HYDRODYNAMIC PRESSURE OF RESERVOIR WATER

1. Abstract

Earthquakes pose a significant threat to dams and other reservoir structures. Beyond the ground shaking itself, the dynamic interaction between the earthquake ground motion and the water in the reservoir creates a phenomenon known as hydrodynamic pressure. This pressure can significantly exceed the static hydrostatic pressure that the dam is typically designed to withstand. Understanding and accurately predicting hydrodynamic pressure is crucial for ensuring the safety and stability of dams during seismic events.

2. Introduction

Dams are critical infrastructure, providing vital services like water storage, irrigation, flood control, and hydropower generation. However, earthquakes can subject dams to immense forces, potentially leading to catastrophic failure. One critical factor in dam safety during earthquakes is the hydrodynamic pressure exerted by the reservoir water.

2.1. Hydrodynamic Pressure vs. Hydrostatic Pressure

Under normal conditions, water in a reservoir exerts hydrostatic pressure, which increases linearly with depth. This pressure is relatively constant over time. In contrast, hydrodynamic pressure arises due to the dynamic interaction between the earthquake ground motion and the water. As the dam vibrates due to the earthquake, the water within the reservoir also experiences movement. This movement creates additional pressure that can fluctuate rapidly and significantly exceed the static hydrostatic pressure.

2.2. Factors Affecting Hydrodynamic Pressure

The magnitude of hydrodynamic pressure depends on several factors, including:

- **Earthquake characteristics:** The intensity, duration, and frequency content of the earthquake ground motion play a crucial role. Stronger earthquakes and those with frequencies that resonate with the natural frequencies of the dam-reservoir system will induce higher hydrodynamic pressures.
- **Reservoir geometry:** The shape, depth, and bathymetry (underwater topography) of the reservoir influence the water's response to ground motion. Deeper reservoirs and steeper slopes can amplify hydrodynamic pressures.
- **Dam properties:** The dam's material properties, such as its stiffness and mass, affect its interaction with the reservoir water. More flexible dams are more likely to experience higher hydrodynamic pressures.

2.3. Consequences of Hydrodynamic Pressure

Excessive hydrodynamic pressure can have several detrimental effects on dams:

- **Increased structural loads:** The additional pressure acts on the dam face, adding significant forces that the dam needs to be designed to withstand.

- **Potential for cracking and damage:** High pressure can cause cracks and damage to the dam's concrete or embankment material, compromising its structural integrity.
- **Overtopping risk:** In extreme cases, hydrodynamic pressure can contribute to overtopping, where water rises above the dam's crest, potentially leading to catastrophic failure.

3. Engineering Analysis and Mitigation Strategies

To ensure dam safety during earthquakes, engineers employ sophisticated analytical methods to predict hydrodynamic pressure. These methods involve complex mathematical models that consider the dam-reservoir interaction and the dynamic behavior of the water under seismic loading.

Several mitigation strategies can be implemented to address hydrodynamic pressure:

- **Dam design considerations:** Dams can be designed to be more resistant to hydrodynamic pressure by incorporating features like thicker sections, buttress walls, and upstream slopes with a gentler inclination.
- **Upstream slope protection:** Protecting the dam's upstream slope with erosion-resistant materials can help dissipate the energy of water waves generated by the earthquake.
- **Early warning systems:** Real-time monitoring of dam behavior and reservoir water levels can provide valuable information for making informed decisions during an earthquake event.

4. Closed-Form Equations of Hydrodynamic Pressure of Resevoir Water

According to the Italian Ministerial Decree of 1982, the actions of water, the effects of which are to be added to those of inertia of the masonry mass, are assimilated to a continuous distribution of pressure normal to the upstream face of intensity:

$$\mathrm{p} = \mathrm{C}\,\gamma\,\mathrm{c}\,y_{\mathrm{o}} \quad (1)$$

where C is the coefficient of seismic intensity, g is the weight per unit volume of water, y_0 is the difference between the maximum height of the reservoir and the height of the most depressed point of the natural riverbed in correspondence with the upstream face of the structure, while c can be calculated using the function:

$$c = \frac{c_m}{2}\left[\frac{y}{y_0}\left(2-\frac{y}{y_0}\right)+\sqrt{\frac{y}{y_0}\left(2-\frac{y}{y_0}\right)}\right] \quad (2)$$

where y is the difference between the maximum height of the reservoir and the height of the generic point of the face to which the pressure p indicated above is associated and c_m is a coefficient dependent on the angle of inclination of the face with respect to the vertical. Referring to Figure 1, the resultant of the pressures acting at different heights can be obtained both numerically (approximate solution) and in closed form (exact solution). To calculate this resultant in exact form means to add all the areas of prssure p_i dy between the dimension y = 0 and the generic dimension y, which consists of solving the integral (by substitution and twice by parts) :

$$S_d(y) = \int_0^y p(y)dy = \frac{c_m}{2} C\gamma \left[y^2 - \frac{y^3}{3y_0} + \frac{y - y_0}{2}\sqrt{2yy_0 - y^2} + y_0^2 \cdot \arcsin\sqrt{\frac{y}{2y_0}} \right] \quad (3)$$

For y=y0 we get the overall resultant of the pressures:

$$\boxed{S_d(y_0) = \int_0^{y_0} p(y)dy = \frac{c_m C\gamma}{24}(8 + 3\pi)\cdot y_0^2} \quad (4)$$

The point of application of the resultant Sd (y) with respect to the dimension y=0 can be evaluated as (5):

$$y'(S_d(y)) = \frac{\int_0^y y \cdot p(y)dy}{\int_0^y p(y)dy} = \frac{\frac{2}{3}y^3 - \frac{y^4}{4y_0} + \left(\frac{y^2}{3} - \frac{y_0 y}{6} - \frac{y_0^2}{2}\right)\sqrt{2yy_0 - y^2} + y_0^3 \cdot \arcsin\sqrt{\frac{y}{2y_0}}}{y^2 - \frac{y^3}{3y_0} + \frac{y - y_0}{2}\sqrt{2yy_0 - y^2} + y_0^2 \cdot \arcsin\sqrt{\frac{y}{2y_0}}}$$

The point of application of the resultant *Sd (y)* with respect to the dimension *y=y0* is obtained by difference:

$$y(S_d(y)) = y_0 - y'(S_d(y)) \quad (6)$$

On the other hand, the point of application of the overall resultant *Sd (y0)* , with respect to the dimension *y = 0 ,* is equal to:

$$y'(S_d(y_0)) = \frac{\int_0^{y_0} yp(y)dy}{\int_0^{y_0} p(y)dy} = \frac{1 + 3\pi}{8 + 3\pi}\cdot y_0 \quad (7)$$

In the same way as *Sd (y)* , the point of application of *Sd (y0)* with respect to the dimension *y=y0* is also obtained by difference:

$$\boxed{y(S_d(y_0)) = \frac{7}{8 + 3\pi} y_0 \cong 0{,}40 \cdot y_0} \quad (8)$$

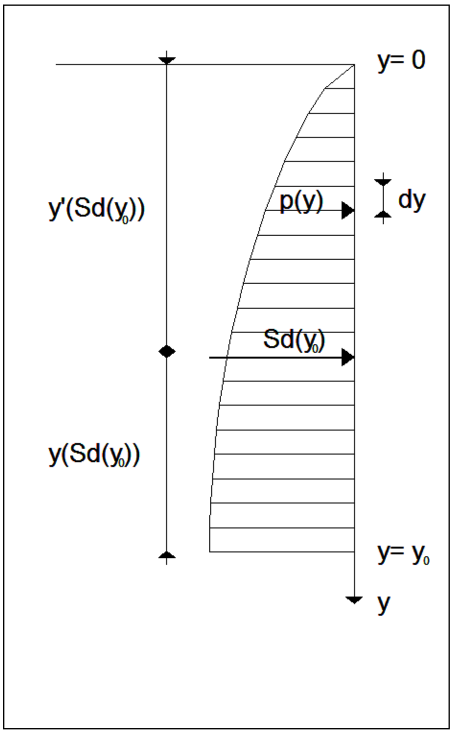

Figure 1 – Hydrodynamic pressure

5. Conclusion:

Hydrodynamic pressure is a critical factor in dam safety during earthquakes. By understanding its causes, effects, and mitigation strategies, engineers can design and maintain dams that are more resilient to seismic events. Further research on advanced analytical methods and innovative dam design solutions is crucial for ensuring the safety and sustainability of dams in earthquake-prone regions.