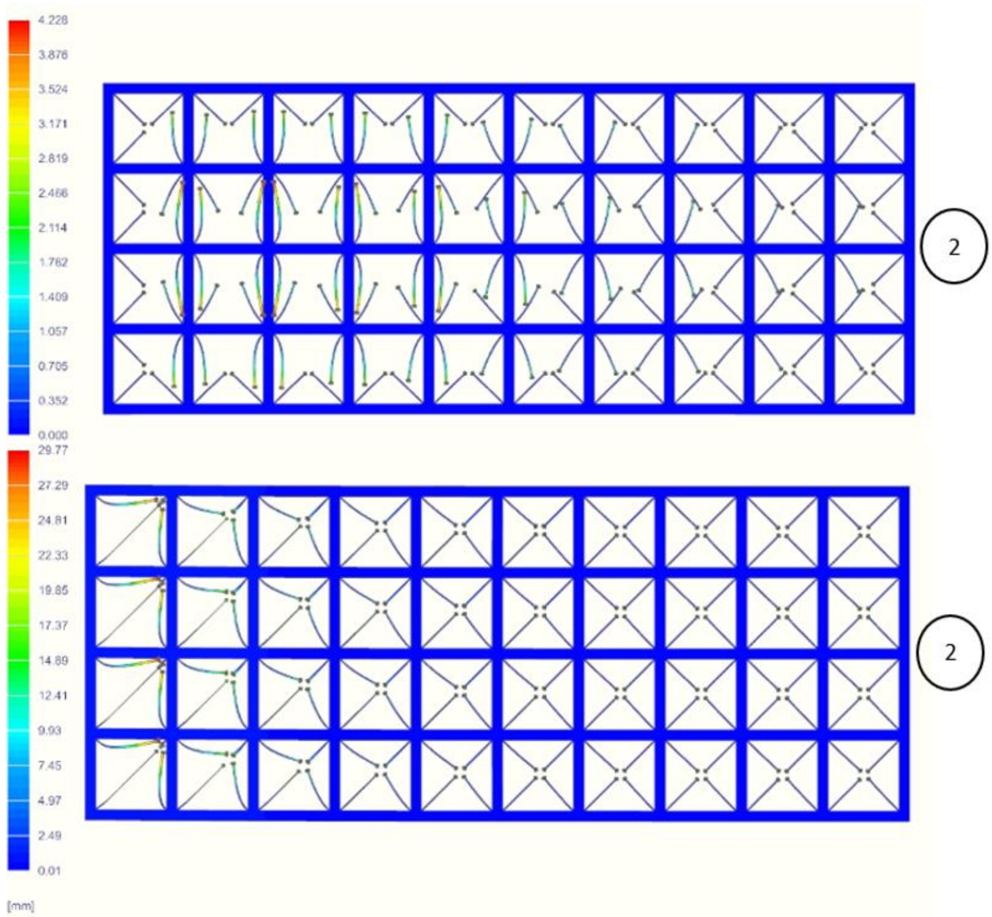

Figure 19 - Natural mode (up) and response shape (down) associated with point 2 in figure 17.

The mode shown above is the mode closest to 130 Hz; however, 157 other modes around 130 Hz were calculated with small frequency differences between them. Defining the natural frequency of the resonator at 130 Hz assumes that it is perfectly clamped at its base, which is not the case in reality. Along the structure, the stiffness of the cells itself can influence the stiffness of the resonators, explaining the large number of natural frequencies so close to each other.

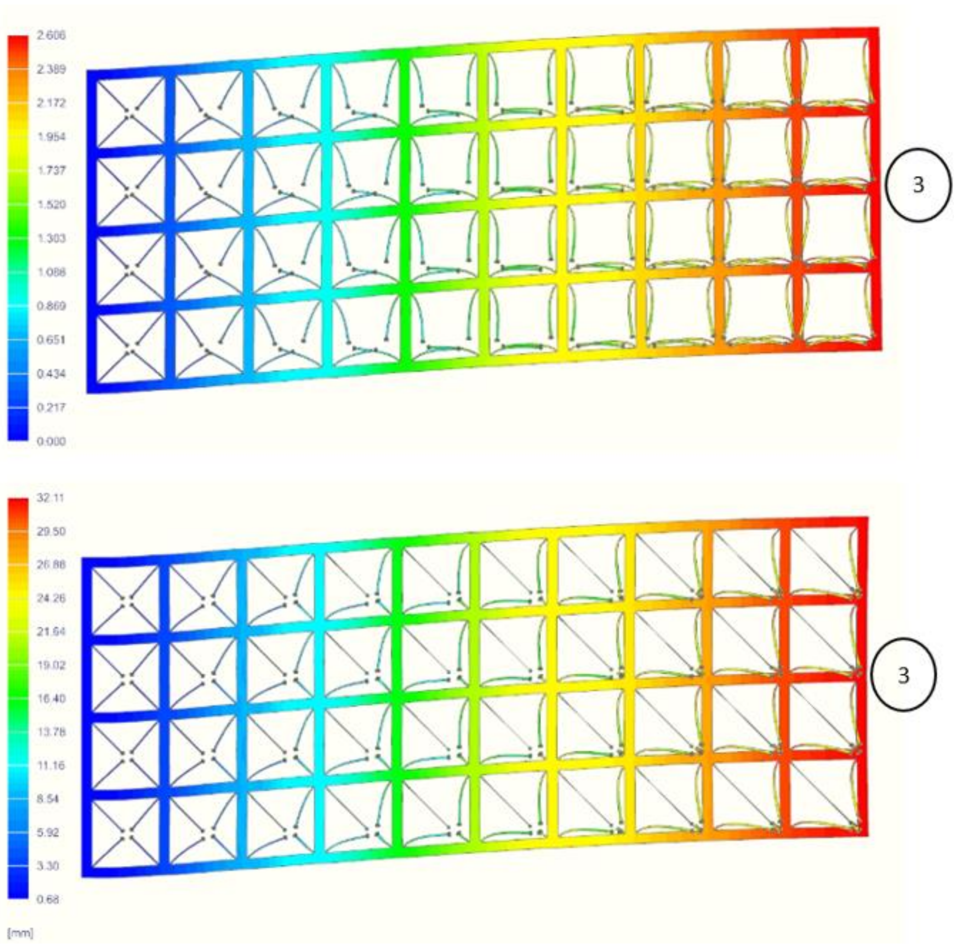

Figure 20 - Natural mode (up) and response shape (down) associated with point 3 in figure 17.

CONCLUSION

From the numerical simulations, it was possible to show that, for a 2D structure, it is possible to control the dynamic response of a cellular structure at the desired frequency using local resonators. For this purpose, no modifications were made to the cellular structure itself, only to the resonators, by inserting them and modifying the masses at their ends to regulate the frequency to be damped.

As shown in Figures 12, 13, 14, and 15, the insertion of the resonators brought, in addition to the desired damping, an amplification of the response at frequencies prior to the attenuated one. For structures that are excited over a wide spectrum of frequencies, such a phenomenon could render the use of the resonators unfeasible as presented. However, for structures operating at specific frequencies, such as rotating machine supports, for example, this concept could be explored in practical applications.

As shown in Figures 18, 19, and 20, the resonators, in different positions, respond differently from each other at the same excitation frequency, even though they all have the same stiffness and mass properties. This opens up possibilities for methodologies to be explored in subsequent studies for the distribution of both resonators and their masses along the structure to achieve an optimal result in terms of elastic band gaps with a wider bandwidth.

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