Critical speed of railway sections obtained using surface wave analysis tests

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ABSTRACT

In recent years, there has been a global trend towards increasing the commercial speed of high-speed rail lines (HSLs) to cope with expected demand problems. Examples include the new HSLs projects for the UK and California, where the design speed reaches 400 km/h. Therefore, one of the biggest challenges currently faced by the railway sector is the determination of the critical speed value of railway sections. This is true both for future HSLs, due to their high design speeds, and for existing rail lines, due to the intended increase in commercial speed.
The critical speed is that train speed that produces a dynamic amplification in the medium underlying the track, causing an amplification of the vertical movement of the track components and the supporting ground such that the stability of the infrastructure and the safety of passengers is compromised.
This work sets out the methodology necessary to experimentally study the critical speed of a railway section by *in situ* applying surface wave spectral analysis techniques, SASW and MASW. The critical speed is determined by the lowest phase velocity of the local minima of the modal dispersion curves, or by the minimum of the apparent dispersion curve. Both methods are equivalent.
This paper also presents the results obtained on some Spanish HSLs, both on tracks under construction and in operation. In the cases studied, the ballast layer is the one that presents the lowest shear wave velocity and the minimum of the dispersion curve, so it is the layer that determines the critical speed.

Keywords: critical speed; SASW; MASW; railway sections.

1. Introduction

In recent years, there has been a trend towards increasing the commercial speed of high-speed rail lines (HSLs) to meet anticipated demand problems. Examples are the new HSL projects for the UK and California, where the design speed reaches 400 km/h. In addition, the possibility of increasing the maximum commercial speed on existing HSLs is under consideration in several countries. Therefore, one of the biggest challenges currently facing the rail sector is the determination of the critical speed value. This is true both for future HSLs, due to their high design speeds, and for existing HSLs, due to the expected increase in commercial speed.

The critical speed is that train running speed which produces a dynamic amplification in the underlying medium of the track. It is a condition that resembles resonance, causing an amplification of the vertical movement of the track components and the supporting ground such that the stability of the infrastructure and the safety of the passengers themselves are compromised. This type of amplification is due to a coupling between the velocities of the train and the waves of the underlying medium.

Since 2017, the Geotechnical Laboratory of CEDEX (Public Centre for Technical Assistance and R+D+i of the Spanish Ministries of Transport and Environment). has opened a line of research to study the phenomenon of critical speed both from a theoretical and experimental point of view. In 2022, an informative conference was held at the CEDEX facilities in which a novel proposal was presented to study the critical velocity phenomenon in an integral way (CEDEX 2022). It is a simple, reliable, and robust method based on a novel theoretical model that has been corroborated with numerical modelling and the application of the method to known real cases. To apply the calculation method, it is necessary to have real data on some mechanical properties of the railway sections obtained from *in situ* geophysical measurements, among which those derived from the surface wave spectral analysis stand out.

This paper briefly presents the methodology required to experimentally determine the critical speed of a railway section. Furthermore, the results obtained on a railway section with a conventional embankment of a HSL are presented.

2. Method used to determine the critical speed of railway sections

2.1. Background information

The need to achieve high running speeds, as well as the requirement that journeys can be made in short periods of time, means that new HSLs have a straighttrack layout, which means that on many occasions it is unavoidable for them to pass through areas with soft soils.

The existence of actual cases where excessive vibrations have been detected in the railway track and surrounding ground during the passage of trains at speeds as low as 200 km/h over soft ground has put the focus on the so-called critical speed phenomenon. At such locations, the vertical deflection experienced by the rail was up to twice that of the static case. Fig. 1 presents different real cases of very high rail deflections associated with the critical speed phenomenon. The information contained in this figure and a detailed explanation can be found in Woldringh and New (1999), one of the first papers entirely devoted to this phenomenon. This paper studies, among others, the case of Ledsgard, Sweden, which is the most significant known case of critical speed (Madshus and Kaynia 2000).

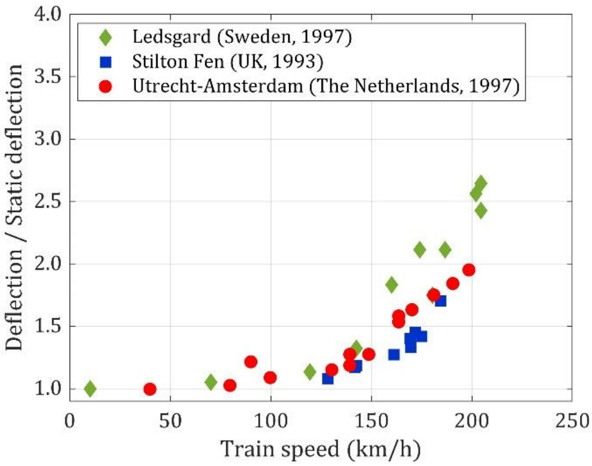


Figure 1. Association of high rail deflections with critical speed (From Woldringh and New 1999).

The effects on the track due to the critical speed phenomenon manifest before the train reaches such speed. These effects include vertical displacements of the track and ground, impacting passenger comfort, increasing maintenance, and causing stability and safety issues. It underscores the importance of knowing the critical speed to take preventive measures.

The method proposed by CEDEX is based on a novel theoretical model, supported by numerical modelling and the application of the method to known real cases, such as those in Fig. 1.

Next, some relevant aspects of the theoretical model; the calculation method; and the experimental part, which provides the necessary data for the model, are briefly discussed.

2.2. Theoretical model of the critical speed

It has been known since the end of the last century (Woldringh and New, 1999), that vertical surface waves, or Rayleigh waves, play a crucial role in the degree of dynamic amplification of the deflection experienced by rails. Although some authors have related the critical speed to the dispersion curves associated with the railway track system (Sheng et al. 2004; Alves Costa et al. 2015; Mezher et al. 2016), there were still some doubts about the relationship between critical speed and surface waves (Estaire and Crespo-Chacón 2024). Moreover, it was not known which of all the velocities involved in the propagation of these waves was the critical speed. The first rigorous theoretical demonstration in this respect is reported by Kausel, Estaire and Crespo-Chacón (2020).

In this paper, it is firstly solved the general equation of a beam on an elastic foundation (BEF) and it is shown that the critical speed is that phase velocity for which the dispersion curve reaches the minimum; where:

a) the phase velocity is defined as the velocity at which the wave peak propagates,
b) the dispersion curve is that which relates the phase velocity to the frequency (ω) or wavelength (λ).

Furthermore, it is shown that the phase velocity and the group velocity coincide at this point, the latter being the velocity at which the energy of the wave packet propagates. It means that when the critical speed is reached, the train moves at the same speed at which the energy propagates through the waves in the underlying medium. Fig. 2 summarises these results.

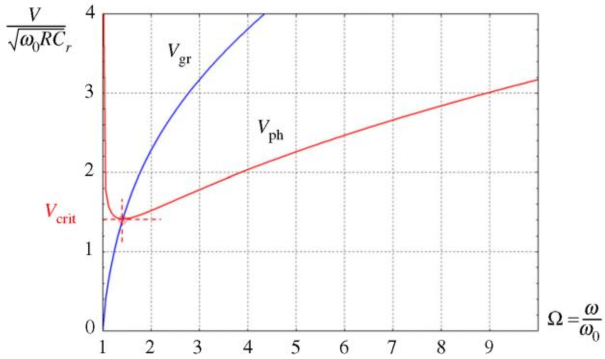


Figure 2. Summary of the results obtained by Kausel, Estaire and Crespo-Chacón for a BEF (2020).

Additionally, this study extends beyond solving the BEF case, including multilayer systems like a railway section (Fig. 3), covering layers such as ballast, sub-ballast, form layer, embankment, and natural ground, the latter possibly being stratified.

In multilayer systems, the dispersion curve isn't unique; multiple oscillation modes emerge, each with an associated modal dispersion curve. Conclusions of the study include:

1. Critical speed equals the lowest phase velocity of local minima in modal dispersion curves.
2. Critical speed depends on the geometry and mechanical properties of different layers via dispersion curves.
3. Critical speed is intrinsic to the studied section, independent of train load spatial configurations. Therefore, the critical speed does not depend on the frequencies generated by the passing train.
4. The influence of track grid components (rails, fastenings, under-rail pads, and sleepers) on critical speed is negligible.

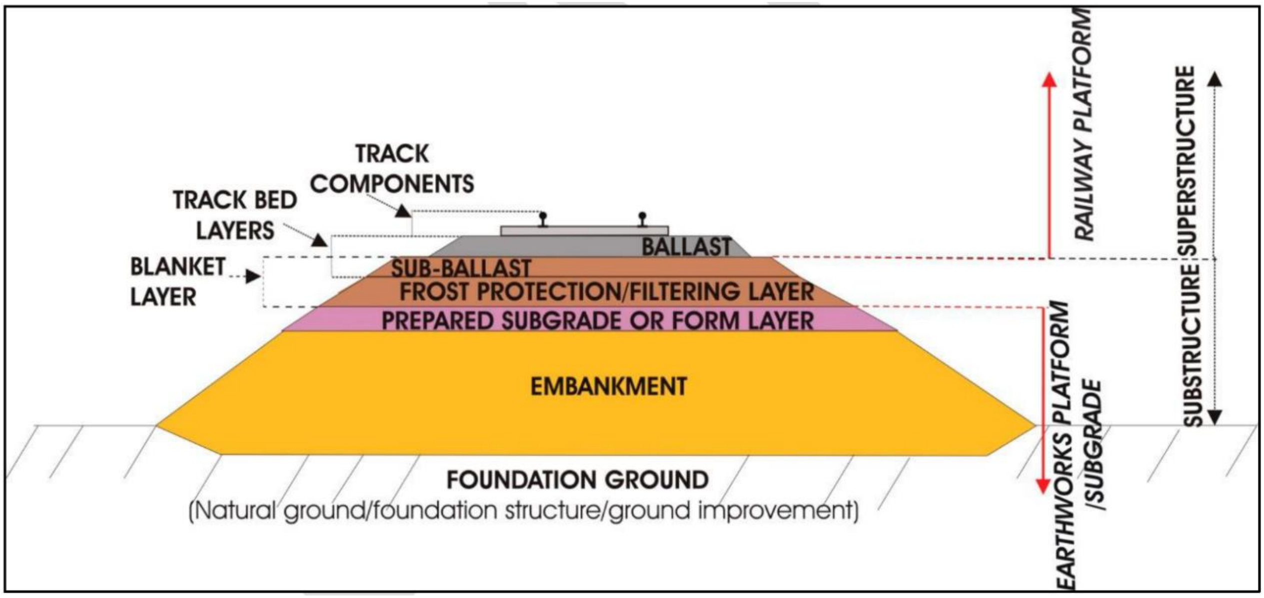


Figure 3. Cross section of a track on ballast and platform. From: UIC-IRS 719 (2020).

2.3. Use of dispersion curves to determine the critical speed of railway tracks

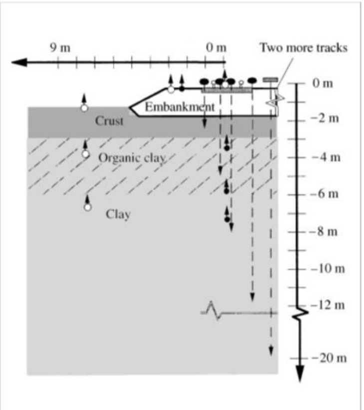
Using the theoretical model as a basis, a method has been developed to determine the value of the critical speed of railway sections, which consists of the following steps:

a) Development of the layer model for the railway section: Definition of parameters for each layer, such as thickness, shear wave velocity, density, Poisson's ratio, and damping ratio. Key parameters are thickness (H) and shear wave velocity (V_S).

b) Calculation of dispersion curves for surface waves: Theoretical derivation of modal dispersion curves for Rayleigh-type surface waves associated with the layer model. It is also possible to work with the apparent dispersion curve, resulting from the superposition of different modes of oscillation.

c) Determination of the critical speed from dispersion curves: The critical speed is determined by the lowest phase velocity of local minima in modal dispersion curves or the minimum of the apparent dispersion curve. Both methods are equivalent.

An example applying this calculation method to the Lesgard case is presented in Fig. 4, showing the cross-sectional view of the railway section, its layer model, and the associated modal and apparent dispersion curves. The critical speed obtained from them is about 64 m/s (Estaire and Crespo-Chacón 2024). This result is in line with what was observed in Ledsgard and with that obtained by other authors with different methods that take into account the configuration of the train (Madshus & Kaynia 2000).



Layer	Thickness (m)	V_s (m/s)
Embankment[1]	1.40	250.0
Crust	1.20	60.0
Organic clay	3.0	40.0
Clay	2.0	59.5
	2.0	68.5
	2.0	77.5
	2.0	86.5
	2.0	96.5
	∞	100.0

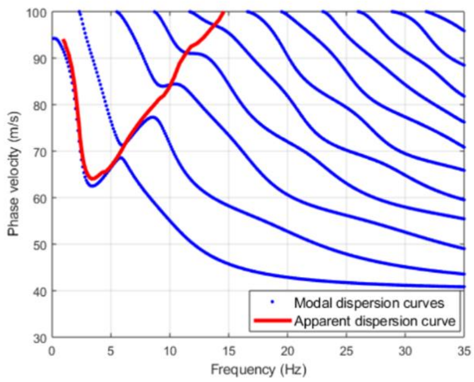


Figure 4. Example of railway section (left), corresponding layer model (center) and modal and apparent dispersion curves together with critical speed (right). Application to Ledsgard case. From: Estaire and Crespo-Chacón (2024).

2.4. Geophysical measurements necessary for the critical speed determination

Due to the direct relationship between the critical speed and the dispersion curve, having a dispersion curve obtained directly from a single test over the whole railway section would be ideal. Unfortunately, in practice, this is not straightforward. This would require high-resolution characterisation of the thin surface layers and the ability to penetrate several meters into the foundation of the section. The spacing of sensors and energy in each case is very different. An alternative way to determine the critical velocity of a section as in Fig. 3 is to obtain a layered model of the section and, therefore, a ground profile containing, among other parameters, the thickness (H) of the layers that make up the section and the value of the shear wave propagation velocity (V_S), which must be obtained in the field by applying *in situ* geophysical techniques.

This geophysical characterisation of the ground is most conveniently carried out using surface wave analysis methods, the objective of which is to obtain the dispersion curve of the surface waves generated during the test (Foti et al. 2017). From this dispersion curve, a model of the variation of V_S with depth (z) is determined to generate a theoretical dispersion curve that is as close as possible to the one obtained experimentally. Therefore, the final result of these geophysical tests is a ground profile defined by layers of a given thickness and shear wave propagation velocity (V_S-z).

The surface waves studied with these techniques are Rayleigh. The Rayleigh wave phase velocity (V_{Phase}) is directly related to V_S, through a constant (k) that depends on Poisson's ratio (ν). Eq. 1 and Eq. 2 show, respectively, the relationship between the two velocities and the dependence of the constant on Poisson's ratio (Sheriff 1995). The constant varies between 0.92 and 0.96, being very common a value of $k = 0.93$.

$$V_S = k\ .V_F \quad (1)$$

$$k = \frac{1}{(0.874 + 0.197\,\nu - 0.056\,\nu^2 - 0.0276\,\nu^3)} \quad (2)$$

Thus, from the Rayleigh wave dispersion curve and using an appropriate inversion algorithm, the profile of *Vs* versus depth, and *vice versa*, can be determined.

There are several methods of surface wave analysis, but, based on the experience of the authors, the following are recommended for the study of critical speed:

- Spectral Analysis of Surface Waves (SASW). This is a method that provides high resolution in the thin layers below the ballast layer (Fig. 3) and allows the data to be validated as the test is carried out. Despite being a laborious test, it is ideal for characterising all the layers of the cross-section, including the thinner surface layers.
- Multichannel Analysis of Surface Waves (MASW). It is a much faster test than SASW, but in the authors' experience, it is not very practical for characterising very thin surface layers, less than 1 m thick. It is indicated if the aim is to know the distribution of layers of the natural ground and embankments in construction with thicknesses greater than 1 m.

3. Results obtained on Spanish HSLs

3.1. Measurement campaigns on tracks during construction

During 2010 and 2013, several *in situ* campaigns were carried out to obtain the mechanical properties of tracks that were being built at that time in Spain. It is worth noting that in those years Spain was leading Europe in the construction of HSLs becoming in 2022 the third country in the world, after China and India, with the most kilometers of HSLs built (UIC 2023). Among the tests carried out during those years at different points of the Spanish network, several SASW tests were performed to determine the value of V_S in the different layers of the railway sections (Fig. 3). The aim of that work was not to study the critical speed but to make a comparison of the values of sections built with different materials and to compare the values of *Vs* with other geotechnical tests such as plate load tests, penetrometers or *in situ* density and moisture tests.

The compilation of this data has helped to establish representative values of the V_S associated with these layers, which are of great help in the study of critical speed. A summary of the V_S values obtained in those field campaigns is presented in Table 1. In addition to the range of values obtained, a representative value has been assigned to each layer, by rounding the most common value.

Table 1. V_S values obtained for the different layers of the track cross-section (field campaigns 2010 to 2013)

Layer	Thickness ΔZ (m)	V_S (m/s) representative values	V_S (m/s) range of values
Subbalast	~ 0.3	350	275-400
Form layer	~ 0.3	350	275-375
Upper embankment	~ 0.5	300	225-350
Embankment	2-7	300	225-300
Natural ground	>5	>300	>300

In all the areas studied the natural ground had higher V_S than the embankment. At that time, no areas of soft soils or areas where ground improvement techniques had been applied were studied.

3.2. Case study: Determining the critical speed on a track in operation

3.2.1. Introduction

During May 10th and 11th, 2022, a series of surface wave tests were conducted to determine the total dispersion curve of a railway section at kilometer point 91+500 of the Madrid-Barcelona HSL. In March 2022, a monitoring campaign was carried out at this location to verify that the section at this point behaved as expected without any anomalous vibrations. The track section at this point is like the one shown in Fig. 3, consisting of ballast and a platform with an embankment approximately 3 meters high.

To obtain the dispersion curve of the entire railway section, it would be ideal to study a point on the surface of the track, at the level of the sleepers, and characterize the different layers that make up the section until reaching the natural ground beneath. Unfortunately, this is challenging on a track in operation. For the signals generated during a SASW test to penetrate several meters and reach the natural ground, using sources capable of generating long wavelengths and transmitting a significant amount of energy is necessary. Typically, strong impacts generated by the fall of large masses are employed for this purpose, using impact deflectometers or accelerated masses with hydraulic systems. This is hardly feasible on a track in service, both due to the logistics of conducting the test and the potential damage to the track components.

Therefore, to characterize the entire railway section, measurements were separately taken at different points on the ballast layer, embankment, and natural ground supporting the railway structure. The objective was to obtain velocity profiles at each point and then combine them into a single-layered model containing the velocity profile of the entire railway section. From this model, the aim was to obtain the dispersion curve associated with that whole section and, therefore, to determine the critical speed.

Fig. 5 shows three photographs taken during the tests in the different areas. On the left, a test on the ballast pavement; in the center, the embankment (the red line indicates the position of the sensors and sources, parallel to the track and between the service duct and the shoulder of the embankment); on the right, the service path where the tests to characterise the natural ground were carried out.

The results obtained for each of the tests performed on the different locations are presented separately below.

Figure 5. Photos taken during the performance of the tests carried out to study the ballast (left); the embankment (center) and the natural ground.

3.2.2. Tests performed to study the ballast layer.

The ballast layer is undoubtedly the most difficult layer to characterise, as the signals obtained during the test are not as clear as in the natural ground or the embankment. This may be because the ballast layer is at the limit of being considered a continuous medium, given the ratio between the particle size (around 5 cm) and the thickness of the layer (around 50 to 60 cm in total). The number, distribution, size, and shape of the ballast particles determine the number and strength of the contacts between the particles and this, in turn, determines the value of the wave propagation velocity through the particle structure. This has a greater influence on the characterisation of the upper part of the ballast layer as the thickness is reduced and is about the same order of magnitude as the particle size. Thus, in those first centimeters, significantly different values can be obtained between closely spaced points. Therefore, 5 different SASW tests were carried out at different points near each other, but only in 3 of them did the quality of the signals allow a reliable interpretation.

Due to the sensor separations used to perform the test, in addition to the ballast, the layers just below it, i.e. the sub-ballast and the form layer, can be studied (Fig 3).

The measurements were carried out following the experience acquired in the CEDEX Track BOX (CTB). CTB is a facility whose main objective is to test complete sections of both conventional and high-speed railway tracks at a 1:1 scale (Estaire et al. 2017). The procedure followed to perform the tests on the ballast layer is described in detail by Ruiz et al (2024), and can be summarised as follows: measurements were carried out by placing accelerometers on the surface of the sleepers and hitting with small hammers a small metal bar that rested directly on some ballast particles located between the sleepers, close to the sleepers where the accelerometers had been fixed. Signals were also generated by tapping directly on the sleepers next to the sensors.

Fig. 6 shows the results obtained at the three points analysed. In the graph on the right, in addition to the layer models of the three points, their average is shown. In the graph on the left, the dispersion curve associated with the average is shown together with the experimental curves of the three points. The values shown in the right-hand graph are those of the representative model. These V_S values are of the same order of magnitude as those obtained by other authors using the same geophysical technique (Ruiz et al., 2024; Stark et al., 2018; Hwang and Park, 2014). The ballast layer at that point has a total thickness of ΔZ=0.60 m; the depth starts from the ballast surface in the area at the outer end of the sleeper, and is subdivided into two sub-layers: the first of ΔZ=0.14 m and V_S=115 m/s; the second of ΔZ=0.46 m and V_S=225 m/s. This two-layer subdivision of the ballast layer was also reported by Hwang and Park (2014) and also coincides with the results in the CTB (Ruiz et al. 2024).

Below the ballast layer is the half-space, required for the model, with a propagation velocity of the shear wave of V_S=360 m/s. This half-space would include the sub-ballast, the form layer, and the upper part of the embankment.

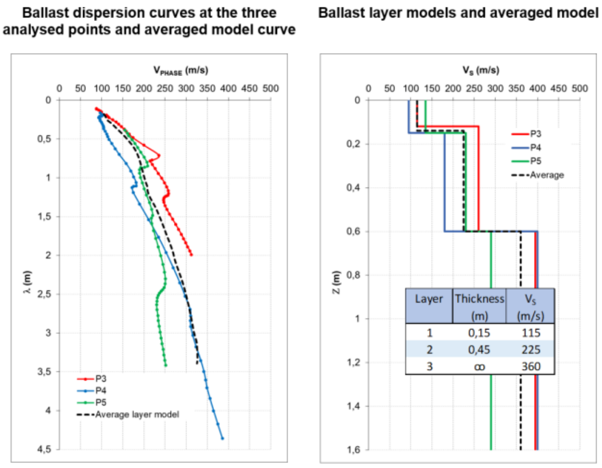


Figure 6. Results obtained at all points analysed in the ballast and their average. Dispersion curves (left) and layer models (right).

3.2.3. Tests carried out to study the embankment.

To study the embankment, SASW tests were carried out in the area outside the service duct running parallel to the track, i.e. outside the ballast layer and about 3 m from the rails. Measurements were made parallel to the track by placing the sensors and sources between the service duct and the embankment shoulder. The sensors used were accelerometers and geophones and the sources used were a vibrator and a sledgehammer. The horizontal spacings tested were: 0.50 m; 1.0 m; 2 m and 4 m.

The results obtained for the embankment are shown in Fig 7. The experimental and theoretical dispersion curves associated with the chosen layer model can be seen on the left. This layer model is presented on the right side of the figure.

In this test, a velocity for the embankment of V_S=425 m/s with a thickness of ΔZ=2.30 m was obtained. Despite this speed being somewhat higher than the values in Table 1, that were obtained for other tracks under construction, it seems compatible with an embankment of a high-speed line that has been in service for more than 15 years.

Due to the location of the test, a first lower velocity layer of 0.60 cm was obtained, but it should not be considered representative of the embankment as it is conditioned by the presence of the service duct and the beginning of the embankment shoulder.

To fit the model, it was necessary to include a half-space of V_S=850 m/s at about 3 m depth, which is unrealistic given the distribution of the natural ground and will not be considered in the final model.

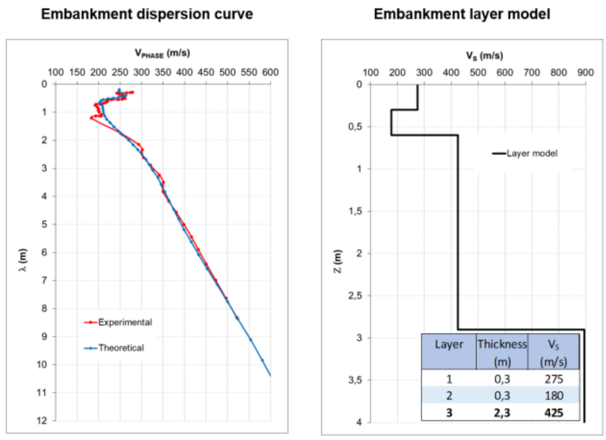


Figure 7. Embankment results. Dispersion curves on the left and layer models on the right.

3.2.4. Tests carried out to study the natural ground.

On the service path that runs parallel to the railway track, SASW and MASW measurements were carried out to characterise the natural ground. This was the only area where MASW tests were carried out.

In the SASW test, measurements were made for different horizontal sensor spacings, from 0.25 m to 10 m, to ensure that the dispersion curve with high vertical resolution between 0.25 m and 20 m could be obtained. For this purpose, different combinations of sensors and sources were used: accelerometers and a vibrator were used for sensor spacings below 2 m and geophones and impacts (sledgehammer and deflectometer) were used for spacings above 2 m.

The MASW tests were performed with a line of 24 geophones spaced 2 m apart and hitting with a sledgehammer at distances of 10 and 15 m from the first geophone in the line.

The results obtained in both tests were very similar and the velocity profile obtained with SASW and MASW tests are presented on the right side of Fig. 8. The values in the table correspond to the SASW test results.

The left side of Fig. 8 shows the experimental and theoretical dispersion curves obtained from the layer model of the SASW test.

These tests were carried out on a path that has a prepared bearing layer for the passage of heavy vehicles. Therefore, the first meter of this model should be discarded in the full section layer model since under the embankment, the natural ground does not have this kind of prepared material. In the lower part of the model, the presence of very hard soil or rock is detected, which is to be expected as this is an area with abundant limestone close to the surface.

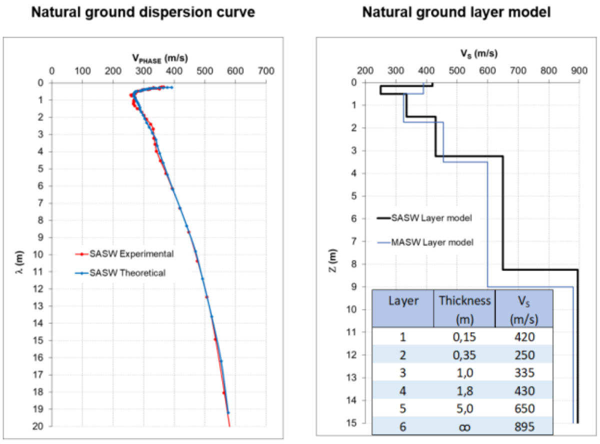


Figure 8. Natural ground results. Dispersion curves for the SASW test on the left and layer models of both SASW and MASW tests on the right.

3.2.5. Representative layer model of the entire section

With the information from all the tests carried out, it is possible to construct the representative layer model of the entire railway section at km 91+500, which is shown in Table 2. As mentioned in the previous sections, the surface layers of the embankment and the natural ground have not been considered in the global model.

Table 2. Representative layer model of the complete railway section

Layer	Thickness ΔZ (m)	V_S (m/s)
Ballast (above the sleeper base)	0.15	115
Ballast (below the sleeper base)	0.45	225
Subballast and form layer	0.75	360
Embankment and surface natural ground	4	425
Shallow natural ground	5	650
Deep natural ground	∞	895

In this section, the V_S of all the layers increases in depth.

The minimum speed values correspond to the ballast, which is the most relevant layer for the study of the critical speed. Within the ballast layer, two sub-layers have been distinguished.

The shallowest ballast layer, about 15 cm thick, is located above the base of the sleepers and is therefore above the energy transmission path from the sleepers to the rest of the underlying layers. Therefore, it does not seem appropriate to consider it as part of the railway section with an influence on the problems associated with the critical speed. Furthermore, considering this layer would lead to a very low value of the critical speed (V_C): V_C = $V_{Phase\ min}$=107 m/s = 385 km/h (which can be obtained by applying Eq. 1 with V_S=115 m/s and k =0.93). This critical speed value seems unrealistic, considering that for traffic speeds of 300 km/h, or even slightly higher, there have been no problems reflected in the underlying layers or in the superstructure elements themselves. As mentioned before, vibration amplification phenomena on the track begin to manifest before the critical speed value is reached. Therefore, it is discarded as the critical speed of the site.

Despite ruling out the influence of the first ballast layer on the critical speed, it seems appropriate to study whether this layer could be affected by the trains passing by at high speeds, causing some change in the geometry of the surface of the ballast layer.

If the first layer of ballast is disregarded, the critical speed would be associated with the second layer of ballast beneath the base of the sleepers, so there is no doubt about its influence on issues related to the critical speed. This second layer of ballast has a shear wave velocity of V_S=225 m/s. Applying Eq. 1 with a constant value of k = 0.93 implies a critical speed of V_C =209 m/s, which is equivalent to V_C =750 km/h. This value also coincides with the asymptote of the minimum velocity represented in Fig. 9, where modal dispersion curves and the apparent curve have been calculated for the layered model without considering the first layer of ballast (excluding the first layer from Table 2). This represents a high critical speed compared to the current maximum train speeds, which are close to 300 km/h. With these values, the studied section would have a safety factor (FS) against the critical velocity of FS = 2.50, where $FS = \frac{V_C}{V_{train}} = \frac{750\ km/h}{300\ m/h}$

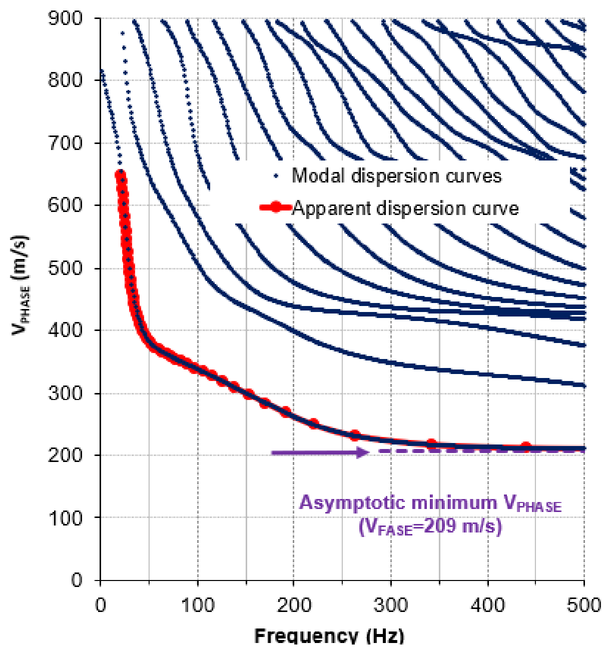


Figure 9. Comparison of the apparent and modal dispersion curves of the section without the first ballast surface layer.

In layered models where the velocity increases with depth, there is no local minimum in the modal curves and the phase velocity minimum (which is equal to the critical speed) corresponds to the horizontal asymptote of the fundamental mode.

3.3. Next steps in the study

Despite the consistency of the results obtained, it seems essential to continue deepening the characterisation of the ballast bed, since it is there, and especially in its most superficial part, where there is a greater dispersion of the experimental data.

It should be checked whether the first superficial layer of ballast located above the base of the sleepers could be affected by the passage of trains at high speeds, causing any changes in the geometry of the superficial part of the ballast shoulder. It would be advisable to gather information from test trains that run at speeds higher than commercial ones.

In the next field campaigns, in addition to the SASW tests on the ballast layer, MASW tests will be carried out by placing an array of sensors on the ballast surface and using the train approach vibrations as a seismic source.

In addition, this method is being applied on high-speed lines under construction that pass through areas with soft soils and where ground improvement treatments are being applied.

4. Conclusions

This paper presents the methodology developed by the Geotechnical Laboratory of CEDEX to determine the critical speed of a railway section. In addition, the actions carried out on a high-speed line to determine the value of the critical speed from the dispersion curve obtained *in situ* are presented. The site chosen was pk 91+500 of the Madrid-Barcelona high-speed line and a dispersion curve and a model of horizontal layers associated with wave propagation velocities increasing with depth were obtained. In this case, the minimum values of this speed correspond to the ballast, which is the most relevant layer for the study of the critical speed. In the ballast layer, two sub-layers can be distinguished, the shallower one, about 15 cm thick, has a particularly low shear wave velocity, V_S=115 m/s, but it is located above the base of the sleepers and is discarded in the critical speed study. The second ballast layer is located below the sleeper support, and therefore certainly influences in the critical speed phenomenon, and has a V_S=225 m/s. If only this ballast layer is considered, discarding the first 15 cm, a critical speed of 750 km/h is obtained, which is compatible with the good performance observed on the track at the studied point.

Despite the consistency of the results obtained, it seems essential to further research into the characterisation of the ballast layer, since it is in the ballast layer where there is the greatest dispersion of the experimental data.

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