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FULL-WAVEFORM SEISMIC MODELING BASED ON A DIGITAL GEOLOGICAL MODEL USING SPECTRAL ELEMENT METHOD ON GPU

ANATOLY V. VERSHININ[1], VLADIMIR A. LEVIN[2] AND YURY P. AMPILOV[3]

[1] Lomonosov Moscow State University
119991, Russia, Moscow, Leninskie gory, 1
versh1984@mail.ru, https://www.researchgate.net/profile/Anatoly-Vershinin

[2] Lomonosov Moscow State University
119991, Russia, Moscow, Leninskie gory, 1
v.a.levin@mail.ru, https://www.researchgate.net/profile/Vladimir-Levin

[3] Lomonosov Moscow State University
119991, Russia, Moscow, Leninskie gory, 1
ampilovy@gmail.com, https://www.researchgate.net/profile/Yury-Ampilov

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Abstract. The paper considers the solution of a three-dimensional problem of modeling of all types of seismic waves propagating in real geological media. The numerical algorithm based on the spectral element method (SEM). The main advantages of SEM (high order space discretization, explicit time integration scheme) are presented in comparison with the classical approach based on the finite element method (FEM). The features of the massively parallel implementation of the algorithm on modern MultiGPU systems (based on A100 GPU) using CUDA technology are considered. The efficiency of parallelization on hybrid systems with different SEM orders and parameters of the numerical time integration scheme is analyzed. The results of solving a three-dimensional problem of modeling the propagation of seismic waves in a heterogeneous geological media with faults and sharply varying properties of layers are presented. Analysis of the numerical convergence of SEM for dispersive waves of the Rayleigh type is performed. Local and non-local non-reflective boundary conditions on the artificial boundary of the computational region are considered. The 3D computational model is constructed using a detailed digital geological model built for one of the Arctic regions. It was converted to an unstructured hexahedral mesh to perform SEM calculations using CAE FIDESYS software. The model is further generalized for typical seismic-geological conditions of Western Siberia, so that on the basis of such modeling it is possible to conduct a wide range of studies on the possibilities of seismic exploration to study the main oil and gas reservoirs in this region. The solution was sought on a hexahedral mesh consisting of 5.5 mln spectral elements of the 5th order with a total number of SEM nodes 1.2 billion. The output results of full-wave modeling are stored in the SEG-Y format, suitable for all types of industrial seismic processing. The analysis of the obtained model seismograms and wave fields is carried out. The conclusion is made about the practical significance of the conducted research.

1 INTRODUCTION

The formulation of three-dimensional dynamic problems of elasticity theory is considered in relation to full-wave modeling of seismic exploration in geological media. The algorithm is based on the spectral element method (SEM) for the numerical solution of problems in inhomogeneous three-dimensional media with sharply changing rock properties. The main advantages and features of SEM are presented (high order of discretization in space, explicit time integration scheme) in comparison with the classical approach based on the finite element method (FEM) [1, 2, 3]. The technical features of the massively parallel implementation of this algorithm on GPUs using CUDA technology are considered. The efficiency of parallelization on hybrid systems is analyzed for various SEM orders and parameters of the numerical time integration scheme.

The results of solving a three-dimensional problem of modeling the propagation of seismic waves in a heterogeneous geological environment with faults and sharply changing properties of interlayers are presented. The obtained seismograms and wave fields are analyzed.

2 GEOLOGICAL MODEL

The problem of propagation of three-dimensional seismic waves in a heterogeneous thin-layered medium with arbitrary interfaces and discontinuities in rock continuity is solved. To calculate the wave field, we used a geological model built based on data in a real subsoil area of the Russian Arctic, which was quite well studied by 3D seismic exploration and deep drilling. Vertically, the model (Fig. 1) covers the section of the sedimentary cover from the relief surface to the roof of the folded base of the platform. In the lower interval there are faults with displacements varying laterally and vertically; the orientation of the structural grid is chosen in accordance with the strike of the tectonic sutures. The dimensions of the modeled area in plan are chosen in such a way as to obtain at the output a volume of 3D seismic data suitable for full processing (16 × 12 km laterally and from 0 to - 4100 m vertically).

The spatial mesh and mechanical properties model were built in a specialized geological modeling software package with vertical rescaling performed at the final stage. The model zones and subzones are divided into cell layers in accordance with the accepted regional sedimentation concept. The final dissection of the section allows one to reflect all the most significant details of the structure of the section and maintain a balance between detail, speed and the very possibility of performing further calculations. The 3D grid of the constructed model contains 144 layers of 39,961 significant cells in each layer (average lateral grid resolution 70×70 m). The average vertical resolution of the grid ranges from 13-15 m in the hydrocarbon productive zone of the section to 25-60 meters in other parts. In total, the mesh contains about 6 million significant cells.

The modeled section includes several oil and gas bearing complexes with proven productivity and regional screens. The modeling area was selected in such a way as to include both parts of open hydrocarbon deposits and underlying water-saturated zones. Modeling of the lithology of the section is based on the results of deep drilling - GIS interpretation materials and laboratory studies of core material. In the upper part, a zone of distribution of permafrost rocks with varying thickness is modeled. The final model contains data on the longitudinal and transverse velocities of elastic waves, rock density, Young's modulus and Poisson's ratio, as well as the absorption decrement of the medium.

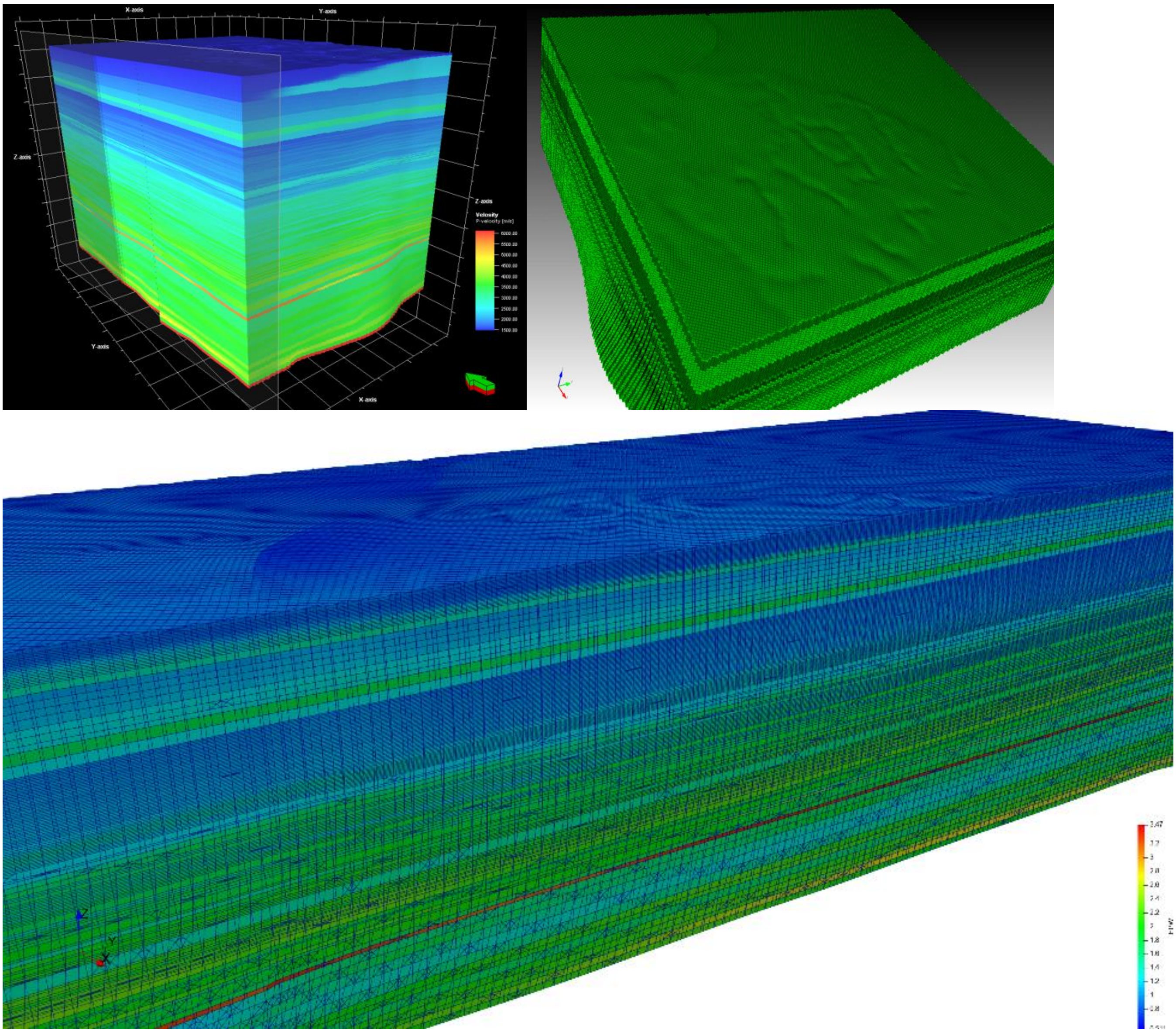


Figure 1: Geological model (left) and hexahedral mesh (right and bottom) for spatial discretization.

The model preparations were made in compliance with all the basic industry requirements for geological models used to calculate hydrocarbon reserves and design developments. Quality control of the completed geological modeling was carried out using a comparative analysis of previously obtained isochron maps for the main OGs and maps calculated on the constructed model.

3 MATHEMATICAL MODEL

The mathematical formulation of the problem consists of the equations of the linear dynamic theory of elasticity in displacements [4, 5], written in a three-dimensional Cartesian coordinate system [10], boundary conditions (approximate non-reflective conditions on the outer boundary of the region and a free surface with a given time-dependent mass source on it) and zero initial conditions. The setting of frequency-dependent absorption in rocks is also provided.

Let $u(x, t)$ be the displacement vector at point x at time t, ρ - density, σ - stress tensor, $S = S(x, t)$ - source function. Let us write out a system of differential equations describing the propagation of elastic waves [4, 5]:

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \nabla \cdot \hat{\boldsymbol{\sigma}} + \mathbf{S} \tag{1}$$

Let's consider elastic materials that obey Hooke's law:

$$\hat{\boldsymbol{\sigma}} = \hat{\mathbf{c}} : \hat{\boldsymbol{\varepsilon}} \tag{2}$$

where $\hat{\boldsymbol{\varepsilon}} = \frac{\nabla \mathbf{u} + \mathbf{u}\nabla}{2}$ is the deformation tensor, $\hat{\mathbf{c}}$ is the Hooke tensor of rank 4.

Due to the symmetry of the Hooke tensor, it can be represented as a matrix using the substitution of indices [4, 5] (Voigt notation): (1, 1)→1, (2, 3) →4, (2, 2) →2, (1, 3)→5, (3, 3) →3, (1, 2) →6. Then Hooke's law will take the form:

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & c_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{pmatrix} \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{pmatrix} \tag{3}$$

In the case of an isotropic medium, the components of the Hooke tensor depend only on 2 quantities, which are the Lamé coefficients (parameters) λ, μ :

$$C_{ij} = \begin{pmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{pmatrix} \tag{4}$$

In this case, Hooke's law takes the form:

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij} \tag{5}$$

On the external artificial boundary of the computational domain, local non-reflective boundary conditions were specified, having the following form:

$$-\sigma \cdot \vec{n} = \rho c_p \frac{\partial \overrightarrow{u_n}}{\partial t} + \rho c_s \frac{\partial \overrightarrow{u_t}}{\partial t}$$
$$\overrightarrow{u_n} = (\vec{u} \cdot \vec{n}) \vec{u}$$
$$\overrightarrow{u_t} = \vec{u} - \overrightarrow{u_n} \quad (6)$$

where Cp and Cs are the velocities of longitudinal and transverse waves for the isotropic case.

For the anisotropic case, we respectively have:

$$-\sigma \cdot \vec{n} = \rho (c_P (\vec{n} \cdot \vec{v}) \vec{n} + c_{SV} (\vec{t}_1 \cdot \vec{v}) \vec{t}_1 + c_{SH} (\vec{t}_2 \cdot \vec{v}) \vec{t}_2) \quad (7)$$

here t_1 and t_2 are unit orthogonal vectors tangent to the outer absorption boundary with a unit outer normal n, Cp is the speed of a quasi-longitudinal wave propagating in the direction n, Csv is the speed of a quasi-transverse wave polarized in the direction t_1, and Csh is the speed of a quasi-transverse wave polarized in the direction t_2. The given local absorption boundary condition is based on one-way interaction, which completely absorbs waves incident at right angles to the boundary; it is less effective for waves tangential to the boundary [9]. This condition is true for the case of a transversal isotropic medium with horizontal or vertical symmetry; a more general case of anisotropy can be reduced to the case in which the medium becomes transversally isotropic at the absorption boundary.

4 NUMERICAL DISCRETIZATION AND SOFTWARE IMPLEMENTATION

To numerically solve equations in an inhomogeneous three-dimensional medium, we used the spectral element method (SEM) [6, 7, 11], which has a high (spectral) order of approximation of the solution in space and a completely explicit time integration scheme due to the diagonal mass matrix. For the case of the geological model considered in this work (Fig. 1), the solution was sought on a hexahedral grid consisting of 5,634,343 spectral elements of the 5th order with a total number of interpolating spectral element nodes of the order of 1.2 billion. An important feature is the adaptation of the grid to the layered geometry of the problem, allowing one to accurately simulate discontinuities in the parameters of the medium when a wave passes from layer to layer. For approximate non-reflective boundary conditions at the outer boundary of the region, this work used a combination of local characteristic conditions [7] and an absorbing layer with an increasing degree of absorption when approaching the boundaries of the region [9,16].

The use of high-order spectral elements and the explicit Newmark scheme [2] of time integration made it possible, on the one hand, to reduce the total number of degrees of freedom in the discrete model (in comparison with the classical finite element method [2, 3], 1-2 spectral elements per minimum wavelength), and on the other hand, increase the time integration step (determined from the Courant condition [1, 9, 13]) due to the use of larger elements. For the software implementation of the SEM, a matrix-free algorithm [14] of spatial integration was used (assembling the vector of internal and external forces [2, 12, 13]) with an explicit time integration scheme on a multi-level GPU architecture (Fig. 2). Within this implementation, the spectral element grid is naturally mapped onto a grid of multiprocessors on the graphics card, and, accordingly, each spectral element is mapped to a thread block, within which individual nodes within the element are processed by their corresponding threads within the block.

This approach makes it possible to effectively use the capabilities of shared memory (Fig. 2) for caching data inside the spectral element when constructing a vector of internal forces on it, which significantly increases the throughput of the parallel version of the algorithm, the performance of which is limited precisely by the speed of access to the global graphics memory (memory bounded), rather than the processing speed of the GPU cores (compute bounded). Thus, the size of a CUDA block is automatically determined by the order of the spectral element, and the number of blocks is constant for a fixed SEM mesh. In addition, the use of atomic operations on CUDA when performing assembly (assembling a global vector of nodal forces and a mass matrix from local vectors and matrices on spectral elements) made it possible to perform calculations for all SEM elements in a massively parallel mode without the need to perform internal synchronizations and/or) grid coloring to prevent read - after - write collisions.

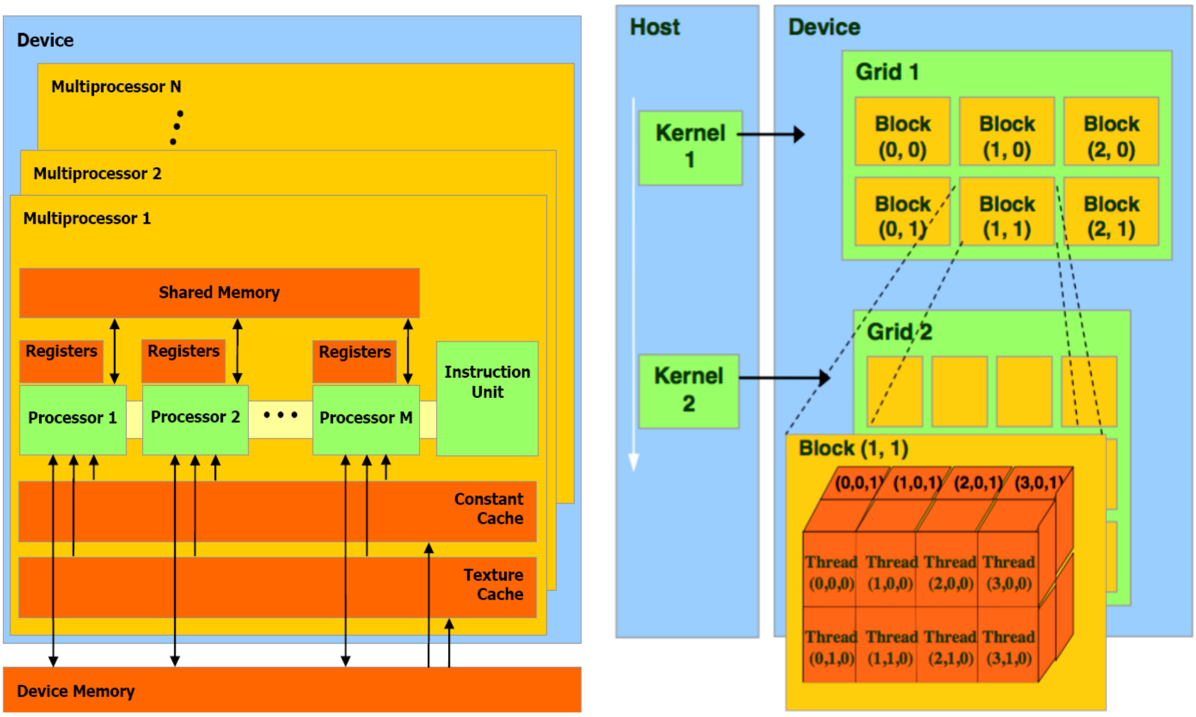


Figure 2: Hardware and software structures of the GPU

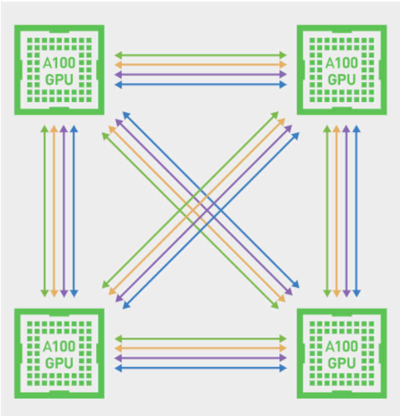
The use of an explicit Newmark scheme for numerical integration over time implies multiple (several tens of thousands of iterations over time) calls to kernels for assembly over space and integration over time. To optimize this procedure and reduce overhead costs for launching kernels, CUDA graphs were used, which made it possible to combine sequential executions of assembly and integration operations over time (step of the Newmark scheme [2, 9]) into a single graph of operations on CUDA, performing several integration steps in time for one call to the graph. By using these approaches, a total performance increase of about 100 times was obtained in comparison with the implementation of the SEM using OpenMP on an Intel Xeon central processor. The results obtained indicate the effectiveness of applying CUDA technology to the considered problem.

5 CALCULATION RESULTS

In order to perform numerical calculations of wave fields (Fig. 3) and seismograms (Fig. 4), computational kernels of the CAE Fidesys package [15] based on CUDA technology [8] were used, which allows using the computing capabilities of massively parallel computing devices - to carry out calculations We used an HPE Mercury workstation based on 4 NVIDIA Tesla A100 graphics accelerators, installed at the Department of Computational Mechanics of the Faculty of Mechanics and Mathematics of Lomonosov Moscow State University with the following specification:

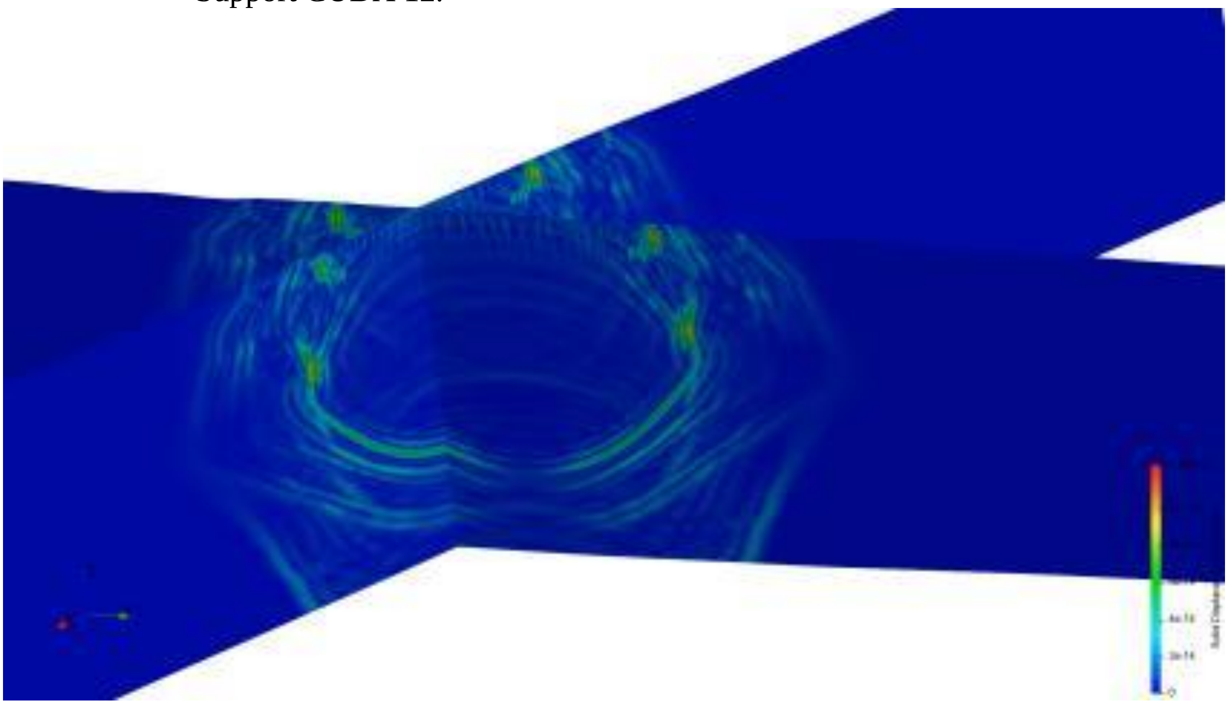
HPE Apollo 6500 Gen10:

- 4 xTesla A100 NVLink 3.0
- Memory bandwidth for inter- GPU exchanges - 600 GB/s
- NVIDIA technology support Grid for remote launch of CUDA applications and 3D rendering.



NVIDIA Tesla A100:

- Operations with double accuracy - 9 , 7 Teraflops
- Stack HBM2 e memory volume 80 GB
- 108 multiprocessors , 6912 cores CUDA
- Global memory bandwidth - 2039 GB/s
- Support CUDA 12.



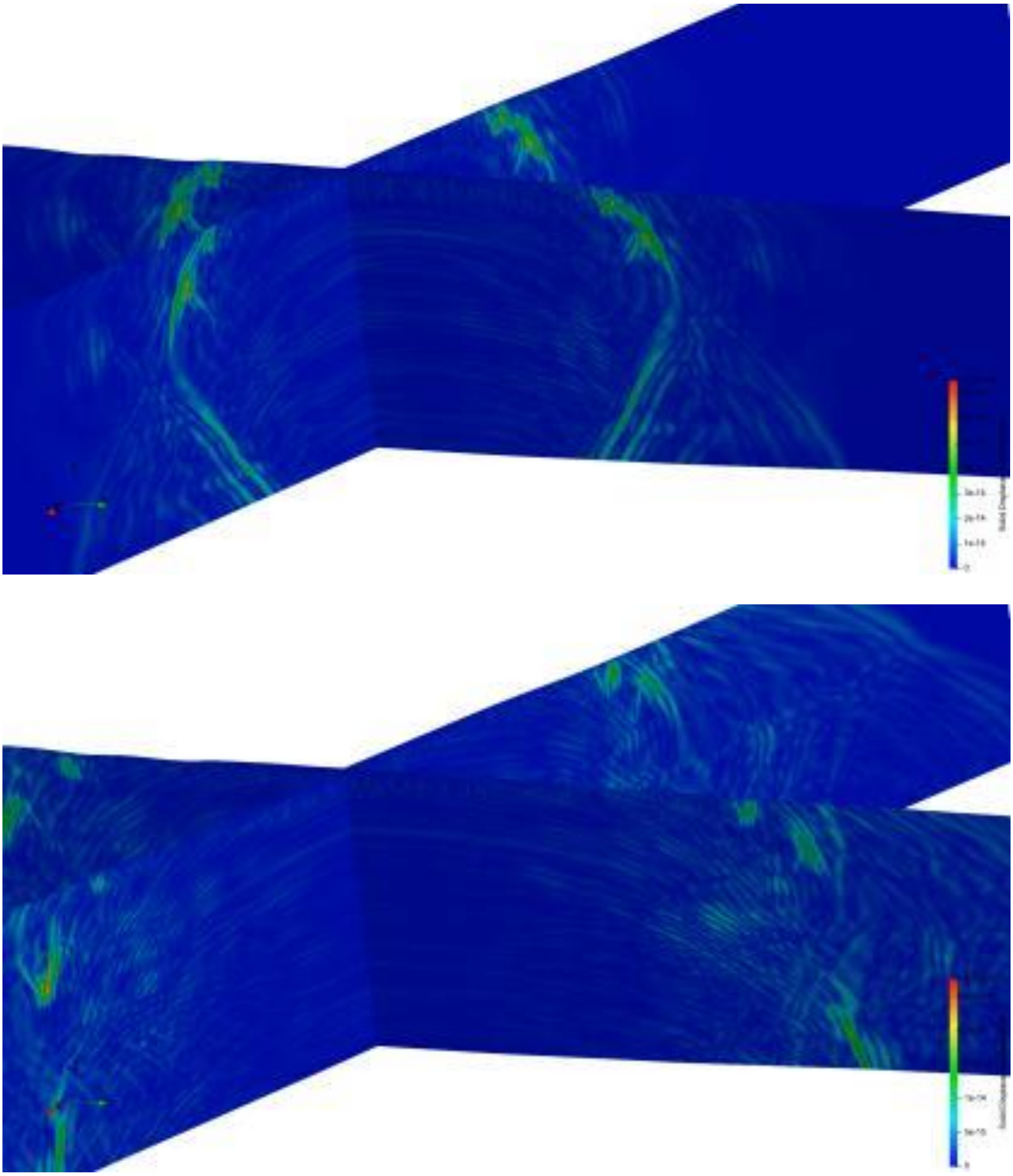


Figure 3: Wave fields at different time moments.

The total calculation time was about 5 hours - 50,000 time integration steps, 0.3 seconds were spent to calculate one time step on the 6th order SEM mesh of spatial approximation, consisting of 1.2 billion nodes. Both surface Rayleigh waves and a full set of volumetric reflected and refracted waves, longitudinal, transverse and exchange waves were calculated, considering all possible facts of diffraction and multiple reflections.

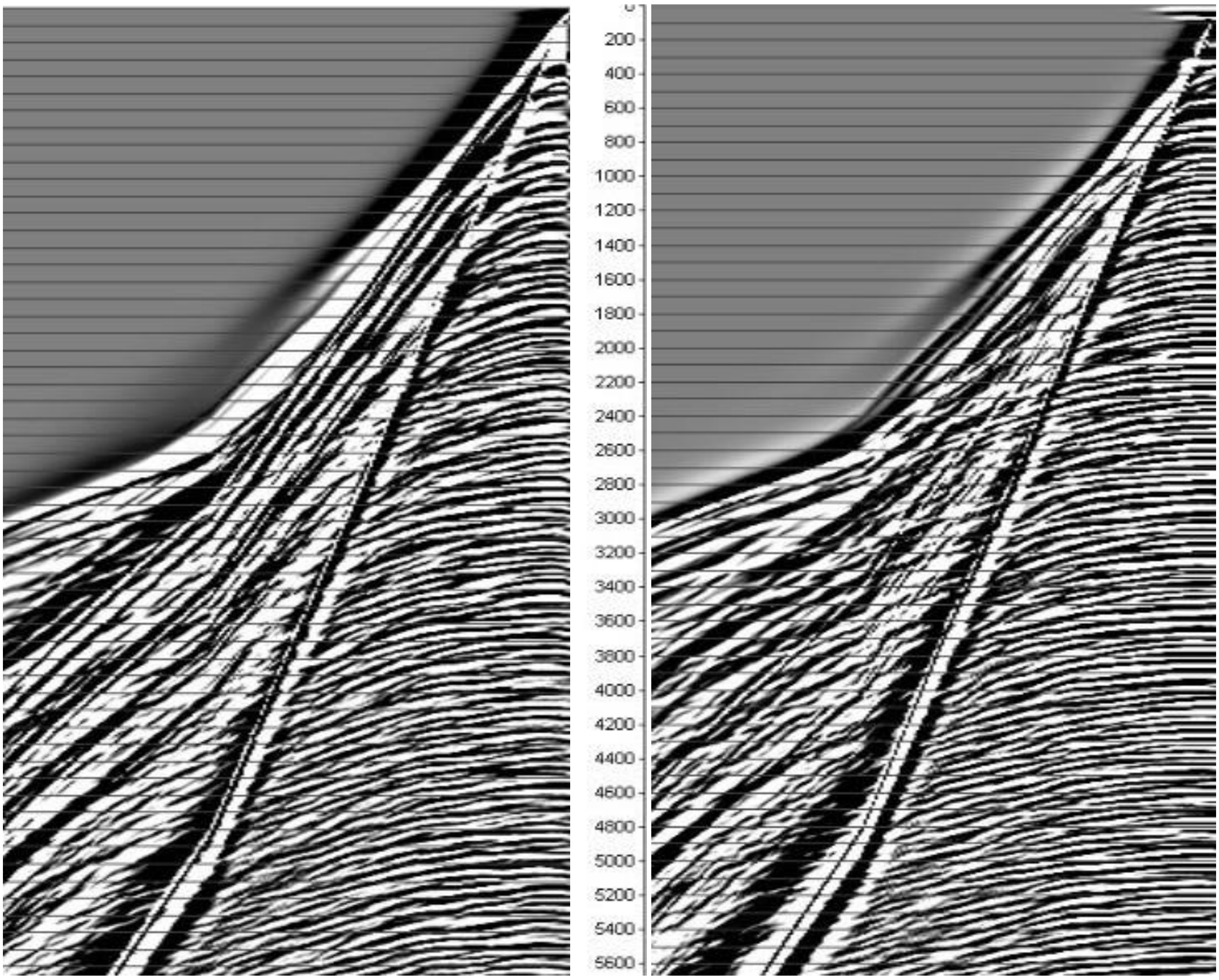


Figure 4: Seismograms of the vertical and horizontal displacement velocity of particles of the earth's surface, obtained for the vibration source located on the surface in the center of the model (Fig. 1)

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7 CONCLUSIONS

Until now, the main types of applied modeling in seismic are carried out using the simplified ray method, and, as a rule, for monotypic waves. Wave modeling is used extremely rarely and mainly for simplified homogeneous layered models of media with large layer thicknesses [16, 17]. Such models cannot adequately reflect the properties of the real geological environment.

In this work, full-wave modeling using the spectral element method was carried out for the first time for a detailed model containing about 6 million cells and 1 billion calculated SEM nodes. Moreover, both surface Rayleigh waves and a full set of volumetric reflected, refracted waves, longitudinal, transverse and exchange, were calculated. All possible facts of diffraction and multiple reflections are considered - in fact, everything that happens in a real environment.

Such full-fledged wave modeling is very important for studying the capabilities of modern methods for processing and interpreting seismic data, which are based on simplified assumptions about the structure of the medium. We assume that this technology will be widely introduced into everyday seismic exploration practice.

The scope of this modeling can include a wide range of applied problems, including:

- Creation of reference models of the environment for the main oil and gas regions for the purpose of a detailed study of wave pattern formation;
- Research into the real possibilities of modern methods of processing and interpreting seismic data;
- Use of full-wave models by seismic processing customers (in the “hidden” source data mode) for objective technical pre-qualification of performers within tenders;
- 4-D seismic signal modeling for developed fields, underground gas storage facilities (UGS) and potential CO2 storage reservoirs

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