Effective friction angle of intermediate grain size soils from variable rate piezocone penetration test

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ABSTRACT

The effective friction angle (ϕ') is one of the most sought soil properties in geotechnical analysis and design that controls the fundamental soil strength behaviour. In this paper, A recently compiled database on intermediate grain size soils that were tested by variable rate piezocone penetration tests (VRCPTU) is used to examine an effective stress limit plasticity solution using piezocone data developed by NTH (Norwegian Institute of Technology) towards prediction of the geomaterials' effective stress friction angle. It is shown that penetration rate during probe advancement influences the CPTU measured data and the drainage conditions can be indicated by corresponding normalized cone resistance and normalized porewater pressure measurement. Further analysis revealed that an input parameter termed the angle of plastification (β) from the CPTU solution correlates with soil contraction and dilation tendency and can be used to evaluate the ϕ' of investigated soils, together with other normalized CPTU measurements. The tentative suggested formula of β for intermediate soils undergone VRCPTU testing is given based on the investigated database. A comprehensive testing program on the Yellow River Delta (YRD) silts in China and a silty geomaterial from northern Italy in Europe including benchmark laboratory triaxial tests on high quality samples, field and calibration chamber variable rate CPTU are showcased to demonstrate the verification of the NTH solution for evaluating ϕ' of intermediate grain size soils.

Keywords: effective friction angle; piezocone penetration test; intermediate grain size soils; the angle of plastification

1. introduction

Current engineering approaches for analysing the strength behaviour of coarse-grained materials (such as sands) and fine-grained materials like clays have been broadly investigated. However, intermediate soils such as silts, clayey silts are less investigated in terms of their strength and deformational characteristic (Jaeger 2010). The effective friction angle ϕ' is one of the most sought soil properties for any types of geomaterials (Ouyang and Mayne 2021). The investigation of the effective friction angle of intermediate soils is of significant importance in elucidating the soil effective stress strength, controlling foundations' bearing capacity, and determining slope stability. Piezocone penetration tests (CPTU), serving as an effective tool for acquiring in-situ soil stratification, classification, soil properties, have been widely employed in site characterization. Previously, an effective stress limit plasticity solution for CPTU has been developed at the Norwegian Institute of Technology (NTH), detailed by Senneset and Janbu (1985), Senneset Sandven, and Janbu (1989), Sandven (1990), Sandven and Watn (1995), and modified by Ouyang and Mayne (2018, 2019, 2023) is used to evaluate the effective friction angle of clays.

In piezocone penetration testing (CPTU), a standard penetration rate of 20 mm/s is adopted. This rate is recognized to induce a fully drained condition in sands and fully undrained behaviour in low-permeability clays. While the effective friction angle of the soil should be theoretically unaffected by drainage conditions, the influence of drainage conditions which change through the variable rate piezocone penetration test (VRCPTU) can impact the recorded test data (tip stress, sleeve friction, and porewater pressure).

Therefore, this paper aims to investigate the influence of VRCPTU data in intermediate soils under different drainage conditions towards the evaluation of the effective friction angle using the aforementioned CPTU based NTH solution.

2. Variable Rate Piezocone Penetration Test

The most direct approach to study the transition region from undrained to partially drained to fully drained response is to conduct CPTU tests at different rates in the same soil. In fact, a special type of variable rate series of CPTU called "twitch testing", or alternatively called VRCPTU within a single sounding has been developed by varying the penetration rate and measuring corresponding testing readings under different drainage conditions (Randolph 2004). These tests have been conducted both in the laboratory using centrifuge, chamber deposits and in the field.

To compare the readings taken at different rates, a nondimensional velocity parameter (V) has been defined by Finnie and Randolph (1994):

$$V = \frac{v \cdot d}{c_h} \quad (1)$$

where v is the penetration rate, d is the probe diameter, and c_h is the coefficient of consolidation of the soil tested.

For the CPTU, a "field decision chart", as shown in the Fig. 1, has been developed to facilitate simple and efficient estimates of drained or undrained penetration rates with a 10 cm^2 cone DeJong et al (2012):

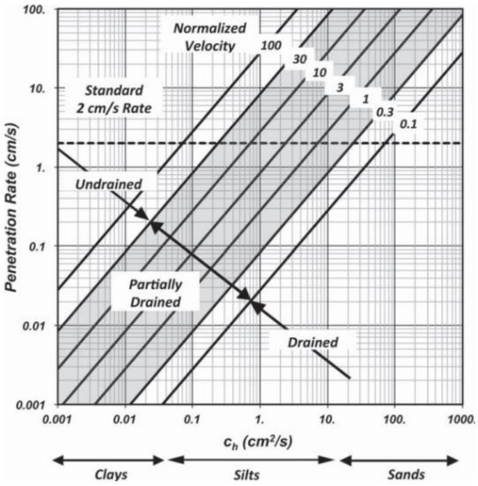


Figure 1. Field decision chart for drainage condition in soils (after DeJong 2012)

For convenience, the measured CPTU data can be presented in terms of the following dimensionless parameters:

$$Q = \frac{q_{net}}{\sigma'_{v0}} \quad (2)$$

$$B_q = \frac{\Delta u_2}{q_{net}} \quad (3)$$

Where $q_{net} = (q_t - \sigma'_{v0})$ is the net cone tip resistance and $\Delta u_2 = (u_2 - u_0)$ is the excess porewater pressure. It's often presented as the variations of Q and B_q with normalized penetration rate (V) in a chart, as suggested by Randolph and Hope (2004). In Fig.2, it is shown that the penetration resistance during testing influenced by the velocity of penetration (v), the diameter of the probe (d) and the coefficient of consolidation (c_v) of the soil. Finnie and Randolph (1994) suggested transition points of $V < 0.01$ for a drained response and $V > 30$ to ensure an undrained response and similar recommendation were provided by DeJong et al (2012).

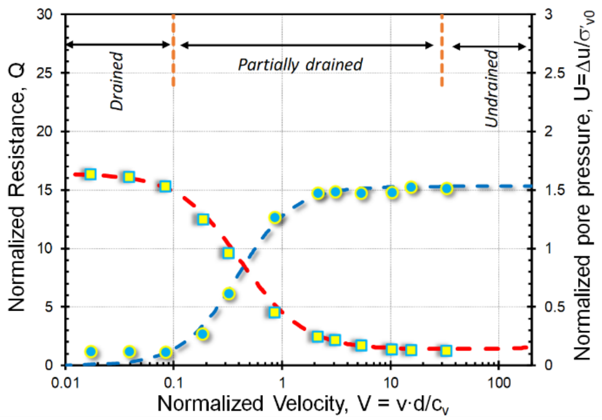


Figure 2. Importance of understanding drainage conditions (after Ouyang and Mayne 2021)

3. Effective stress limit plasticity solution (NTH method)

The general NTH solution for effective stress friction angle (ϕ') of soils using CPTU is given by the following:

$$Q = \frac{\tan^2(45° + \phi'/2) \cdot \exp[(\pi - 2\beta) \cdot \tan\phi'] - 1}{1 + 6 \cdot \tan\phi' \cdot (1 + \tan\varphi') \cdot B_q} \quad (4)$$

Where β is defined as the angle of plastification, which is usually used to represent the failure mode of soil. During the process of CPTU penetration, soil will have a tendency to contraction (soil volume decreases) or dilatation (soil volume increases), as shown in Figure 4. When the angle of plastification (β) is greater than 0°, it means soil shear dilatation; when the plastic Angle (β) is less than 0°, it means soil shear contraction.

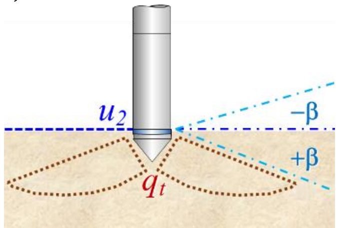


Figure 3. Schematic illustration of angle of plastification β showing failure surface of soils during probe penetration

For clays under undrained loading condition, the angle of plastification is taken as $\beta = 0°$, reflecting the typical scenario of constant volume, in concordance with undrained penetration (Sandven 1990, Ouyang and Mayne 2018, 2019). Details regarding the development of this solution are documented in Sandven (1990).

Ouyang and Mayne (2019) provided an interpretation formula for the above method to evaluate the effective friction angle of clays of various stress history under the condition of $\beta = 0°$, as shown in Fig. 3.

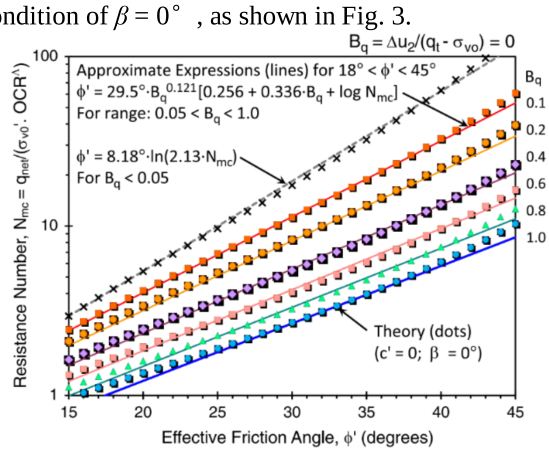


Figure 4. NTH method for evaluating ϕ' from CPTU

4. Variable Rate CPTU Database

The primary objective of this study is to conduct a comprehensive review of data derived from piezocone penetration test (CPTU) conducted at variable penetration rates in soil predominantly composed of intermediate grain size like silt. A meticulously compiled database is presented, incorporating information from nine distinct series of CPTU tests documented in the geotechnical literature and research reports. The soils subjected to CPTU range from silty sands to silty clay to clay. Table 1 includes essential details concerning index parameters, reference sources, and a concise summary of piezocone data, accompanied by laboratory results wherever available.

All investigated soils utilized either electric (analog) or electronic (digital) cone penetrometers, and the associated measurements of cone tip resistances (q_c), pore pressure(u_2) and the sleeve friction (f_s) were duly recorded. The corrected cone tip resistance (q_t) were adjusted to account for porewater effects, following instruction elucidated by Lunne et al (1997), wherein pore pressure(u_2) measured at the shoulder position was employed for correction. For the investigated soils, Fig. 4 demostrasted some VRCPTU data response where the ratio of the normalized cone tip resistance (Q) and its reference value during undrained penetration (Q_{ref}) is plotted against the normalized penetration velocity (V).

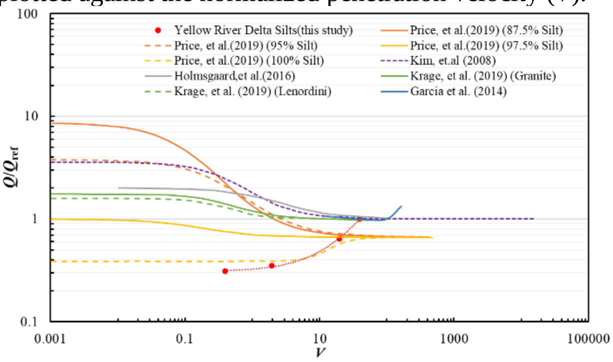


Figure 5. Summary of VRCPT data on silty soils

Price et al. (2019) and Kim et al. (2008) observed Q/Q_{ref} values ranging from 3 to 8, and the tested geomaterials are 80% silts mixed with 20% kaolin, 95% silts mixed with 5% kaolin, and silty clay. Krage et al. (2019) and Holmsgaard et al. (2016) conducted tests on sandy silts, observing the Q/Q_{ref} ratio in the range of 1 to 2. According to their findings, the lower Q/Q_{ref} values are attributed to the smaller values associated with silts and sands, indicating smaller volume changes during loading.

In particular, experiments conducted by Price et al. (2019) on 97.5% silts mixed with 2.5% kaolin, 100% silts, Garcia et al. (2014) on silty sand, and the tests performed in this study on silty soil from the Yellow River Delta (YRD silt) yielded Q/Q_{ref} values generally less than 1. Compared to others, this phenomenon could be resulted from dilative tendencies observed in the soils during the penetration of CPTU.

It is worth noting that Dienstmann et al. (2018) claimed a significantly higher drained-to-undrained ratio (Q/Q_{ref}) in tailings compared to natural soils. Simultaneously, the magnitude of the ratio Q/Q_{ref} is also related to fundamental soil parameters such as the consolidation coefficient and overconsolidation ratio (OCR).

Based on the above analysis, it is evident that the volume of the soil changes, either in terms of dilation or contraction, for VRCPTU conducted in intermediate soils. To address this and distinguish the applicability of the formula, the soil classification index is introduced, this study derived an empirical correction formula for the angle of plastification (β) which is an input parameter in the NTH solution to calculate the soils' effective stress friction angle based on the data collected (Table.1):

$$\beta = 192.59 \ln I_c - 177.79 \tag{5}$$

Where I_c is the soil type index based on the Robertson 1990 classification diagram and defined as follows:

$$I_c = \sqrt{((3.47 - lgQ_{tn})^2 + (1.22 + lgF_r)^2)} \tag{6}$$

Robertson (2009) updated the normalized tip resistance Q_{tn} using a normalization with a variable stress exponent:

$$Q_{tn} = (q_t - \sigma_{v0}/p_a) \times (p_a/\sigma'_{v0})^n \tag{7}$$

In Eq. (7), p_a is the atmospheric reference pressure, n is the stress exponent, and defined by

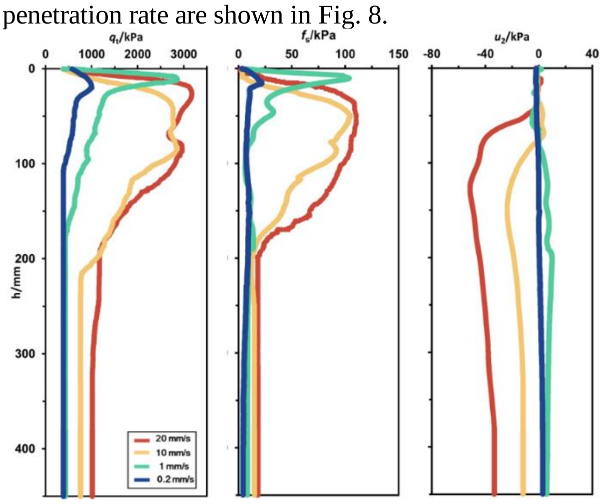
$$n = 0.381 I_c + 0.05(\sigma'_{v0}/p_a) - 0.05 \tag{8}$$

where $n \leq 1.0$.

Equation [5] is only applicable when $1.5 \leqslant I_c \leqslant 3$, suggesting soils are mainly showing characteristics of sands, silts, and clays such that the calculated β is in the range of -90° to +90° .

5. Case study: VRCPTU in Yellow River Delta, China

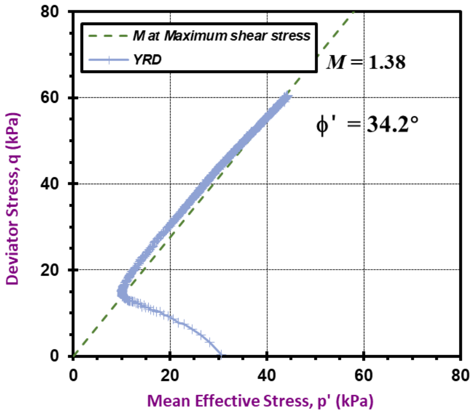
To illustrate the aforementioned empirical correction formula for the angle of plastification (β),Yellow River Delta (YRD) silts located to the south of the Bohai Sea are subjected to VRCPTU chamber test, and the CPTU soundings are illustrated in Fig. 5. Results of laboratory index tests showed the YRD silt having a natural water content ≈ 25%, liquid limit about 29.6%, and plasticity index (PI) around 9.8%. The VRCPTU penetration test of the YRD silty soil showed that as the penetration rate increased from 0.2 mm/s to 20 mm/s, the tip resistance q_t gradually increased, and the pore pressure Δu gradually decreased, while the measured values were similar when the penetration rates were 1 mm/s and 0.2 mm/ s. In particular, when the penetration rate was 20 mm/s, the soil accumulates negative pore pressure. Because CPTu data in the early stage was unstable, the 300 mm-350 mm data was selected as the reference value for data interpretation, and the CPTu-calcualted ϕ' for each penetration rate are shown in Fig. 8.



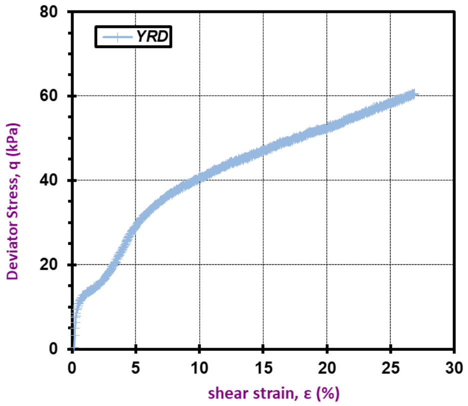
(a) (b) (c)

Figure 6. Representative CPTU sounding on YRD (Yellow River Delta) silt in calibration chamber: (a) cone resistance; (b) sleeve resistance; and (c) porewater pressure

Fig. 6 depicts the effective stress path and stress-strain curve for undrained triaxial tests conducted on remodeled YRD silty soil samples. The average effective friction angle (ϕ') interpreted based on the criterion for maximum deviatoric stress (q_{max}) for triaxial tests is ϕ' = 34.2°, with an effective cohesion intercept c' = 0. Furthermore, the stress-strain curves from the triaxial tests indicate a tendency for shear dilatancy in the soil.



(a)



(b)

Figure 7. Triaxial test results of YRD (Yellow River Delta) silts (a) stress path; (b) stress-strain curve

Post data reduction involves application of Eq. (5) to (8), aiming to determine the effective friction angle using VRCPTU measurements, and the calculated effective friction angle (ϕ') of the Yellow River Delta silts is illustrated in Fig. 7. Benchmark reference laboratory triaxial tests measured ϕ' are compared in the same figure. The analysis reveals that the effective friction angle, computed using VRCPTU data with a β value of -8.9° exhibits a good agreement with the lab results.

According to the above results, the input values of β decrease with increasing penetration rate of CPTU, indicating a greater degree of soil dilation as the drainage condition approaches undrained shear. When the penetration rate is 20 mm/s, the value of β is -8.9°. This negative value corresponds to the occurrence of negative pore pressure in the CPTU calibration chamber. The paper speculates that the negative value of β is associated with soil dilation during CPTU penetration.

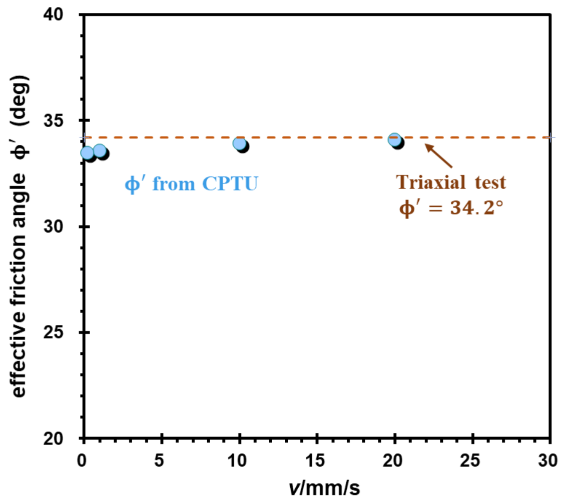
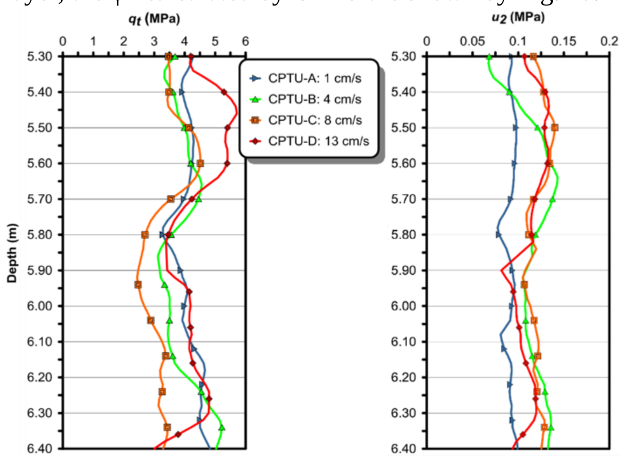


Figure 8. CPTU calculated ϕ' compared with laboratory benchmark triaxial measured ϕ' of YRD silts

6. Case study: VRCPTU in Mirabello Site, Italy

The paper further analyses an in-situ VRCPTU in Northern Italy to examine the accuracy of the empirical correction formula for the angle of plastification (β). The experiments were conducted in silty soils at the Mirabello Site, a village established on the abandoned levee of the local Reno River in northern Italy. The experimental setup consists of four CPTU tests spaced two meters apart to mitigate potential misinterpretations of data arising from horizontal spatial variability. The cone penetration speed (v) remained constant for each test, with values of 10, 40, 80, and 130 mm/s, respectively. Readers are referred to Garcia et al. (2014) for detailed description of the testing program.

To better focus on sand and silt mixtures, the analysis was applied to the piezocone data obtained from thin soil layers within the depth range of 5.3 to 6.4 meters. According to the average value of CPTU data of this soil layer, the ϕ' calculated by CPTU are shown by Fig. 10.



(a) (b)

Figure 9. Representative CPTU sounding at Mirabello Site (data after Garcia et al. 2014): (a) cone resistance; (b) porewater pressure.

The results indicate a general trend of increasing u_2 and decreasing q_t as the cone velocity varies from 10 mm/s to 80 mm/s. However, at 130 mm/s, the q_t values are consistently higher compared to those observed at 80 mm/s, while the u_2 values exhibit a reduction. The reason behind this phenomenon could be due to the reason that, beyond a certain rate, the soil reaches a fully undrained state. Consequently, there exists a penetration rate threshold between partial drainage and complete undrained conditions, leading to a transition from minimum to maximum penetration resistance. As pointed by Garcia et al. (2014), based on the experimental results, it seems reasonable to assume that a penetration rate equal to 80 mm/s, where q_t is minimum, corresponds to a fully undrained behaviour with negligible viscous effects.

Following the calculation method for the angle of plastification (β) from the previous section, the effective friction angle (ϕ') of the Mirabello site is obtained, as shown in the Fig. 9, which shows good agreement with the triaxial test-determined ϕ' = 34.0°, The formula proposed in this study can once again demonstrate its capability to eliminate the discrepancies in data interpretation arising from variations in CPTU raw data due to different penetration rates and the resulting variations in drainage conditions. The penetration rates in this case are relatively high compared to the standard penetration rate, resulting in negative predicted values for β. Further in-situ tests are required to investigate intermediate grain size soils behavior more comprehensively.

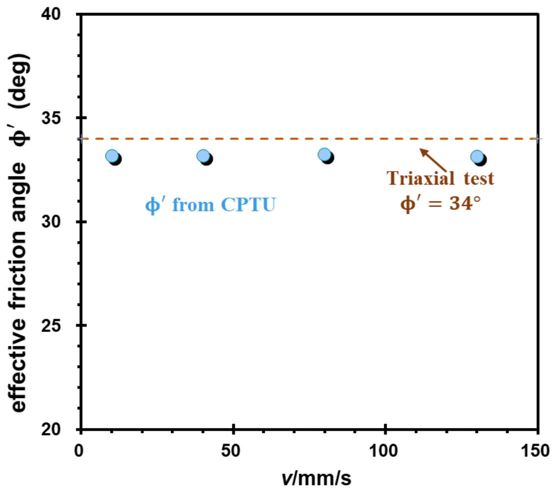


Figure 10. CPTU calculated ϕ' compared with laboratory benchmark triaxial measured ϕ' of Mirabello silty soils

7. Conclusions

This paper examined an effective stress limit plasticity CPTU method to evaluate the effective stress friction angle (ϕ') of silty soils. The following conclusions are drawn:

- When evaluating the effective friction angle (ϕ') of intermediate soils using the NTH effective stress limit plasticity solution from VRCPTU data, possible volume change of the soil (dilation/contraction) during testing should be considered.
- Through calibration with collected databases, a formula for directly interpreting the angle of plastification (β) which is a modifier input for the CPTU method is obtained.
- Two VRCPTU case studies, YRD silts in China and silty soils in northern Italy demonstrated the accuracy of the proposed modification to the CPTU method in evaluating the effective stress friction angle of intermediate soils.

The presented formula proves to be accurate and demonstrates its valuable applicability across various CPTU penetration rates, as evidenced by the thorough calibration with high-quality laboratory samples in the mentioned case studies. It should be pointed out that high quality samples should be collected whenever possible and pair with site-specific calibration of the CPTU-based solution to ensure reliability and a higher degree of confidence when it comes to geotechnical site characterization.

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Table 1. CPTU Database

number	site	Soil type	rates, mm/s	ch, cm2/s	d, mm	Mean PI, %	Qu	Qd	Qt	Bq	NTH φ', °	Lab φ', °	Reference Source
1	lab Centrifuge	100% silts (100S)	0.01-20	0.2	6	0	400	240	230-400	0.01	52.54	33.44	Price et al. (2019)
		97.5% silts 2.5% kaolin		NA			26	36	24-36	0.01	33.73		
		95% silts 5% kaolin		NA		6	7	30	2-30	NA	NA	28.73	
		80% silts 20% kaolin		0.06			0.9	7.9	0.9-13	NA	NA		
2	SR18	Silty clays	0.05-20	0.0582	11.3	16.1	7	25	8.8-3.7	0.01-0.58	27.9	29.00	Kim et al. (2008)
3	Dronninglund	Sandy silts	0.5-60	1.6	NA	5	55	110	65-118	0.01-0.1	42.5	41.00	Holmsgaard et al. (2016)
4	Granite	SP/SM	0.2-200	NA	34.3 or 43.7	NA	24	42	100-90	<0.01	37	NA	Krage et al. (2016)
	Lenordini	SM				NA	30	48	64-52	<0.01	35.0	NA	
5	Mirabello	Silty sands	10-130	0.28	35.7	NA	57	60	61.74-49.9	<0.02	35.3	34.00	Garcia Martinez (2014)
6	Millstream	silty clay to clay	20	0.167	63.5	27	NA	NA	29.94	0.01	36.6	34.00	Shi, W et al. (2016)
7	Refneveien	Clayey silt	20	NA	71.6	NA	NA	NA	27.79	0	31.54	33.96	Carroll R et al. (2017)
	Skibbereen	Sandy silts	20	NA	17.8	NA	NA	NA	21.35	0	30.8	27.43	
8	Halden	silt, sandy clayey	2-20	0.16	NA	NA	NA	NA	21-24	0.07-0.09	33.06	36.00	Carroll R et al. (2018)
9	YRD	Sandy silts	0.2-20		10	9.8	12.4	11	11-35.5	-0.03-0.01	35.05	34.20	This study