Evaluating the dynamic liquefaction potential of tailings: a comparison of simplified methods

Lisandro Roldan[1,2*], *Florencia* Spinazzola[1,2] and *Mauro Giuliano* Sottile[1,2]

[1] *SRK Consulting Argentina, Chile 300 (C1098), Ciudad de Buenos Aires, Argentina*
[2] *Universidad de Buenos Aires, Laboratorio de Mecánica de Suelos, Av. Las Heras 2214 (C1126), Ciudad de Buenos Aires, Argentina*
[*] *Corresponding author: lroldan@srk.com.ar*

ABSTRACT

The state of the practice to evaluate the dynamic liquefaction potential of a soil column entails the use of simplified methods that compares the cyclic stress ratio with the cyclic resistance ratio. One of the most used methods is the Boulanger & Idriss 2014, which relies on cone penetration test data to estimate in-situ cyclic resistance ratio and cyclic stress ratio distributions, considering corrections factors for soils with fine contents. Saye, Olson and Franke in 2021 presented a novel method to assess liquefaction susceptibility using cone penetration test data on soils ranging from non-sensitive clays to clean sands. The procedure was developed using +400 documented case records of liquefaction and non-liquefaction in clean sands, silty sands, sandy silts, and low plasticity fine grained soils. Although promising, this method is not widely used in the industry yet. This paper presents a comparison between both methods for tailings. It uses cone penetration test soundings performed in tailings with a wide range of mineralogy and fine contents, combined with variations in peak ground accelerations and magnitudes. This analysis shows that Saye, Olson and Franke's method is generally more robust, particularly for the analysis of fine tailings, as Boulanger & Idriss' method relies on site specific data or the engineer's judgement to define an Ic cut-off value that screens out clay-like soils from the liquefaction analysis.

Keywords: CPTu, liquefaction, simplified methods, common-origin, tailings.

1 Introduction

Liquefaction occurs when a saturated granular material instantly loses its strength and stiffness, due to an increase of pore pressure. When induced by an earthquake, it is defined as dynamic or seismic liquefaction. Over the last years, there have been several Tailing Storage Facilities (TSF) catastrophic failures, leading to high death tolls, environmental crisis and economic losses. According to Rico et al. (2008), the main causes of failure are earthquakes, overtopping, seepage and foundation instability. As a consequence, the safety of many TSFs around the world is being reviewed and stabilization projects are being design to meet minimum requirements by international guidelines.

As an intrinsically brittle process, it is important to assess the tailings liquefaction potential rather than relying on the observational method (Jefferies and Been, 2016). There are several methods to quantify this potential and one of the most widely used is the Boulanger & Idriss simplified method (Boulanger and Idriss, 2014), which compares the cyclic stress ratio (CSR) with the cyclic resistance ratio (CRR) to compute a factor of safety (FoS).

Boulanger and Idriss method (BI) relies on standard penetration tests (SPT) or cone penetration tests (CPTu) data to estimate in-situ CRR and CSR distributions. Although it has been used satisfactorily for years, it is biased towards clean sands as it evolved from empirical correlations (Kishida, 1996, Seed, 1984) that were developed for clean soils and later extended to low-plasticity soils with reduced fine contents by using correction factors. The limitation of this method regarding the soil gradation and plasticity is generally ignored by practitioners.

A novel method (SOF) has recently been published with a similar approach, but developed using +400 documented case records of liquefaction and non-liquefaction in clean sands, silty sands, sandy silts, and low plasticity fine grained soils. As a result, it does not need to incorporate a fine content correction as the effect of fines is implicitly included in the method (Saye, et. al. 2021).

The objective of this work is to compare the performance of BI and SOF methods considering CPTu soundings performed in tailings from several TSFs around the world. Using data from real site tests allows to enclose the problem to a smaller range of input values, disregarding unrealistic parameters and allowing a more objective comparison.

Section 2 summarizes and compares the key elements and considerations from each method. Section 3 presents a summary of the CPTu database from various TSFs, along with a brief description of the tested materials. Section 4 defines a range of loading conditions and compares both methods for some CPTu soundings. Section 5 compares statistically the the results of both methods. Finally, Section 6 presentes a conclusion of the findings.

2 Literature review

2.1 Dynamic liquefaction potential evaluation

Both methods (BI and SOF) rely on the same concepts. They are based on statistical studies on historical cases where liquefaction was or was not observed. From these analyses, a cyclic resistance surface is determined as a function of some input parameters obtained from CPT testing performed in the same sites of study. If the cyclic stress induced by an earthquake is higher than the cyclic resistance, then it is assumed that the soil liquefies. In its ratio form, a factor of safety against dynamic liquefaction can be calculated as shown in Eq. (1). When FoS is below one at a certain depth in the CPT test, that portion of the soil will likely liquefy.

$$FoS = CRR/CSR \tag{1}$$

These methods have evolved from Seed's original work (Seed, 1984), with definitions varying among the different authors. As they rely on multiple correlations used to fit historical data into a limit state function, it is important to be consistent with the hypothesis, definitions and correlations used by each method.

The section below summarizes the formulas used in BI and SOF to compute the CSR and CRR. For consistency, the notation is adapted from the original sources.

2.2 Boulanger and Idriss method (2014)

2.2.1 Earthquake-induced cyclic stress ratio (CSR)

The cyclic stress ratio considering the corrections for magnitude (M) and effective vertical stress (σ'_{v0}) is estimated using Eq. (2),

$$CSR_{M=7.5,\sigma'_{v0}=1} = 0.65\frac{\sigma'_{v0}}{\sigma_{v0}}\frac{a_{max}}{g}\frac{r_d}{MSF\cdot K_\sigma} \tag{2}$$

where σ_{v0} is the vertical total stress at a given depth, a_{max}/g is the maximum horizontal acceleration (in g) at the ground surface, r_d is the shear stress reduction factor, MSF is the earthquake magnitude scaling factor and K_σ is the overburden stress correction factor.

The r_d factor recommended by the method is calculated through Eq. (3), Eq. (4) and Eq. (5),

$$r_d = \exp[\alpha(z) + \beta(z)M], \tag{3}$$

$$\alpha(z) = -1.012 - 1.126\cdot\sin(z/11.73 + 5.133) \tag{4}$$

$$\beta(z) = 0.106 + 0.118\cdot\sin(z/11.28 + 5.142) \tag{5}$$

where z is the depth below the ground surface in meters. The recommended MSF equation is shown in Eq. (6) and Eq. (7),

$$MSF = 1+(MSF_{max} - 1)\left[8.64\cdot\exp\left(\frac{-M}{4}\right) - 1.325\right] \tag{6}$$

$$MSF_{max} = 1.09 + (q_{c1Ncs}/180)^3 \leq 2.2 \tag{7}$$

where q_{c1Ncs} is the normalized corrected tip resistance considering the fine content. Finally, the K_σ is determined using Eq (8) and Eq (9),

$$K_\sigma = 1 - C_\sigma\cdot\ln[(\sigma'_v)/P_a] \leq 1.1 \tag{8}$$

$$C_\sigma = \frac{1}{(37.3 - 8.27(q_{c1Ncs})^{0.264})} \leq 0.3 \tag{9}$$

2.2.2 Cyclic resistance ratio (CRR)

The cyclic resistance ratio at a reference magnitude $M = 7.5$ and stress $\sigma'_{v0} = 1atm$ ($CRR_{M=7.5,\sigma'_v=1}$) is obtained from the CPTu results through Eq. (10),

$$\begin{aligned}CRR_{M=7.5,\sigma'_v=1} &= \exp[q_{c1Ncs}/113 + (q_{c1Ncs}/1000)^2 \\ &-(q_{c1Ncs}/140)^3 + (q_{c1Ncs}/137)^4 - 2.6 + \sigma\cdot\Phi^{-1}[P_L]]\end{aligned} \tag{10}$$

This probabilistic version of the correlation considers an uncertainty parameter with σ being the uncertainty in the CPT results (usually 0.2 or 0.24, depending on the soil input parameters uncertainty), Φ^{-1} is the inverse of a standard cumulative normal distribution and P_L is the probability of liquefaction, usually 15%. The fine content correction is obtained considering an additive correction factor (Δq_{c1N}) in Eq. (11),

$$q_{c1Ncs} = q_{c1N} + \Delta q_{c1N} \tag{11}$$

The adjusting expression for CPT is Eq. (12),

$$\begin{aligned}\Delta q_{c1N} = (11.9 + q_{c1N}/14.6)\cdot\exp[1.63 - 9.7/(FC+2) \\ -(15.7/(FC+2))^2]\end{aligned} \tag{12}$$

with the fine content (FC) expressed in percentage. FC can be estimated from the CPT test interpretation of the soil behavior type (I_c), using Eq. (13) and considering the uncertainty of the correlation with $C_{FC} = 0.0, -0.29, +0.29$. For this study, $C_{FC} = 0.0$ is used.

$$FC = 80(I_c + C_{FC}) - 137 \tag{13}$$

The method uses I_c from Robertson and Wride (1997) definition as function of the normalized tip and sleeve friction ratios, Q and F, respectively. Although it is not explicitly included in the method's equations, BI suggests to use a cut-off value of I_c to screen out clay-like soils from the analysis. A value of 2.6 is commonly used, but it is recommend to repeat the analyses with different cut-off values to assess its impact on the results.

The corrected tip resistance for the overburden stress effect is calculated with Eq. (14),

$$q_{c1N} = C_N \frac{q_c}{P_a} \tag{14}$$

C_N is an overburden correction factor recommended by Idriss and Boulanger (2008) which is calculated iteratively thorugh Eq. (15) and Eq. (16),

$$C_N = \left(\frac{P_a}{\sigma'_{v0}}\right)^m \leq 1.7 \tag{15}$$

$$m = 1.338 - 0.249(q_{c1Ncs})^{0.264} \tag{16}$$

In Eq. (14), the uncorrected tip resistance, q_c, is indicated as in Idriss and Boulanger (2014). The authors state that there is a small impact in using q_c instead of q_t (cone tip resistance corrected for area effects and pore pressure, u_2) for sands but recommend performing the correction whenever u_2 data is available. In this study, q_t is employed in Eq. (14).

2.3 Saye, Olson and Franke (2021)

2.3.1 Earthquake-induced cyclic stress ratio (CSR)

The definition for the $CSR_{M,\sigma'_{v0}}$ in SOF's method is the same as BI, although the coefficients r_d, MSF and K_σ are computed according to Youd (2001) with Eq. (17), Eq. (18) and Eq. (19),

$$r_d = \begin{cases} 1.0 - 0.00765 \cdot z & \text{for } z < 9.15m \\ 1.174 - 0.0267 \cdot z & \text{for } 9,15 \leq z < 23m \end{cases} \tag{17}$$

$$MSF = \frac{10^{2.24}}{M^{2.56}} \tag{18}$$

$$K_\sigma = \left(\frac{\sigma'_{v0}}{P_a}\right)^{f-1} \tag{19}$$

where f is an exponent which is function of the site condition and relative density (D_r).

2.3.2 Cyclic resistance ratio (CRR)

The main difference with BI's method is the determination of CRR. Instead of relying on the correction of the normalized tip stress of the CPT test, the authors re-calibrated the strength correlations with respect to the normalized uncorrected stress tip (q_{c1}/P_a) and a new soil classification parameter, Δ_Q.

Δ_Q is simply the slope of a wide range of linear functions in the $Q_t - f_s/\sigma'_{v0}$ space, when the origin is $Q_t = -10$ and $f_s/\sigma'_{v0} = -0.67$, as shown in Eq. (20),

$$\Delta_Q = \frac{Q_t + 10}{f_s/\sigma'_{v0} + 0.67} \tag{20}$$

Here the normalized corrected tip resistance (Q_t) is computes from Eq. (21),

$$Q_t = \frac{q_t - \sigma_{v0}}{\sigma'_{v0}} \tag{21}$$

The CRR is re-calibrated to Eq. (22) and Eq. (23),

$$CRR_{M=7.5,\sigma'_{v0}=1} = 10^{m_{CRR}(q_{c1}/P_a)-1.34+\sigma\cdot\Phi^{-}1[P_L]} \tag{22}$$

$$m_{CRR} = \frac{\Delta_Q}{178 \cdot \Delta_Q - 3349} \leq 0.1 \text{ for } \Delta_Q > 20 \tag{23}$$

where q_{c1}/P_a is computed through Eq. (24) and Eq. (25),

$$\frac{q_{c1}}{P_a} = C_q \frac{q_c}{P_a} \tag{24}$$

$$C_q = \left(\frac{P_a}{\sigma'_{v0}}\right)^n \leq 1.7 \tag{25}$$

with $n = 0.5$ regardless of the soil type and q_c is the uncorrected tip resistance.

Note that the authors recommend considering a $P_L = 35\%$ for the results to be comparable with BI's method.

3 CPT database

To do a practical comparison of both methods, data from real CPTu campaigns in several TSFs around the world were chosen. Table 1 summarizes the heights, tailings minerals and number of selected CPTu tests for each TSF.

Table 1. Data sources

Site	TSF Height	Minerals	Description	N° CPT
A	30m	Copper, uranium, silver, gold	Low plasticity clay to silt	8
B	65m	Copper, gold, silver and zinc	Silty sand to Silt	3
C	33m	Niquel, copper, gold and silver	Clay to silty sand	5
D	75m	Iron	Silty material	9
E	70m	Fluorite	Non-plastic clay to silty sand	3
F	17m	Andalusite	Silty. Non-plastic to high plasticity	7

For all the mentioned sites, only tests performed in shallow (depths up to 20m) and saturated tailings were considered.

4 Simplified dynamic trigger analysis

4.1 Ground motions

The analyses were carried out considering a wide range of M and peak ground accelerations (PGA), in order to assess the influence of the loading conditions in the results. It is especially relevant the effect of M, as the expressions from BI correlations change for different magnitudes.

For M, three cases were studied: 6.5, 7.5 and 8.0. Such values are typical in examples available in the literature, particularly $M = 7.5$ which yields $MSF = 1$ in both methods. For PGA, a wide range between 0.05 and 1.0 was chosen.

4.2 Cyclic stress ratio analysis

Figure 1 shows the interpretation for two CPTu performed on typical interlayered coarse-fine tailings; these were taken from the database for site D and E. Only data below the water table is plotted, as the applicability of the methods is restricted to saturated materials. The raw data (f_s and q_c) are plotted together with D_r (after Bray, 2022) and FC, along with the parameters and factors employed in the CSR calculations with both methods (r_d, MSF and K_σ). The correlation employed for FC is the one used by BI, Eq. (13). The comparison of the plotted parameters of both methods explains the differences observed in the calculated CSR, shown later in section 4.4.

Regarding r_d, SOF uses a unique correction depending on the depth, while BI additionally depends on M. For shallow depths, SOF's coefficient is similar to high magnitude BI's coefficient; as depth increases, SOF's curve approaches the lower magnitude BI's curve.

Another source of discrepancy is MSF. SOF's coefficient is a constant value that depends only on M, Eq. (18), resulting in much higher values for high magnitudes and much lower values for low magnitudes, when compared with BI's coefficients. BI's coefficient, on the other hand, depends also on q_{c1Ncs}, which, in turn, depends on q_c and FC. However, noticeable changes in these parameters entail little variations in BI's MSF (e.g. refer to the difference between FC and q_c for site D and E).

Finally, the overburden correction factor, K_σ, shows similar values in both methods. This was achieved by selecting a proper value of f, Eq. (19), according to the tailings D_r. Note that SOF's database consisted mostly in cases where $K_\sigma \approx 1$, so it was excluded from their analysis. In this work, values in the range of 0.85-1.10 were obtained.

These findings reinforce the recommendation of using the same equations as per those presented by the different authors, as each of them was determined from different calibrations process and assumptions.

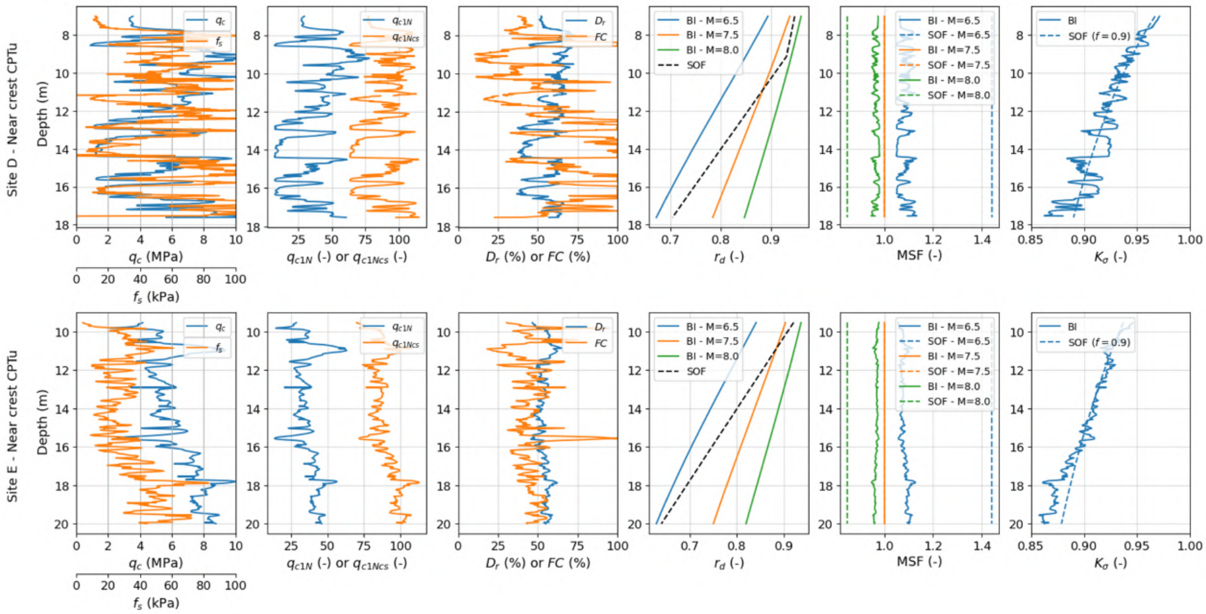

Figure 1. Raw data and coefficients for CSR computation for two representative tests.

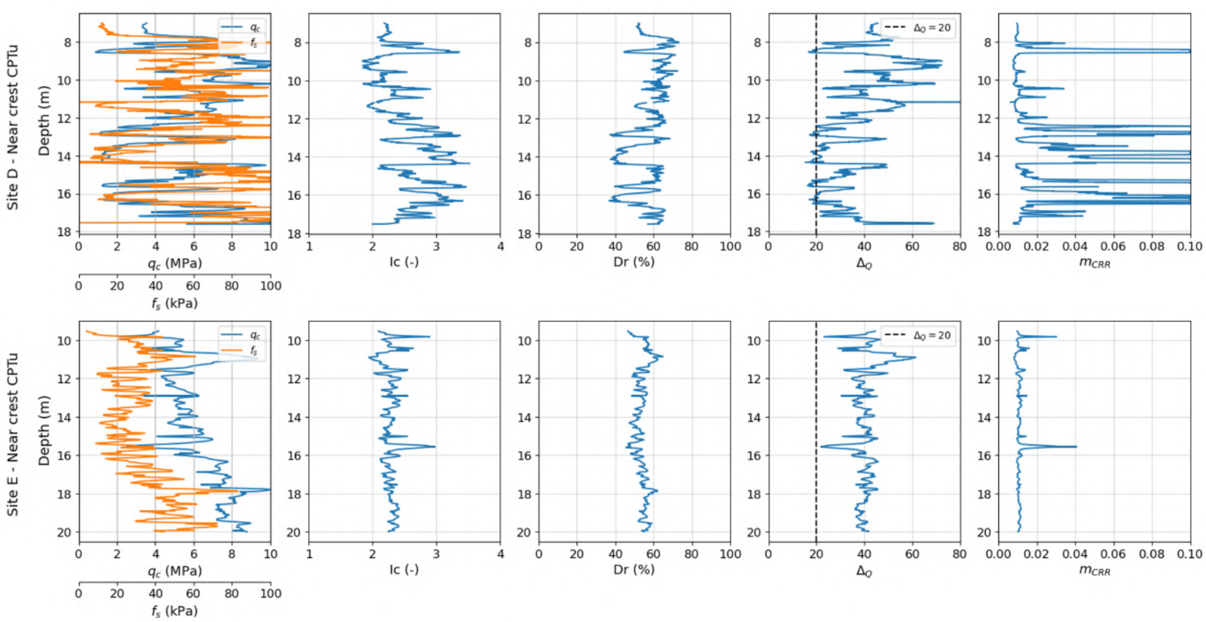

Figure 2. Influence of FC in CRR computation for two representative tests. SOF method.

4.3 Cyclic resistance ratio analysis

SOF's method relies on the computation of Δ_Q. When $\Delta_Q > 20$, SOF's method can compute a m_{CRR} and then a CRR. The resulting CRR may or may not be similar to BI's results, but when the conditions are met so that $\Delta_Q < 20$, SOF's method assumes that the material is not susceptible to liquefaction and therefore no m_{CRR} or CRR is computed. This is shown in Figure 2 for site D, where portions of the m_{CRR} are missing when this occurs. Note as well that when Δ_Q is very close to 20, m_{CRR} spikes as it becomes asymptotic in Eq. (23).

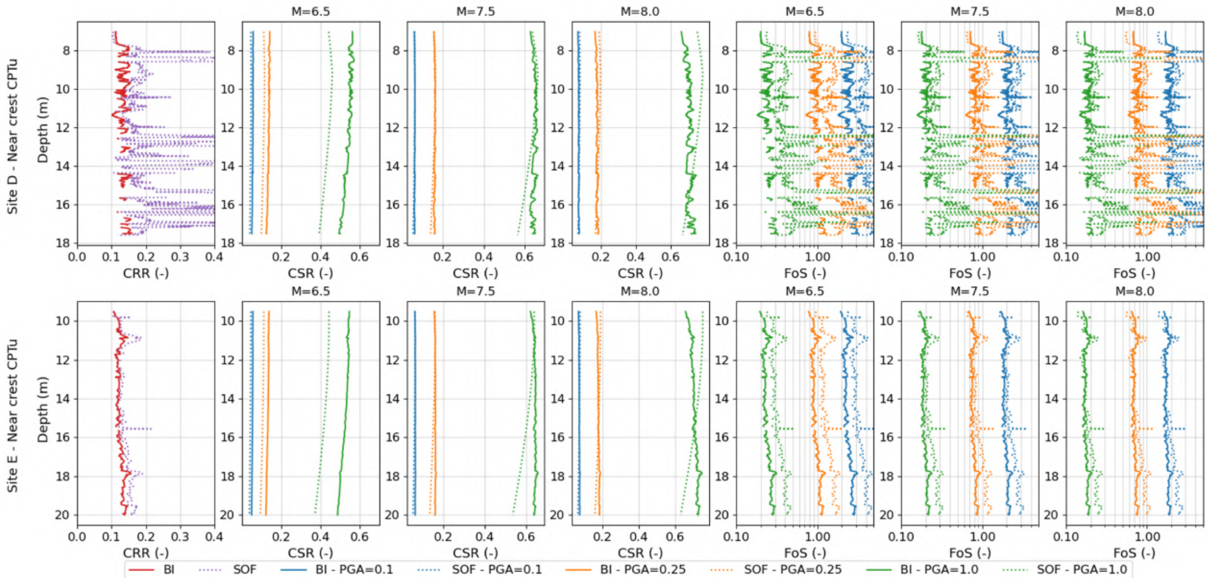

Figure 3. Comparison on CRR, CSR and FoS for two representative tests.

It is important to remark that even though FC plays an important role in the calculation of Δ_Q and therefore CRR, the in-situ state of the material is also important. Even the finest materials may be susceptible to liquefaction (or cyclic mobility) if they are loose/soft enough, while coarser materials may not be susceptible if very dense. This is illustrated in terms of state parameter (calculated after Plewes, 1992) in Figure 4.

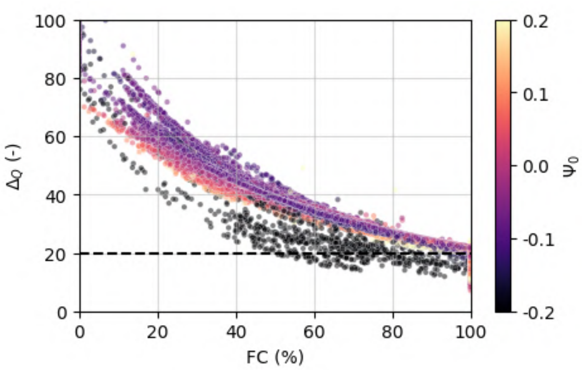

Figure 4. Effect of I_c and in-situ state in Δ_Q.

BI's method computation is straightforward as it depends only on q_{c1Ncs}. Additionally, it contemplates a screening out of clay-like materials by including an I_c cut-off value (points with higher I_c are considered as non-liquefiable). For this part of the study, an $I_c = 2.6$ was chosen as cut-off. In the first column of Figure 3, the resulting CRR for the same CPTu as in the previous section are shown.

For coarse tailings (Site E) the results for both methods are very similar. However, for finer portions of the sounding as in Site D (see I_c plot in Figure 2), the CRR differences are more evident. In general, SOF's method yields higher CRR than BI's method.

As both methods consider in some way a cut-off or asymptotic formulation, the CRR curves for both methods show empty sections. Those portions of the sounding are considered as non-liquefiable. In general, these portions coincide for both methods. However there are some points where BI's method yields non-susceptibility to liquefaction but SOF's method still computes a low value for CRR.

4.4 Factor of Safety

Figure 3 shows the CRR, CSR and FoS variation with depth obtained with both methods. Nine curves of CSR and FoS are shown, depending on selected M and PGA. As expected, both methods entail lower FoS with increasing M (plots from left to right) and increasing PGA (curves from blue to green).

Comparing both methods, some differences in FoS values are found, which are attributed to the discrepancies in CSR and CRR between the methods, as pointed out in sections 4.2 and 4.3, respectively. In some cases, they arrive to very similar values of FoS, as the second CPTu shown in Figure 3 (Site E). However, in many other cases, as shown with the first CPTu on the same figure (Site D), the methods can differ.

In most of the cases, SOF's method results in higher

FoS than BI's method. This is partially explained by the higher CRR. However, in some portions with $I_c > 2.6$ but also $\Delta_Q > 20$, SOF's method computes a CRR while BI's method does not, resulting in $FoS < 1$ for SOF while BI neglects liquefaction.

The differences in the CSR that have higher impact on FoS are related to M, mainly due to the MSF factor calculation, as discussed in section 4.2. For M lower than 7.5, SOF's method results in lower CSR than BI's, while for M higher than 7.5, the opposite occurs. Note that when M is 7.5, both methods are better aligned (as the effect of MSF factor is null) and then the FoS are similar for both methods in coarse tailings.

5 Statistical analysis

When the methods are compared in terms of proportion of points that result in $FoS \leq 1$, the impact of the selection of I_c cut-off can be appreciated. In Figure 5 (left) the results for $M = 7.5$ and different PGA values are plotted for three sites. For BI's method, three cut-off values of I_c were considered: 2.6, 3.0 and no cut-off. The sites were chosen such that a wide range of tailing gradations are represented (Figure 5, right).

For fine tailings (Site A), the methods yield completely different results. SOF's method show a soft transition between 0% of liquefied points for $PGA = 0.05$ to 80% for $PGA = 0.6$ (still increasing for higher PGAs). Instead, BI's method shows a high dependency on the I_c cut-off value. For $PGA = 0.6$, when points with $I_c > 2.6$ are screened out, no liquefaction is expected; when points with $I_c > 3.0$ are screened out, only 20% liquefaction is expected, and without cut-off, 100% liquefaction is expected. In general, SOF's method yields more liquefaction than BI's method with cut-off.

For coarse tailings (Site E), the methods become comparable. For high PGAs, all the methods yield between 80% and 100% liquefaction. In fact, for $PGA > 0.4$, SOF's method and BI's method with I_c cut-off value of 3 yields the same percentages. The main differences are observed in the intermediate PGA range ($0.15 < PGA < 0.3$) where SOF's method yields lower percentages than BI's method. For example, for a $PGA = 0.2$, SOF's method finds that only 25% of points liquefy compared to BI's 60-70%.

The intermediate case (Site D), show results compatible with the previous remarks. The effect of the I_c cut-off value selection plays an important role in the resulting percentages, but not as critical as in site A. Similar results are obtained for SOF's method and BI's method with I_c cut-off value of 3 for high PGAs. For intermediate PGAs SOF's method yields lower percentages than BI's method.

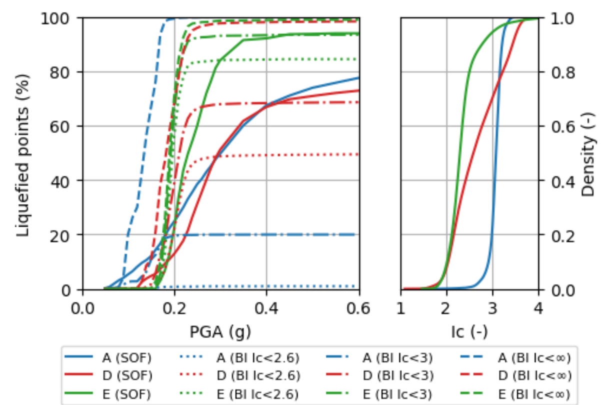

Figure 5. Liquefaction points proportion as function of PGA (left) and I_c distribution for each Site (right).

6 Conclusions

Two simplified methods for assessing dynamic liquefaction triggering were analyzed and compared: BI, after Boulanger & Idriss (2014), which is the most used method in the industry, and a novel method, SOF, after Saye, et. al. (2021). These methods were applied to real CPTu data from different TSFs around the world, comprising a wide range of tailings gradation and mineralogy. Several combinations of seismic magnitudes (M) and peak ground acceleration (PGA) were considered as loading conditions.

After comparing the results of both methods for each analyzed CPTu and performing statistical analyses, the following concluding remarks can be made:

- The calculation of the cyclic stress ratio (CSR) follows the same expression in both methods (although with some differences among the correction coefficients) while the cyclic resistance ratio (CRR) is obtained following different approaches.

- The differences between the methods in terms of CRR are mainly related to the fine content (FC) treatment through out the methods. Similar CRR are obtained for coarse materials.

- There is a strong influence of M in the results: while both methods calculate similar CSR for M=7.5, the difference between the methods increases notoriously when other M is selected; when $M < 7.5$, SOF's methods yields lower CSR than BI's method, while when $M > 7.5$, the opposite is observed.

- For BI's method there is a strong dependency on the adopted I_c cut-off value, being the greatest source of uncertainty for the method when soils with fines are analyzed.

- The greatest benefit of SOF's method is that although it considers a cutoff value for Δ_Q, it is based in case history evidence while BI's method leaves the selection of a I_c cut-off to the engineer's judgement or to site specific data.
- If high I_c values are not screened out from the analysis, BI's method yields unrealistic results and it's not comparable with SOF's method.
- Not only FC is indicative of liquefaction susceptibility, but also the in-situ state. This is well accounted for in SOF's method as fine materials may liquefy (or reach cyclic mobility) if soft/loose enough and the loading conditions are high.
- It is fundamental to use the same set of equations and considerations as in the original publications. The differences between the coefficients for obtaining CSR cannot be ignored, since the methods calibrations were performed using the equations indicated in the corresponding works.

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