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FAST SEISMIC VULNERABILITY ANALYSIS OF HISTORICAL MASONRY STRUCTURES BY MEANS OF A NOVEL LIMIT ANALYSIS-BASED APPROACH

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Summary. Historical masonry structures are widespread all over Europe, and most of them are in seismic areas, such as in Italy, Greece and Portugal. Therefore, a seismic vulnerability assessment is fundamental for the preservation of the historical cultural heritage of such countries. Nowadays, many methods of analysis are available. Unfortunately, they can sometimes be costly in terms of computational burden and the results obtained are strongly influenced by the input parameters adopted. It is notorious, for instance, that FEM non-linear static analyses, especially for big and complex structures, are affected by a quite large scatter of the output. This contribution aims to present a homemade limit analysis-based method to assess the seismic vulnerability of historical masonry structures in a fast and reliable manner. The approach is based on a discretization by means of infinitely resistant hexahedron elements and quadrilateral interfaces where all dissipation occurs. The collapse load is obtained through a kinematic approach. When non-linearities are lumped on interfaces, static and kinematic problems coincide, being one the dual of the other. The first provides a failure mechanism, interfaces plastic multipliers and collapse multipliers, while with the second the internal forces acting on interfaces are retrieved. The case study adopted to benchmark the procedure proposed is a historical palace (Vittorio Emanuele II building) located in Piacenza, Italy. It is a very big masonry structure, characterized by an E-shape plan and by the presence of numerous vaults.

1 INTRODUCTION

Historical masonry structures are generally characterized by ineffective connections between walls, roofs and floors since they were designed only to withstand vertical loads. Consequently, when subjected to seismic actions they do not develop a box-like behavior but rather present local collapse mechanisms [1,2]. Sometimes, out-of-plane failure mechanisms are triggered even if the connections are effective because of the high slenderness of walls [3]. Therefore, to preserve historical structures and guarantee their safety it is necessary to estimate the acceleration that triggers such kind of failures and eventually design effective strengthening interventions.

To reach this aim many approaches are available, such as Finite Element Models (FEM), Discrete Element Models (DEM) and kinematic limit analysis. FEM are the most used allowing the study of both global behavior as well as local failure mechanisms [4,5]. The drawback is that in FEM many parameters are involved in the definition of suitable constitutive models for non-linear materials such as masonry. Among the available constitutive models, the Concrete Damage Plasticity (CDP) model is the most used. If well calibrated, the CDP can precisely approximate masonry behavior as shown in the technical literature [6,7]. However, this model does not allow the hypothesis of a no-tension material, at least a very low tensile strength is needed to guarantee the convergence of the algorithm. Therefore, in some cases, it could lead to overestimating the collapse acceleration of out-of-plane failure mechanisms. Another methodology that is adopted to study masonry structure is based on DEM. This approach is the most accurate for masonry structures since the actual interaction between units can be directly modeled [8-10]. The drawbacks of this approach are the high computational time and the costliness of available commercial software.

For these reasons, the Italian building code [11,12] requires the verification of out-of-plane local collapse mechanisms at least through kinematic linear analysis starting from pre-assigned failure mechanisms. Despite this approach being very fast and simple, it could overestimate or underestimate the acceleration that triggers local failures, especially in the case of complex geometries where it is difficult to identify a priori the portion of the structure involved in the mechanism [13].

In the present work, an upper bound limit analysis approach is developed by the authors to overcome the different drawbacks of the methods cited before. Indeed, the proposed approach developed in the commercial software MATLAB is based on the methodology proposed in [14]. The limit analysis-based approach developed by the authors is fast, avoiding the drawback of DEM models, only the definition of the geometry and the Mohr-Coulomb strength criterion is needed, overcoming the drawback of FEM, and finally, it does not require the selection of pre-assigned failure mechanisms. The methodology is described in Section 2, while in Section 3 the case study used as a benchmark is presented, namely Vittorio Emanuele II palace. The results in terms of collapse acceleration, active failure mechanism and safety verification are presented in Section 4.

2 METHODOLOGY

An upper bound limit analysis-based approach is developed for the assessment of masonry structures according to Italian standards [11,12]. The formulation of a classic limit analysis problem is possible whenever the structure is discretized with infinitely resistant hexahedron elements (see **Figure 1a** and **Figure 1b**), materials are assumed rigid-perfectly plastic with infinite ductility and plastic deformations are lumped at elements' interfaces. The solution to this problem consists in finding the collapse load, the active failure mechanism, and the distribution of internal forces.

Since plastic deformations are lumped at element interfaces, the upper bound coincides with the lower bound. Therefore, the limit analysis problem is written from a kinematic point of view since is more straightforward and then the static counterpart is derived from the self-dual linear programming problem. Hence the primal variables of the problem are six unknowns per hexahedron, namely centroid velocities (U_x^i, U_y^i, U_z^i) and rotation rates ($\phi_x^i, \phi_y^i, \phi_z^i$), as depicted in **Figure 1c**.

Only external volume forces are assumed to act on the structure, which are gravity loads $f_0^{(i)}$

and horizontal forces $f_{\Gamma}^{(i)}$ simulating the seismic actions (dependent on the load multiplier Γ). Two horizontal load distributions are assumed in accordance with the Italian building code, namely inverse linear in height (G1) and uniform (G2).

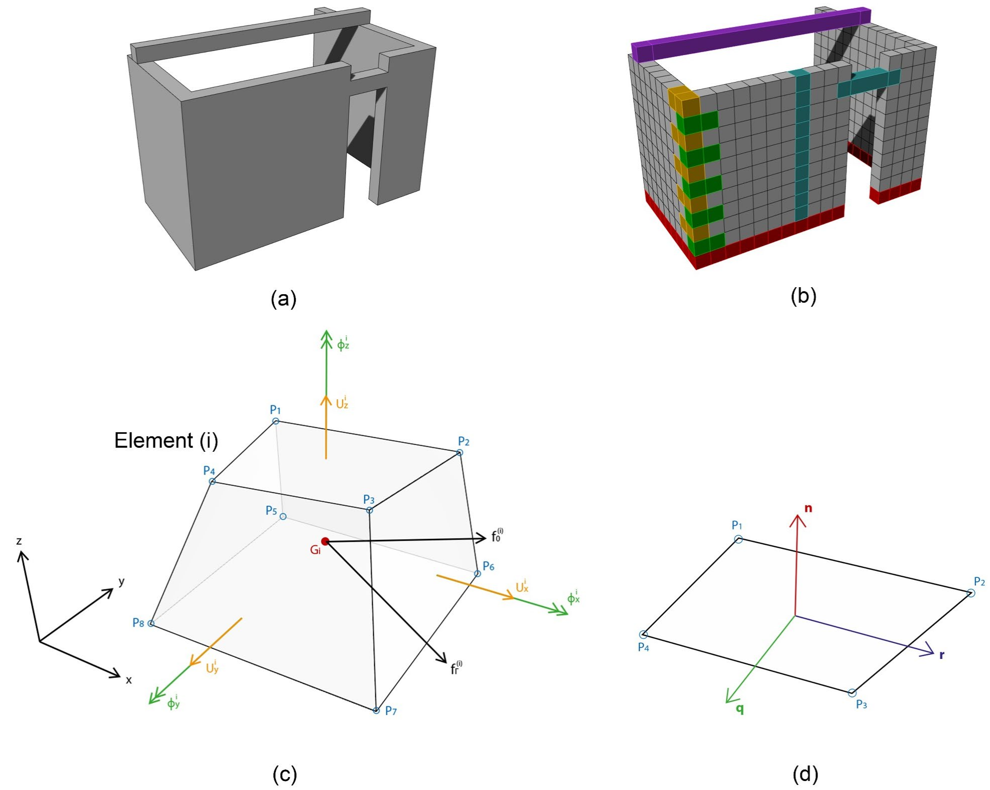

Figure 1: (a) generic structure, (b) discretized structure, (c) generic hexahedron element, (d) local reference frame

The jump of velocities between elements' interfaces are constrained to impose plastic compatibility. Furthermore, a constant stress state is assumed at elements' interfaces allowing the definition of compatibility constraints on the jumps of velocities only at collocation points, that are assumed to be at the vertices of the quadrilateral interface. Once defined a suitable local reference frame $\boldsymbol{n}-\boldsymbol{q}-\boldsymbol{r}$ (see **Figure 1d**), internal actions, jump of velocities, and power dissipation can be evaluated at collocation points.

Hence, the velocity at the collocation point CP_k of element (i) is computed as:

$$\boldsymbol{U}_{CPk}^{(i)} = \begin{bmatrix} \boldsymbol{n}^T \\ \boldsymbol{q}^T \\ \boldsymbol{r}^T \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & z_{Pk}-z_{G_{Ei}} & -(y_{Pk}-y_{G_{Ei}}) \\ 0 & 1 & 0 & -(z_{Pk}-z_{G_{Ei}}) & 0 & x_{Pk}-x_{G_{Ei}} \\ 0 & 0 & 1 & y_{Pk}-y_{G_{Ei}} & -(x_{Pk}-x_{G_{Ei}}) & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{U}^{(i)} \\ \boldsymbol{\phi}^{(i)} \end{bmatrix} = \boldsymbol{R}^{(i)} \boldsymbol{u}^{(i)} \tag{1}$$

$$\boldsymbol{U}^{(i)} = \begin{bmatrix} U_x^i & U_y^i & U_z^i \end{bmatrix}^T$$

$$\boldsymbol{\phi}^{(i)} = \begin{bmatrix} \phi_x^i & \phi_y^i & \phi_z^i \end{bmatrix}^T$$

The jump of velocity at collocation point CPk between two adjacent elements named (i) and (j), assuming that vector $\boldsymbol{n}$ is outward from element (i), is equal to:

$$\Delta \boldsymbol{U}_{CP_k} = \boldsymbol{R}^{(j)}\boldsymbol{u}^{(j)} - \boldsymbol{R}^{(i)}\boldsymbol{u}^{(i)} \tag{2}$$

The Mohr-Coulomb failure criterion with tension and compression cut-off characterizes the interface properties between adjacent elements, and it is simply defined by the tensile strength f_t, the compressive strength f_c, the cohesion c and the friction angle Φ. The presence of different materials and different interface properties can be considered in the proposed approach, as shown in **Figure 1b** with different colors. The plastically admissible strength domain in the local reference frame is defined by:

$$\boldsymbol{A}_{in}^{I} = \begin{bmatrix} \boldsymbol{N} \\ \boldsymbol{Q} \\ \boldsymbol{R} \end{bmatrix} \leq \boldsymbol{b}_{in}^{I} \tag{3}$$

Where N, Q, R are the internal actions acting on the interfaces, computed as the interface area multiplied by the stress components along $\boldsymbol{n}$, $\boldsymbol{q}$ and $\boldsymbol{r}$, respectively.

An associate flow rule is assumed leading to:

$$\begin{gathered} \boldsymbol{R}^{(j)}\boldsymbol{u}^{(j)} - \boldsymbol{R}^{(i)}\boldsymbol{u}^{(i)} - {\boldsymbol{A}_{in}^{I}}^{T}\dot{\lambda}_I = 0 \\ \dot{\lambda}_I \geq 0 \quad \forall I = 1, \dots, N_{in} \end{gathered} \tag{4}$$

Where N_{in} is the number of interfaces.

The balance of powers dissipated by internal and external forces allows the identification of the load multiplier, but to identify one failure mechanism among the infinite set of homothetic collapse deformed shapes it is necessary to define the normalization condition. It is assumed that the power dissipated by the loads dependent on the load multiplier Γ is unitary when $\Gamma = 1$. Therefore, the load multiplier is given by:

$$\Gamma = \sum_{I=1}^{N_{in}} {\boldsymbol{b}_{in}^{I}}^{T}\dot{\lambda}_I - \sum_{i=1}^{N_e} V_i {\boldsymbol{f}_0^{(i)}}^{T} \boldsymbol{U}^{(i)} \tag{5}$$

Where N_e is the number of elements. Following the kinematic approach, the collapse multiplier is the minimum of the load multipliers.

External boundary conditions are defined as velocity constraints of the involved elements (h), as those highlighted in red in **Figure 1b**:

$$\boldsymbol{A}_{bc}^{(h)}\boldsymbol{U}^{(h)} = 0 \tag{6}$$

In conclusion, the limit analysis problem formulation is:

$$\min\left\{\Gamma = \sum_{I=1}^{N_{in}} {\boldsymbol{b}_{in}^{I}}^{T}\dot{\lambda}_I - \sum_{i=1}^{N_e} V_i {\boldsymbol{f}_0^{(i)}}^{T} \boldsymbol{U}^{(i)}\right\} \tag{7}$$

$$s.t.\begin{cases} \boldsymbol{R}^{(j)}\boldsymbol{u}^{(j)} - \boldsymbol{R}^{(i)}\boldsymbol{u}^{(i)} - {\boldsymbol{A}_{in}^{I}}^{T}\dot{\lambda}_I = 0 \quad \forall I = 1, \dots, N_{in} \\ \displaystyle\sum_{i=1}^{N_e} V_i {\boldsymbol{f}_1^{(i)}}^{T} \boldsymbol{U}^{(i)} = 1 \\ \boldsymbol{A}_{bc}^{(h)}\boldsymbol{U}^{(h)} = 0 \qquad \forall h \in b.c. \\ \dot{\lambda}_I \geq 0 \qquad \forall I = 1, \dots, N_{in} \end{cases}$$

The solution is estimated solving a linear programming problem in the commercial software

MATLAB, writing the limit analysis problem in its standard form [14].

Once the collapse acceleration and the active failure mechanism have been identified, it is possible to verify the activation of local failure mechanisms according to the Italian building code. Since Italian standards require the assumption of a no-tension material, the authors followed the approach presented in [15] to assess the collapse acceleration for $f_t = 0\ MPa$. This procedure is necessary to avoid the activation of parasite sliding between adjacent elements or stalling issues of the numerical algorithm that could occur when enforcing the no-tension material hypothesis. It consists of assuming a sufficiently large tensile strength for the first iteration and then performing a series of limit analyses progressively decreasing the tensile resistance. Results plotted in terms of collapse acceleration in function of the tensile strength allow identifying the change of the collapse mechanism. Moreover, looking at the associated deformed shape allows the identification of the occurrence of systematic spurious sliding. Through the linear interpolation of the results, which are not affected by spurious sliding, the collapse acceleration for the no-tension material hypothesis is found.

In accordance with Italian standards, the value of the seismic action that triggers the failure mechanisms corresponds to the spectral acceleration a_0^*:

$$a_0^* = \frac{\alpha_0 \cdot g}{e^* \cdot FC} \tag{8}$$

Where: α_0 is the collapse multiplier, g is the gravity acceleration, FC is the confidence factor and e^* is the fraction of participating mass computed as:

$$e^* = \frac{g \cdot M^*}{\sum_{i=1}^{n+m} P_i} \tag{9}$$

Where $n + m$ is the number of self-weights P_i whose masses, due to seismic action, generate horizontal forces on the elements of the kinematic chain and M^* is the participating mass evaluated as:

$$M^* = \frac{\left(\sum_{i=1}^{n+m} P_i \delta_{x,i}\right)^2}{g \cdot \sum_{i=1}^{n+m} P_i \delta_{x,i}^2} \tag{10}$$

Where $\delta_{x,i}$ is the horizontal virtual displacement of the application point of the i-th weight P_i. Since the code developed by the authors can be applied to an entire structure, to avoid overestimating the spectral acceleration a_0^*, only the elements involved in the local failure mechanism (characterized by a value of $\delta_{x,i}$ greater than or equal to 20% of the maximum one) are considered to compute the participating mass M^*. In addition, the collapse load α_0 in Equation (8) is substituted by $a_g/g = \frac{base\ shear}{vertical\ load}$ to consider the actual distribution of forces applied to the structure (G1 or G2), otherwise the spectral acceleration a_0^* could be overestimated.

The local failure mechanism is not triggered by the design seismic action of the selected limit state if:

$$a_0^* \geq \frac{a_g S}{q} \tag{11}$$

Where: a_g is the peak acceleration at bedrock at the site, S is the soil and topography coefficient and q is the behavior factor (generally assumed equal to 2 for masonry structures).

3 CASE STUDY: VITTORIO EMANUELE II PALACE

The method developed by the authors is benchmarked on a historical masonry building, namely Vittorio Emanuele II palace (**Figure 2**), located in Piacenza in northern Italy. The structure is characterized by an 'E' shape, and it is constituted by three above-ground floors and a basement. The Vittorio Emanuele II palace is the result of the enlargement of the Randani-Tedeschi palace which dates back to 1670 and was characterized by only two floors. Indeed, this structure was enlarged in 1882 and in 1912 the three wings were built. In the last intervention occurred in 1956, the entire building was raised of one floor. Two internal courtyards are created by the presence of the wings, called Old Cloister and New Cloister in **Figure 2**. The oldest portion of the structure presents cloister and cross vaults.

Small portions of the structure are studied and verified according to the Italian building code against possible local failure mechanisms. In particular, for the sake of brevity in this contribution only two portions of the structure are analyzed, namely the central and the external wing shown in **Figure 3** together with the structural model used. In the structural models, floors and roofs are modeled only as distributed masses due to their poor connection with walls.

4 RESULTS AND DISCUSSION

In this section, the results found for the two portions analyzed are shown. In particular, the spectral acceleration a_0^* that triggers the local failure mechanisms is assessed for both horizontal load distributions G1 and G2, as well as for different seismic input angles. Spectral acceleration values are compared with the design seismic demand defined by the Italian building code for the life safety limit state at the site of Piacenza.

The design seismic action is characterized by a return period $T_R = 475$ years, and by a bedrock acceleration equal to $a_g = 0.092\ g$. The soil type where the palace is located is classified as soil class C, while the topography class is T1 since it is a flat area. Therefore, the soil and topography coefficient S is equal to 1.5.

Hence, the failure mechanism is not activated if:

$$a_0^* \geq \frac{a_g S}{q} = 0.677\ m/s^2 \tag{12}$$

4.1 Central wing

The results found for the central wing of the Vittorio Emanuele II palace are reported in this section in terms of collapse acceleration a_g/g function of the tensile strength f_t, in the four main directions of the structure and for both G1 and G2 horizontal load distributions (**Figure 4** and **Figure 5**). The collapse acceleration a_g/g under the hypothesis of no-tension material ($f_t = 0\ MPa$) is found following the procedure explained in the previous section through linear interpolation, disregarding the results associated with a failure mechanism affected by spurious sliding of the blocks.

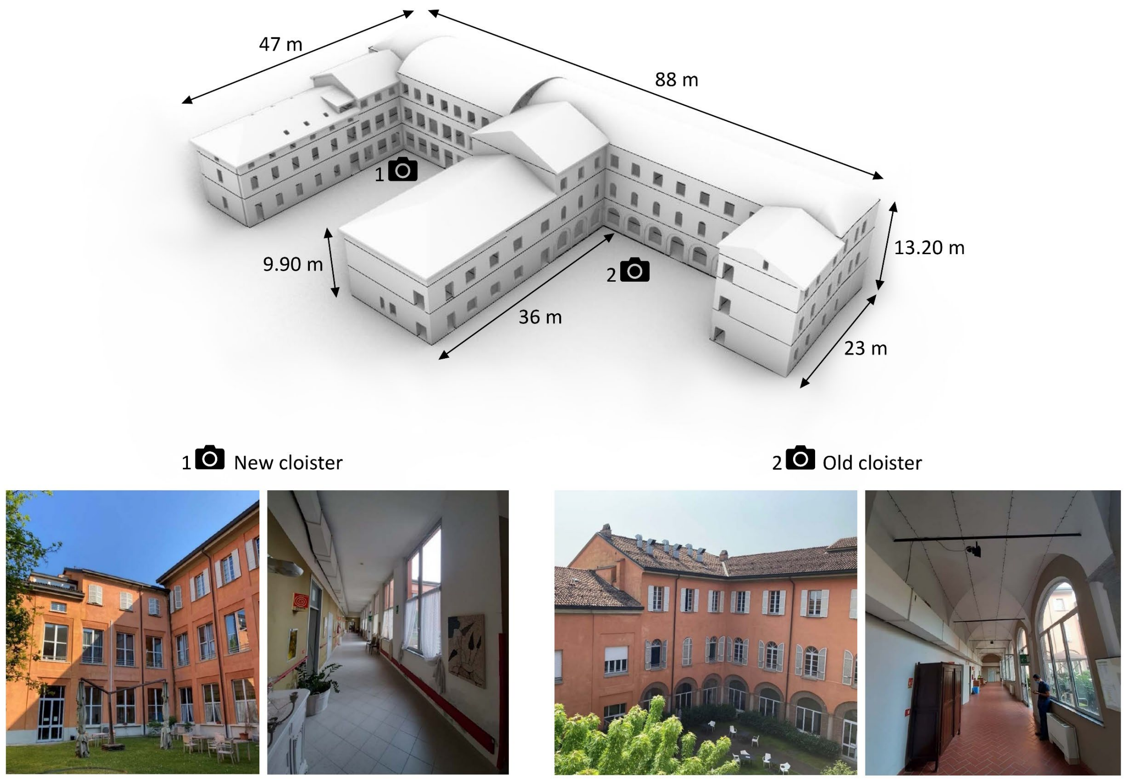

Figure 2: Vittorio Emanuele II palace

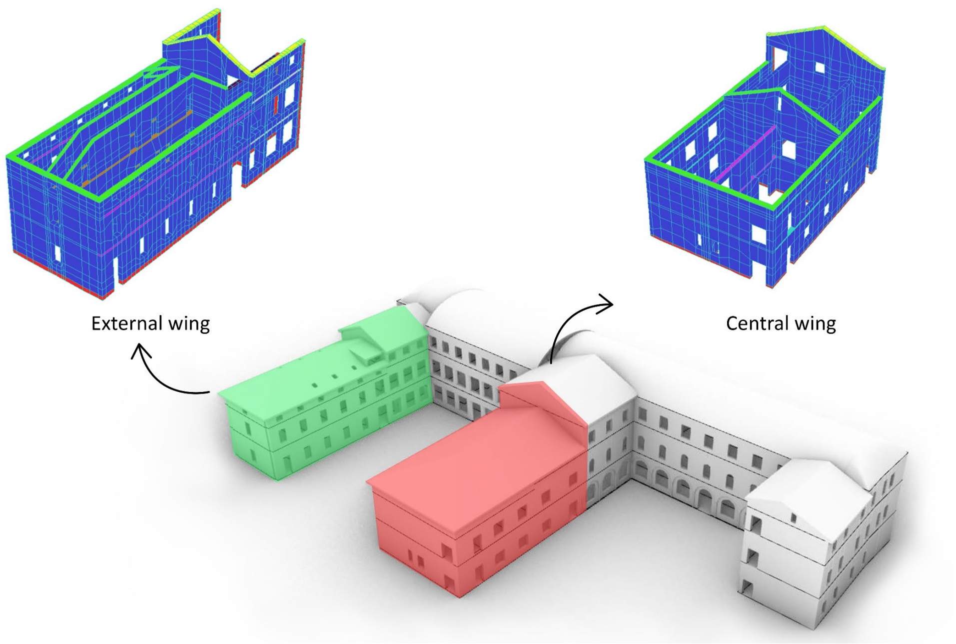

Figure 3: Structural models

Once the collapse acceleration values for $f_t = 0\,MPa$ have been found, the spectral acceleration values a_0^* triggering local failure mechanisms are computed in all directions and for both horizontal load distributions. Results are depicted in **Figure 6** in terms of a_0^* values, which are compared with the design seismic action according to Italian standards, and in terms of active failure mechanism, where the elements involved in the failure mechanism ($\delta_{x,i} \geq 0.2 * \delta_{x,i_{MAX}}$) are highlighted in red.

The central wing of Vittorio Emanuele II palace results to be verified only in the y-positive direction when subjected to an inverse linear horizontal load distribution (G1), while when subjected to a uniform horizontal load distribution (G2) it results to be verified in the x and y-positive directions. The results highlight that strengthening interventions are necessary to avoid local collapses.

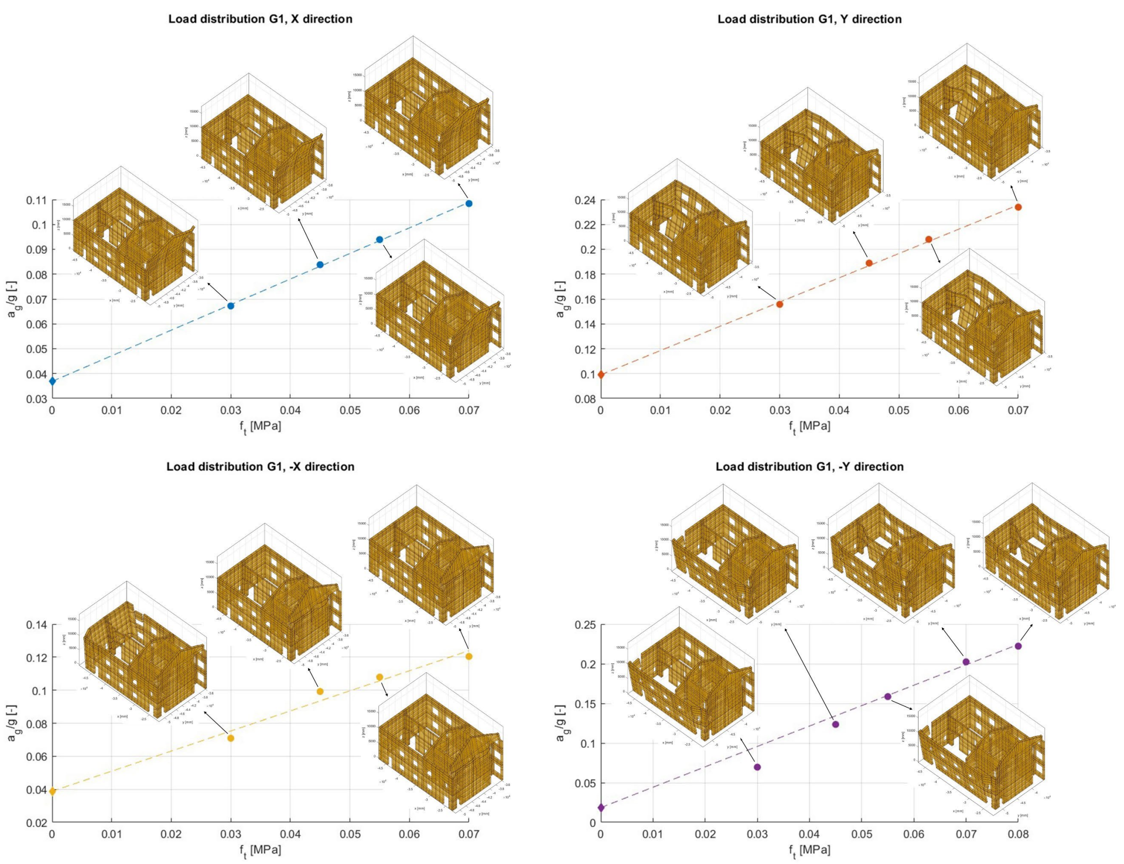

Figure 4: Collapse acceleration vs tensile strength and active failure mechanisms under G1 load distribution.

4.2 External wing

The analysis outcomes for the external wing of Vittorio Emanuele II palace are shown, for the sake of brevity, in terms of the collapse acceleration function of the tensile strength in **Figure 7**. Once again linear interpolation, disregarding the values related to spurious sliding, leads to the identification of the collapse acceleration under the assumption of a no-tension material. In this case, it is evident in the x-negative direction, identified with the yellow color, the sudden decrease of the collapse acceleration value for $f_t = 0.03\ MPa$ due to the occurrence of spurious sliding under both G1 and G2 horizontal loading distributions.

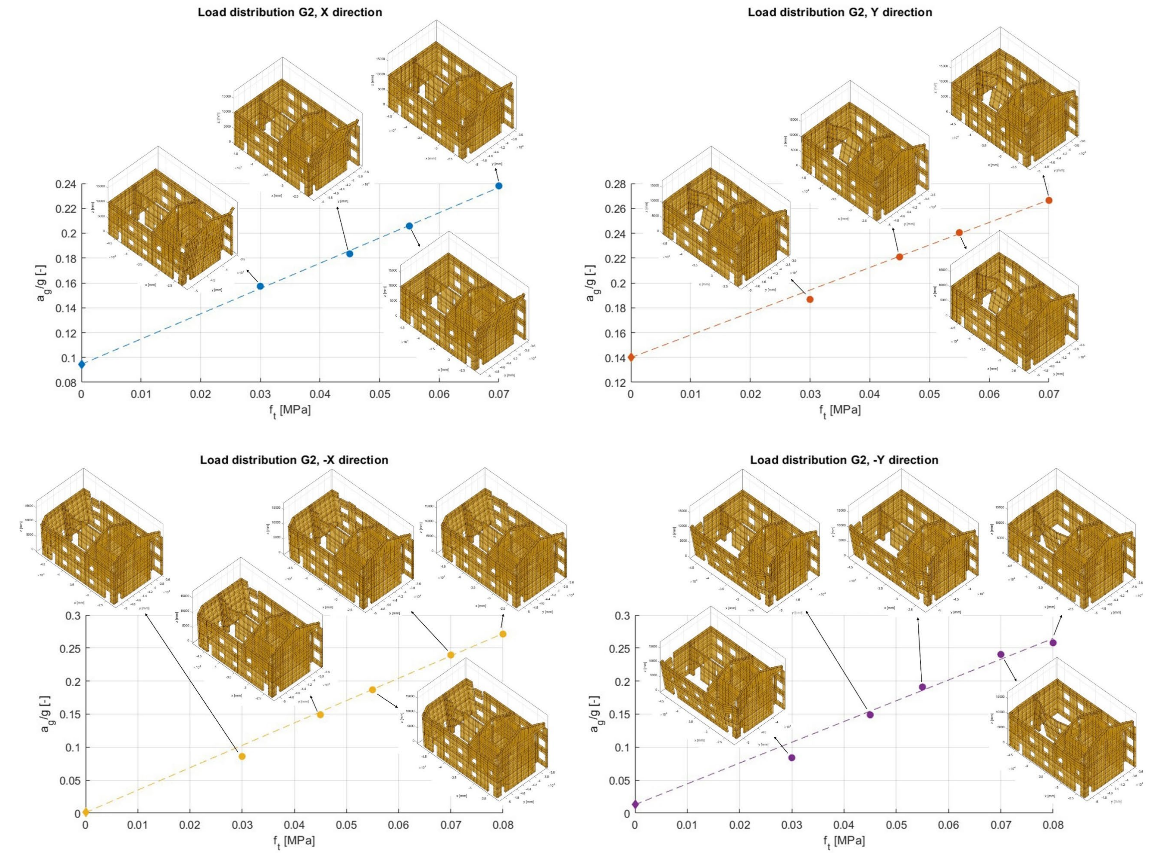

Figure 5: Collapse acceleration vs tensile strength and active failure mechanisms under G2 load distribution.

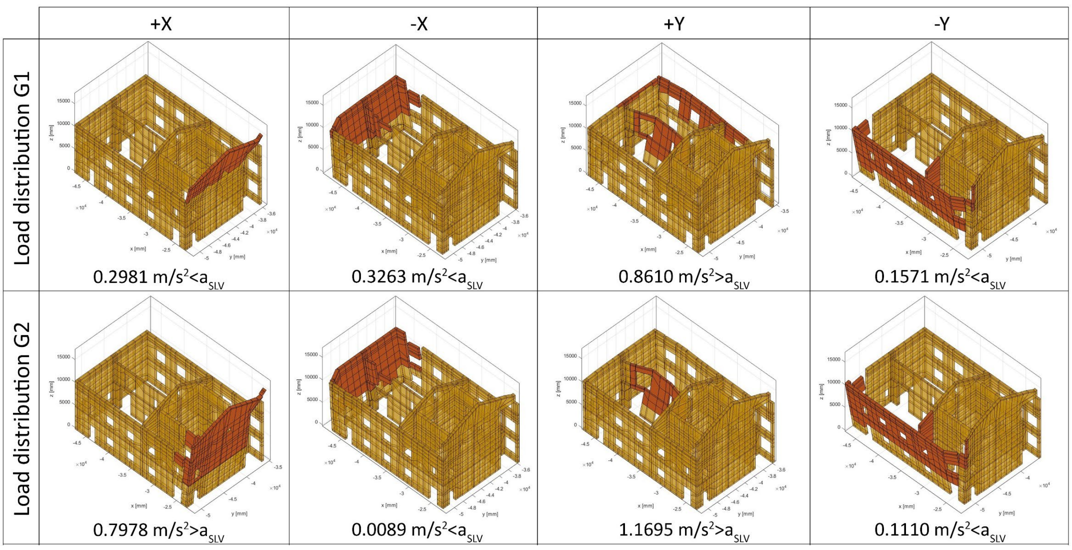

Figure 6: Spectral accelerations triggering local failure mechanisms under G1 load distribution.

From the values found for the collapse acceleration for $f_t = 0\ MPa$, the spectral accelerations triggering local failure are evaluated and listed in **Figure 8** together with the associated failure mechanisms. The structure is verified only in the x-positive direction for both loading distributions. The horizontal load distribution G1 represents the worst loading condition for this portion, except for the x-negative direction where two different collapse mechanisms are triggered. Indeed, load distribution G1 involves only the tympanum, on the contrary, load distribution G2 involves the whole façade and a portion of perpendicular walls.

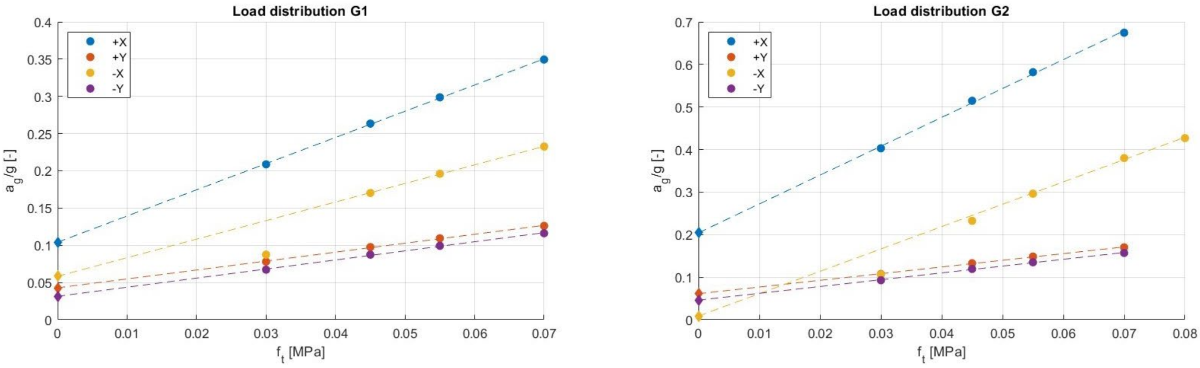

Figure 7: Collapse acceleration vs tensile strength under G1 and G2 load distributions.

5 CONCLUSIONS

In the present work, an upper bound limit analysis-based approach is developed for the safety verification of local collapse mechanisms of masonry structures according to Italian standards. It is based on the discretization of the structure with infinitely resistant hexahedron elements, where plastic deformation occurs only at elements' interfaces. The problem is formulated from the kinematic point of view. Since plasticity is lumped in a finite number of interfaces, the upper bound coincides with the lower bound, therefore the static counterpart can be found solving the self-dual linear programming problem. The results are in terms of collapse acceleration and active failure mechanism.

Since Italian standards require the assumption of the no-tension material for the verification of local failures, the collapse acceleration (computed as base shear divided by vertical loads) under this hypothesis is evaluated indirectly through linear interpolation. The necessity of an indirect evaluation of this value is done to avoid stalling issues of the numerical algorithm and parasite sliding between adjacent elements.

The main advantages of this methodology are several. First of all, the analyses are fast, and the code developed in the commercial software MATLAB by the authors requires only the definition of the mesh and of the parameters determining the Mohr-Coulomb strength criterion. Moreover, the spectral acceleration that triggers the local failure is found without the selection of pre-assigned failure mechanisms. Therefore, it is possible to analyze complex geometries where it is difficult to choose a priori the shape of the local failure mechanism.

The procedure was benchmarked on a case study located in Piacenza, northern Italy, named Vittorio Emanuele II Palace. The results found for two significant portions of the structure were analyzed and it was found that the structure needs strengthening interventions since the design seismic action defined by the building code may activate some local collapses.

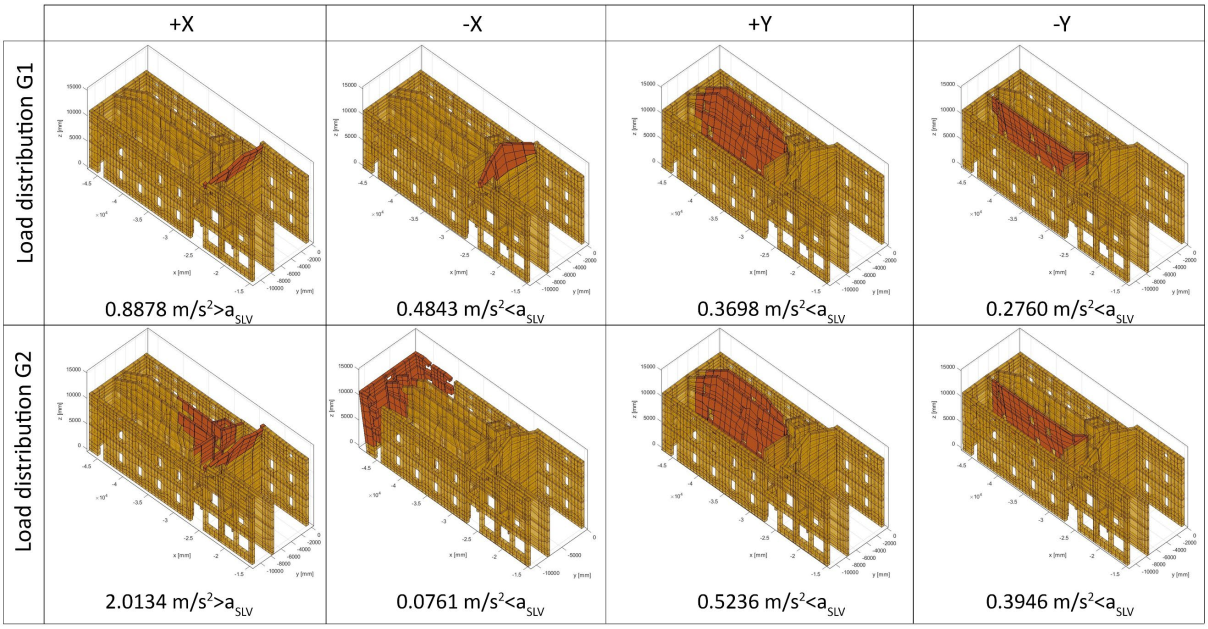

Figure 8: Spectral accelerations triggering local failure mechanisms under G1 and G2 load distributions.

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