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On Critical Densities And Velocities For Pedestrians Entering a Crowd

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Abstract

The problem of pedestrians trying to enter and/or advance against an incoming crowd is studied. The analysis shows that starting with a very limited set of assumptions (elliptical cross-section of pedestrians; constant ratio of forward to lateral separation) one is able to derive from purely kinematic considerations critical densities beyond which it is impossible for pedestrians to enter and/or advance into an incoming crowd. The results obtained indicate that for the common pedestrian size of $a = 0.5\ m,\ b = 0.3\ m$, the limit densities range from $\rho = O(5.0 - 6.06)\ [p/m^2]$, in good agreement with empirical observations.

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1. Introduction

In many situations, pedestrians need to enter a moving crowd. Subway and train stations, airports, sports events, mass concerts and other venues with large crowds are familiar examples. Perhaps the worst case for a pedestrian trying to enter and pass through the moving crowd is shown in Figure 1. The limiting cases are obvious: for low crowd densities and speeds the motion of the incoming pedestrian is basically unimpeded; for very high crowd densities or velocities it may result impossible for the incoming pedestrian to enter and/or advance against the crowd. Clearly, there must be a so-called **critical density and velocity relation** beyond which entering and moving against the incoming stream is impossible.

The attempt to derive such critical values stems from a very immediate need: emergency personnel may have to move against an incoming crowd. Knowing when this is possible and how long it would take to reach the scene of an accident in such a situation are requirements that building and event planners have to meet.

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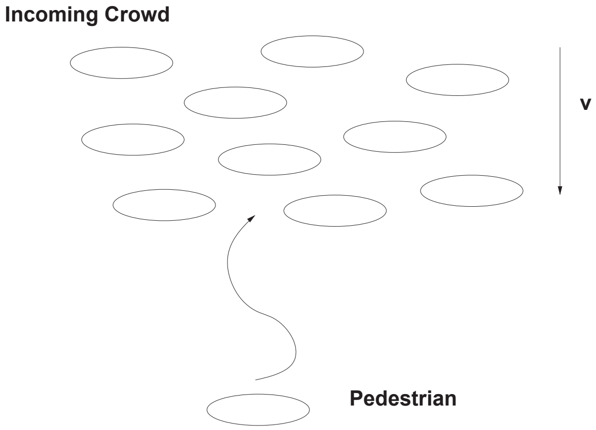

Fig. 1. Pedestrian Trying to Enter an Incoming Crowd

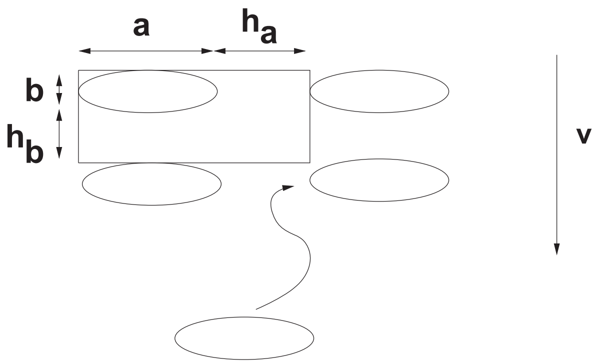

Fig. 2. Pedestrian Dimensions and Row Formation

2. Limiting Densities for Entering a Stationary Crowd

In order to fix the ideas and notation, let us consider pedestrians that have an elliptical cross-section of dimension $E = a \cdot b$ (see Figure 2).

Furthermore, in order to obtain a scenario that is easy to analyze, let us consider the case where the incoming crowd is in row formation as shown in Figure 2. The distances between pedestrians are assumed to be h_a, h_b respectively. Each pedestrian of the crowd then occupies an area of $A = (a + h_a)(b + h_b)$, implying that the density is given by:

$$\rho = \frac{1}{(a + h_a)(b + h_b)} \; [p/m^2] \; . \tag{1}$$

Just to verify: for the typical values of $a = 0.5\ m, b = 0.3\ m$ the limit density ($h_a = h_b = 0$)

$$\rho_l = \frac{1}{ab} \tag{2}$$

is given by: $\rho_l = 6.67\ p/m^2$. Note that this is below the absolute limit density given by

$$\rho_l^a = \frac{4}{\pi ab} \tag{3}$$

which yields $\rho_l^a = 8.49\ p/m^2$, in line with empirical observations (Predtetschenski and Milinski (1971)).

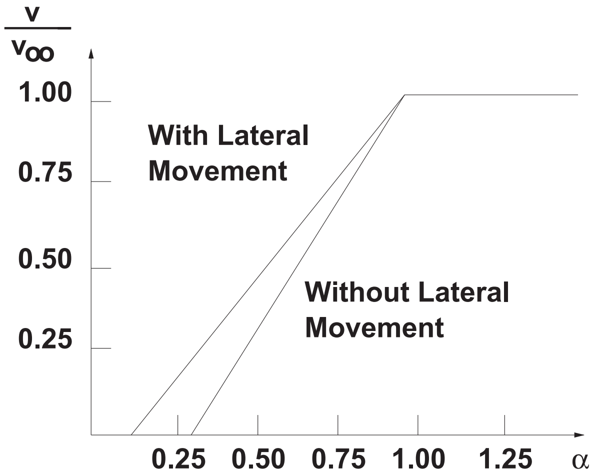

Fig. 3. Velocity vs. Opening Factor α

2.1. Tight Arrangement in Movement Direction

Assume now a completely tight arrangement in the movement direction. Then $h_b = 0$. In this case the density as a function of h_a is given by:

$$\rho_a = \frac{1}{(a + h_a)b} \quad . \tag{4}$$

If we assume an opening between rows of $h_a = \alpha a$ then we have:

$$\rho_a = \frac{1}{ab(1+\alpha)} = \frac{1}{1+\alpha}\rho_l \quad . \tag{5}$$

One is then in a position to establish a maximum possible propagation velocity upstream as a function of the opening factor α. This curve is shown schematically in Figure 3. Note that for lack of any better empirical data, we have assumed a linear decay of velocity with α:

$$v_\alpha = v_\infty \cdot min\left(1, max\left(0, \frac{\alpha - \alpha_{min}}{1 - \alpha_{min}}\right)\right) \quad . \tag{6}$$

Consider first the case when the incoming crowd is rigidly moving with velocity $\mathbf{v}$, i.e. with no lateral movement. For $\alpha \geq O(1.0)$ the movement is free, as the incoming pedestrian has enough room to move between the other pedestrians. For $\alpha = O(0.5)$ the movement slows down to perhaps half the free movement velocity v_∞. Below $\alpha = O(0.4)$ the movement becomes onerous; and below $\alpha = O(0.3)$ the gap between pedestrians is so small that movement is nearly impossible. For the typical values of $a = 0.5\ m,\ b = 0.3\ m$, one can see that for $\alpha = O(0.3)$ the critical density beyond which no entry is possible is given by:

$$\rho_c = \rho_a(\alpha = 0.3) = 5.13 \ \ [p/m^2] \quad . \tag{7}$$

Consider second the case where lateral movement is possible (Figure 4). If we assume that only the pedestrians adjacent to the clearance being opened are moving laterally, the effective opening is $s = 3\alpha h_a$. This implies that the value for α when movement becomes nearly impossible is now $\alpha = O(0.1)$. For the typical values of $a = 0.5\ m,\ b = 0.3\ m$, one can see that for $\alpha = O(0.1)$ the critical density beyond which no entry is possible is given by:

$$\rho_c = \rho_a(\alpha = 0.1) = 6.06 \ \ [p/m^2] \quad . \tag{8}$$

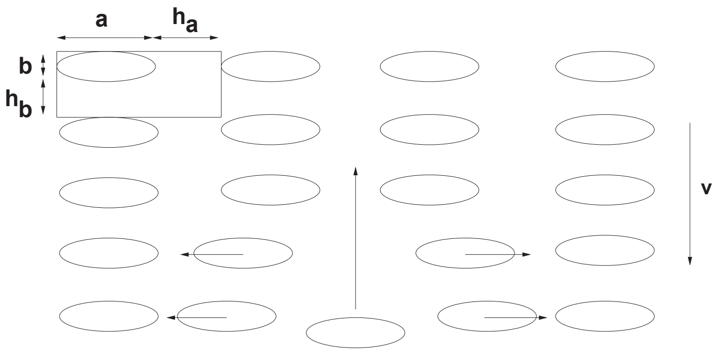

Fig. 4. Lateral Movement of Incoming Crowd

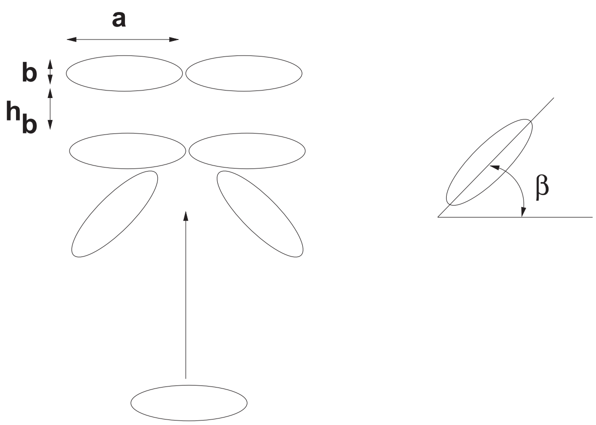

Fig. 5. Rotational Movement of Incoming Crowd

2.2. *Tight Arrangement Normal to Movement Direction*

Assume next a completely tight arrangement normal to the movement direction as shown in Figure 5. Then $h_a = 0$. In this case the density as a function of h_b is given by:

$$\rho_b = \frac{1}{a(b + h_b)} \quad . \tag{9}$$

The only way an incoming pedestrian is going to be able to move against the incoming crowd is if the pedestrians in the crowd can rotate as shown in Figure 5.

If we again consider an opening width αh_a through which the pedestrian can enter the crowd, we have, from geometrical considerations:

$$a = a \cdot cos(\beta) + a \cdot \frac{\alpha}{2} \quad . \tag{10}$$

For $\alpha = 0.3$ this yields $\beta = 31.8^o$. The required distance between pedestrians for the rotation to be possible is then $h_b = a \cdot sin(\beta) = 0.53a$, which for the usual values of $a = 0.5\ m,\ b = 0.3\ m$ yields a limiting density beyond which no entry is possible of:

$$\rho_c = \rho_b(\alpha = 0.3) = 3.54 \ \ [p/m^2] \tag{11}$$

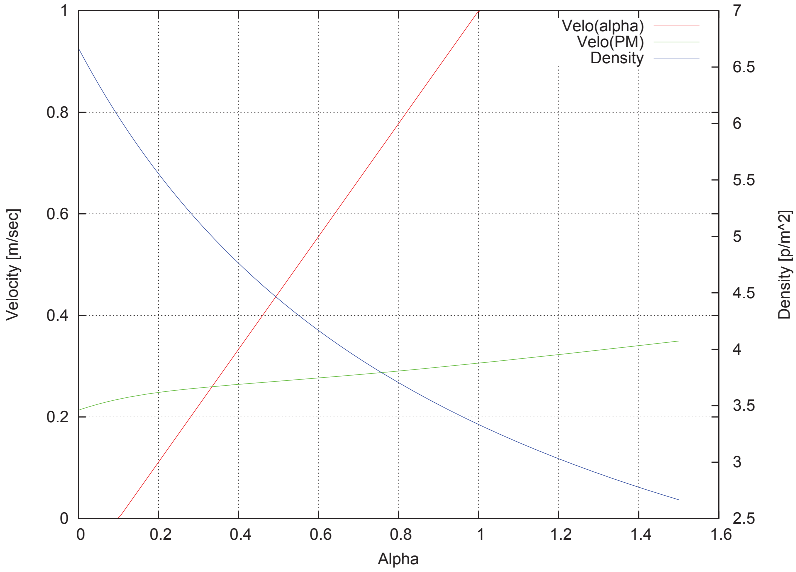

Fig. 6. Velocity and Density vs Gap Factor

i.e. considerably lower than the values obtained before.

We remark that the 'perfect row formation' shown in Figures 2,4,5 is seldomly seen in real life. If the incoming pedestrian would have to 'weave' his way through the crowd, the critical density required to have a possible entry will be lower. Thus, the value given by Eqn.(8) can be regarded as a maximum possible critical density.

3. Entering and Moving Against a Moving Crowd

The incoming crowd is certainly in movement. Therefore, it is not enough to obtain a critical density for passage within a standing crowd, but me must try to ascertain what happens when movement is present. The velocity of the moving crowd will depend on the density, following what is commonly termed as the universal diagram (Predtetschenski and Milinski (1971); Weidmann (1993); Virkler and Elayadath (1994); Daamen et al. (2005); Seyfried et al. (2005); Helbing et al. (2007); Seyfried et al. (2010)). While these can vary significantly depending on the particular situation, let us consider what happens if we assume that the pedestrians are moving in a passage. In this case, the universal diagram from (Predtetschenski and Milinski (1971)), approximated by:

$$v_{PM} = v(\rho) = \frac{1}{60}\left[112\rho_A^4 - 380\rho_A^3 + 434\rho_A^2 - 217\rho_A + 57\right] \;\; [m/sec] \;\; , \tag{12}$$

where ρ_A is the specific area density (area divided by area occupied by pedestrians) yields a very good approximation to reality. Given a density ρ_A, one is then able to obtain the crowd velocity v_{PM}. The resulting curves for the values used above ($a = 0.5\ m,\ b = 0.3\ m$) are shown in Figure 6. This graph is obtained as follows: a) from α, obtain $v(\alpha)$ from Eqn.(6) [red curve]; b) from α, obtain the density from Eqn.(5) [blue curve]; c) from the density, obtain the crowd velocity from Eqn.(10) [green curve].

For the pedestrian entering the incoming crowd to be able to advance, we must have:

$$v(\alpha) > v_{PM} \tag{13}$$

One can see that this happens once $\alpha > (0.34)$, which corresponds to a density of $\rho = O(5.0)\ [p/m^2]$. Thus, we have to lower the critical density for a pedestrian to be able to enter a dense moving crowd to $\rho_c = 5.0\ [p/m^2]$.

4. Conclusions

The analysis described above shows that starting with a very limited set of assumptions:

- Elliptical cross-section of pedestrians;
- Constant ratio of forward to lateral separation;

one is able to derive from purely kinematic considerations estimates for critical densities beyond which it is impossible for pedestrians to enter and/or advance into an incoming crowd.
The analysis shows that for the common pedestrian size of $a = 0.5\ m$, $b = 0.3\ m$, the limit densities range from $\rho = O(5.0 - 6.06)\ [p/m^2]$, in good agreement with our own (Gdoura et al. (2014)) and other empirical observations (Predtetschenski and Milinski (1971)).

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