

RESEARCH ON SIMILARITY LAW OF NONLINEAR SHOCK RESPONSE OF SHIP PLATE FRAME STRUCTURE UNDER UNDERWATER EXPLOSION

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Abstract. The anti-explosion ability of ship grillage structure is an important index to evaluate the vitality of ships. Its model test is a low-cost and effective method to evaluate the vitality of ships and guide the design of ship anti impact structures. In view of the nonlinear and non-stationary process of underwater explosion damage to ship grillage, this paper breaks through the nonlinear effect of transient explosion impact that is not considered in the traditional scale model design, focuses on the one-dimensional nonlinear impact response of ship grillage structure, and carries out the characterization study of the similarity between model experiments and real ships. Considering that the vertical motion of the prototype and the model grillage structure in the model test obey the random walking model, the vertical impact response of the deck grillage is characterized as one-dimensional nonlinear non-stationary Brownian motion, which is described by Hurst index. Based on the classical similarity law, the similarity transformation relationship between the range R and the mean square deviation S is derived, and the Hurst index of the model and the prototype meets the equal relationship; Take a section of grillage structure on a real ship and conduct prototype, 1/2, 1/3, 1/4 and 1/5 one-dimensional nonlinear explosion impact scale simulation tests respectively. The numerical response results show obvious nonlinear characteristics, and the Hurst index of displacement, velocity and acceleration response of the model within the pulse width range is less than 5% compared with the prototype. According to the scale invariance of fractional Brownian motion, the similarity conversion relationship of multiple parameters (displacement, velocity, acceleration and mean square response) is obtained. With the mean square response as the characteristic parameter, the response value of the prototype is converted through this relationship, and compared with the model simulation results, the multi parameter response error under each scale ratio is less than 20%. It provides theoretical and technical support for conducting similar experiments on nonlinear response of underwater explosion shock of ships.

1 INTRODUCTION

As competition for naval equipment intensifies among countries, nations around the world are striving to develop advanced underwater weapons with high speed, large capacity, and precision guidance, which pose a great threat to the vitality and combat effectiveness of ships. Conducting theoretical and experimental research on the destruction mechanism and impact response of ship structures and scaled models under different underwater explosion loads is critical for improving the anti-explosion and anti-impact performance of ships. Based on

experimental data, designing ship structures with targeted approaches is an important means to comprehensively enhance the protection and weapon systems of ships, providing guarantees for the strategic position of China's navy in safeguarding its maritime interests in the future.

Conducting full-scale ship tests is expensive and time-consuming. Therefore, in experimental design, small-scale model experiments are used to predict the impact response of full-scale structures, reducing economic costs, shortening experimental cycles, simplifying experimental environments, etc. The similarity method is widely used in the field of impact dynamics. Underwater explosion shock waves have high peak values and short pulse widths[1-3], which can cause serious damage to ship structures. How to use the similarity method to accurately predict the dynamic response of structures under the action of underwater explosion shock waves has always been a hot topic of concern for scholars around the world[4-6].

Researchers have conducted a significant amount of research on the similarity problem in the field of underwater explosion shock response. Cheng Suqiu et al.[7] used dimensional analysis to derive an empirical formula for underwater explosions that is similar to Cole's law, and ultimately verified the accuracy of this similarity rule through preliminary experiments. Zheng Changyun et al.[8], Zhang Quan[9], Han Lu[10], Cao Yu[11]and others have also used numerical simulation methods (including similarity theory analysis, π theorem derivation, etc.) to verify that the shock response characteristics of the prototype and model structures when subjected to underwater explosions are basically the same for different target objectives. The aforementioned scholars only considered linear features, and few scholars considered nonlinear features. Wang Peng[12] believed that there are theoretical defects in classical similarity theory in the nonlinear field, and considered nonlinear similarity when deducing the similarity relationship between the prototype and model. Pei et al.[13] considered nonlinear factors such as the slender ratio of plate-to-plate and the flexibility coefficient of reinforced ribs, and verified that the response of the model in both linear and nonlinear stages can be effectively equivalent to the prototype. Cheng Ruiqi et al.[14] proposed a similarity conversion method between the model and the prototype based on the shear nonlinear feature parameters of the plate-frame and verified the effectiveness of this shear nonlinear similarity method. Wu Youjun et al.[15] conducted an equivalent scaling of a double-shell cabin section, considering non-linear factors such as the material's ultimate strength. Through numerical simulations and experiments, they verified that the midship bending moment limit of the model can be converted to the actual ship cabin section.

Hurst exponent is an important parameter for describing the nonlinear characteristics of time series, and has been widely used in fields such as nonlinear behavior and nonlinear self-similarity. Scarlat et al.[16] studied the nonlinear similarity law and explored the nonlinear self-similarity characteristics of daily exchange rate time series based on the Hurst exponent. Similarly, Hu et al.[17] introduced the concept of fractals and studied the dynamics and behavior of nonlinear financial time series in the Russian market based on the Hurst exponent. Liu Y, Liu XL, Huang XM, etc.[18-20] studied the nonlinear similarity rule based on the Hurst exponent in the areas of energy consumption, artificial DNA sequences, mathematical Fibonacci sequences, etc., which laid a foundation for using the Hurst exponent in the study of nonlinear shock response similarity rules. When the ship structure is subjected to underwater explosion shock load, the shock wave and the ship structure interact with each other, and the ship structure receives a huge shock load in a very short period of time, which is a complex nonlinear dynamic response process and belongs to a strong nonlinear problem. Currently,

when considering underwater explosion model testing, the design is mainly based on the similarity criteria derived from dimensional analysis, but the influence of nonlinear factors is not considered in this process. With the development of the discipline, in order to further understand the underwater explosion process, it is necessary to analyze the nonlinear problems in it and also to find a method for converting model results into prototype results considering nonlinear factors in the design process of model testing. In this paper, taking the response of ship plate-frame structures under underwater explosion load as the research object, the nonlinear shock response similarity rules are derived, summarized and induced, and a similarity conversion rule applicable to explosion shock nonlinear systems is proposed.

2 HURST INDEX FOR SIMILAR SYSTEMS

2.1 Nonlinear characteristics of plate frame structure response under explosion load

Under the impact load of underwater explosion, the ship impact environment has the characteristics of nonlinear transient motion response, and the non-stationary random characteristics are very obvious[14]. Harold Edwin Hurst (H. E. Hurst), a famous hydrologist, proposed a new statistical parameter, the Hurst exponent, when studying the nonlinear random fluctuation laws of water level. The value of the Hurst exponent corresponds to the correlation of the time-domain trend, and this exponent can characterize one-dimensional nonlinear non-stationary Brownian motion, that is, the vertical impact response of the deck frame[21-26]. As the value of H approaches 0.5, the sequence is a standard random walk sequence, and events are completely independent, indicating that past increments and future increments are uncorrelated, i.e., the nonlinearity of the system[27]. Therefore, the Hurst exponent can be used to characterize the nonlinear characteristics of the one-dimensional impact response of the elastic deformation of the plate-frame structure.

2.2 Hearst index theory derivation of similar systems

For a model, if the scaling ratio is $\lambda=l/L$, where l and L are the dimensions of any corresponding structures of the model and prototype, respectively, then all parameters of the model and prototype satisfy the classical similarity law, as shown in Equation (1):

$$\left\{ \begin{array}{l} x_{\max} = \lambda X_{\max} \\ x_{\min} = \lambda X_{\min} \\ x(t_i) = \lambda X(T_i) \\ \langle x_i \rangle = \lambda \langle X_i \rangle \\ f = \frac{1}{\lambda} F \\ t = \lambda T \end{array} \right. \quad (1)$$

In the equation: X_{\max} and x_{\max} represent the maximum displacement responses of the model and the prototype, respectively; X_{\min} and x_{\min} represent the minimum displacement responses of the model and the prototype, respectively; $X(T_i)$, $x(t_i)$, $\langle X_i \rangle$, $\langle x_i \rangle$, F , f , T , and t represent the corresponding vertical response, average response, sampling frequency, and sampling time

of the model and the prototype at a certain moment.

For both the model and the prototype, there is an initial moment $t = T = 0$. Then, the range and mean square deviation of the model and the prototype satisfy the following similarity conversion relationship:

$$\begin{aligned} R &= X_{\max} - X_{\min} \\ &= \frac{1}{\lambda} [x_{\max} - x_{\min}] \\ &= \frac{1}{\lambda} \hat{R} \end{aligned} \quad (2)$$

$$\begin{aligned} S &= \left\{ \frac{1}{\tau} \sum_{T=1}^{\tau} [X(T) - \langle X_n \rangle]^2 \right\}^{1/2} \\ &= \left\{ \frac{1}{\tau} \sum_{t=1}^{\tau} [\lambda^{-1} x(t) - \lambda^{-1} \langle x_n \rangle]^2 \right\}^{1/2} \\ &= \frac{1}{\lambda} \hat{S} \end{aligned} \quad (3)$$

In the equation, R represents the range of the prototype; S represents the mean square deviation of the prototype; \hat{R} represents the range of the model; \hat{S} represents the mean square deviation of the model; τ represents the total duration of the action.

Substituting equations (2) and (3) into the Hurst empirical formula, we get:

$$\frac{R}{S} = \frac{\frac{1}{\lambda} \hat{R}}{\frac{1}{\lambda} \hat{S}} = \frac{\hat{R}}{\hat{S}} = \left(\frac{\tau}{2} \right)^{\hat{H}} \quad (4)$$

That is:

$$\left(\frac{\tau}{2} \right)^H = \left(\frac{\tau}{2} \right)^{\hat{H}} \quad (5)$$

In the formula, H and \hat{H} represent the Hurst exponent of the prototype and model, respectively.

From equation (5), it can be inferred that in a similar system, the Hurst exponent of both the model and prototype must remain equal, that is $H = \hat{H}$.

2.3 Hurst exponential verification based on complete geometric similarity model

To verify the theoretical derivation of the equation above, a scaled-down simulation experiment was conducted on a section of a deck frame taken from a certain ship. The ship deck frame model is shown in Figure 1, with dimensions of 18m*17.4m*0.02m (corresponding to the size of a cargo hold on the actual ship). Reinforcement ribs are arranged along both the horizontal and vertical directions of the deck frame, with T-shaped sections[28-31].

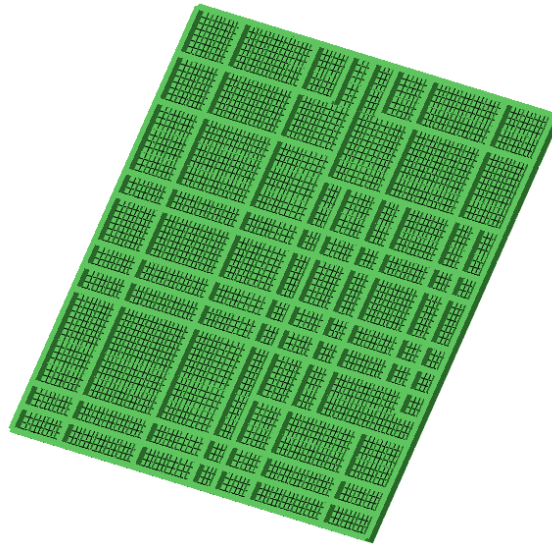


Figure 1: Hull frame model

To perform the calculations, the dimensions of the deck frame, reinforcement rib dimensions, explosive charge radius, stand-off distance, and simulation time were scaled down according to the scaling ratio λ (dimensionless scaling relationship is described in [32]). Table 1 shows the details of five different conditions. According to the classification of underwater explosion types mentioned in literature [9], all conditions set in this experiment are classified as far-field explosions, which means that the deck frame structure only needs to withstand the shock wave load. The simulation experiment was carried out based on Abaqus finite element simulation software. The mesh was divided into a total of 9,264 nodes and 9,640 elements for the ship's deck frame model, with the mesh distribution shown in Figure 1. To ensure that the calculation results are not affected by the number of meshes, the number of meshes was kept consistent for each scaling ratio. Similarly, the water domain (medium for transmitting impact load) was set up with a total of 97,794 nodes and 552,113 elements. The water domain was assumed to be a semi-spherical solid with a diameter of 90m. The center of the upper surface of the water domain was set as the origin with the z-axis pointing vertically upwards and away from the upper surface of the water domain. The explosive charge was set at the corresponding position on the positive z-axis. The water domain structure is shown in Figure 2.

Table 1: Hull plate frame scaling simulation test conditions

Model	Side length/m	Plate thickness/m	Stiffener size	Equivalent weight/kg	Detonation distance/m	Calculation time/s
Prototype	18*17.4	0.02	$\frac{0.012 \times 0.05}{0.016 \times 0.25}$	500	30	1
$\lambda = 1/2$	9*8.7	0.01	$\frac{0.06 \times 0.025}{0.08 \times 0.125}$	62.5	15	0.5
$\lambda = 1/3$	6*5.8	0.0067	$\frac{0.04 \times 0.017}{0.0053 \times 0.083}$	18.5	10	0.33
$\lambda = 1/4$	4.5*4.35	0.005	$\frac{0.03 \times 0.0125}{0.04 \times 0.0625}$	7.8125	7.5	0.25
$\lambda = 1/5$	3.6*3.48	0.004	$\frac{0.0024 \times 0.01}{0.0032 \times 0.05}$	4	6	0.2

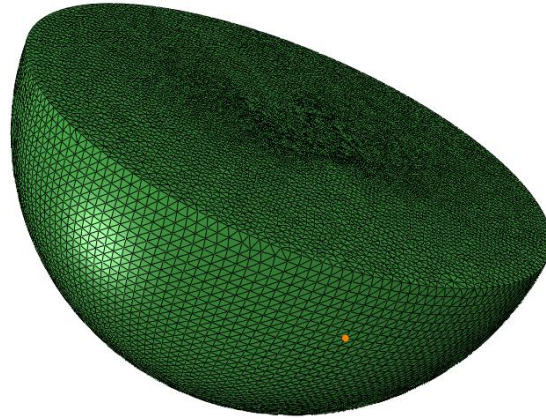


Figure 2: Page layout

The material properties of the deck frame structure and water domain are set as shown in Table 2.

Table 2: Material properties

Component	Parameter	Numerical value
Water area	Buckling modulus (N/m ²)	2.14×10^9
	Density (kg / m ³)	1025
Plate-frame construction (Steel)	Young's modulus (Pa)	2.1×10^{11}
	Poisson's ratio	0.3
	Density (kg / m ³)	7850

The calculation time for the prototype condition is set to 1 second, with a sampling frequency of 100 kHz, resulting in a total of 1,000,000 data points in the response curve. To ensure that the number of sampled data points is consistent and eliminate the influence of the difference in the number of sampled data points on the Hurst exponent, the sampling frequency for the condition with a scaling ratio of 1/2 is adjusted to 200 kHz, resulting in 1,000,000 data points as well. The sampling frequency is also adjusted for other scaling ratio conditions to ensure that there are 1,000,000 data points within the calculation time period[33-36]. The shock wave load from underwater explosions exhibits strong nonlinearity within the pulse width[37-40]. Therefore, the Hurst exponent of displacement, velocity, and acceleration during the shock wave pulse width was calculated for each scaling ratio condition. Table 3 shows the pulse width of the shock wave for the prototype and each scaling ratio condition (extracted result in the z direction from the origin), as well as the results of the Hurst exponent calculation. The changes in displacement, velocity, and acceleration values for each model during the shock wave pulse width are shown in Figure 3.

Table 3: Statistics of Hurst exponents for similar models at different scales

Scaling ratio	Shock wave pulse width (μs)	Displacement		Speed		Acceleration	
		Hurst index value	Error with prototype	Hurst index value	Error with prototype	Hurst index value	Error with prototype
Prototype	24	0.698786	—	0.711439	—	0.626169	—

$\lambda = 1/2$	12	0.689262	-1.36%	0.74665	4.95%	0.615858	-1.65%
$\lambda = 1/3$	8	0.692606	-0.88%	0.70909	-0.33%	0.635037	1.42%
$\lambda = 1/4$	6	0.68715	-1.67%	0.707745	-0.52%	0.65439	4.51%
$\lambda = 1/5$	4.8	0.696839	-0.28%	0.714675	0.45%	0.648963	3.64%

From the table, it can be seen that in the conditions designed based on complete geometric similarity models, the Hurst exponent errors of displacement, velocity, and acceleration response for the model and prototype are all less than 5%, which can be considered approximately equal. Therefore, for similar systems, the Hurst exponent of the shock response of the model and prototype is approximately equal, and the Hurst exponent is around 0.7, indicating non-linear characteristics, with more obvious non-linear features in acceleration. The displacement, velocity, and acceleration results over the entire calculation period are shown in Figure 4.

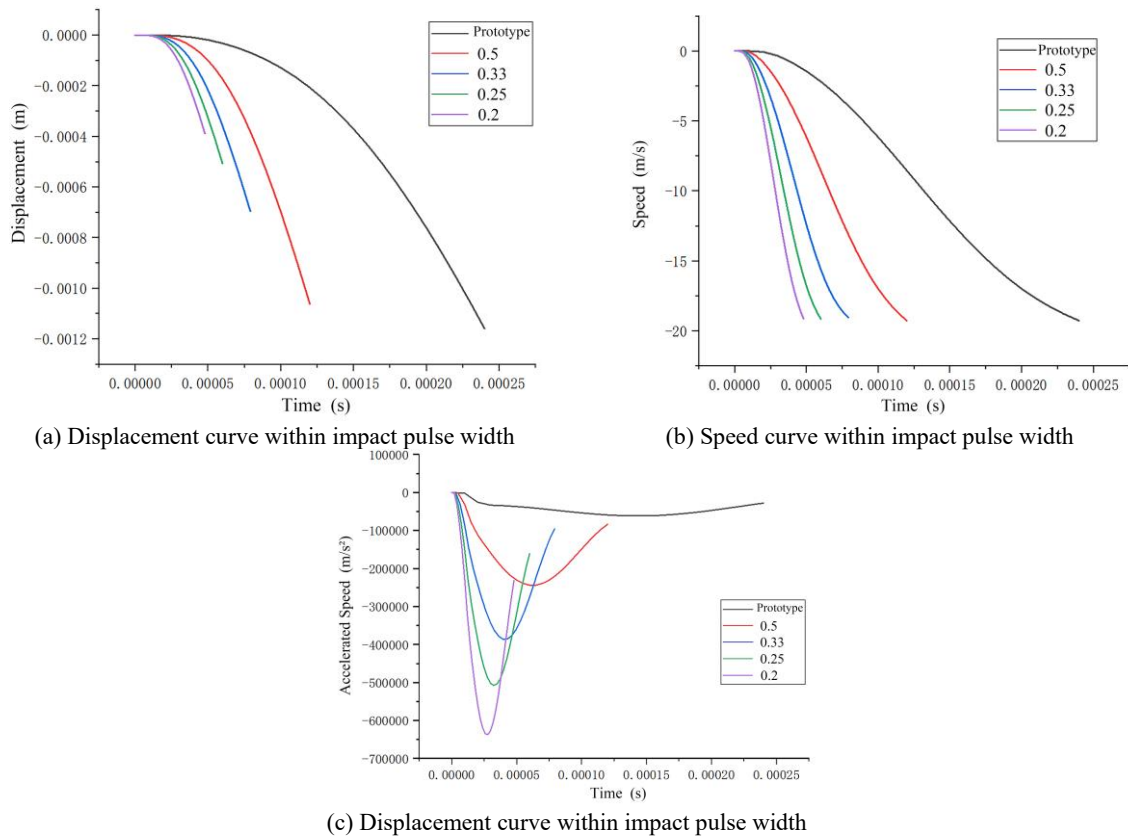
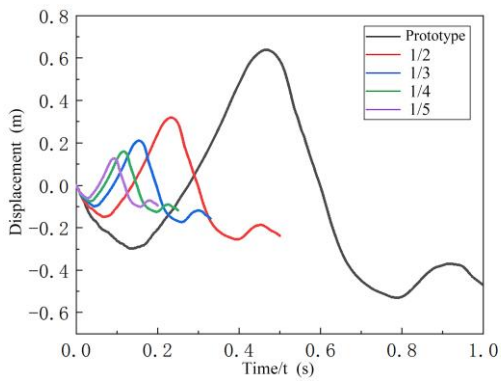
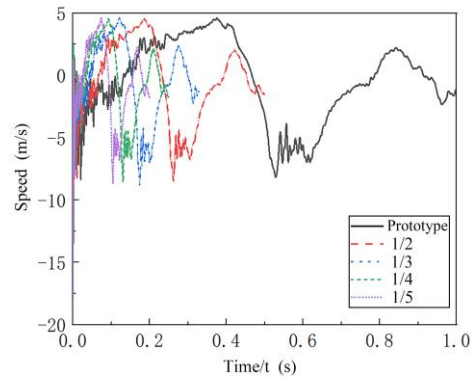


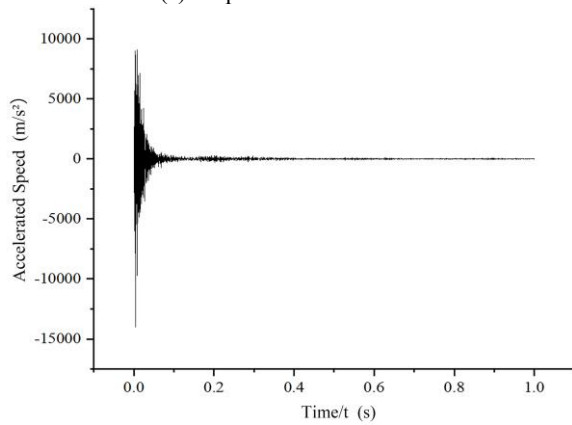
Figure 3: Calculation result curve of different scaling ratio within the impact pulse width



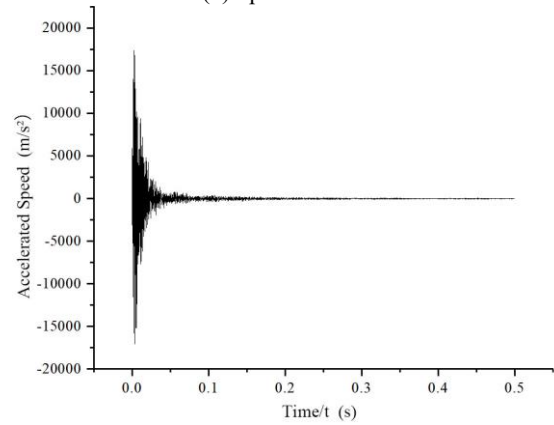
(a) Displacement curve



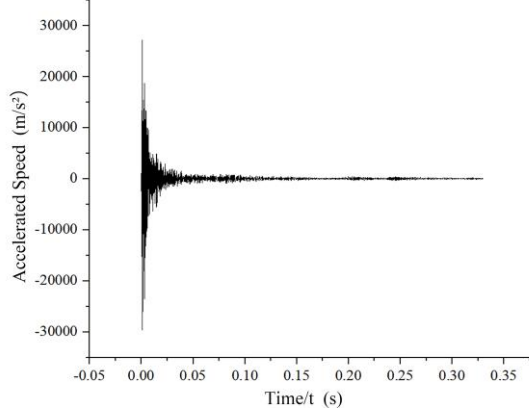
(b) Speed curve



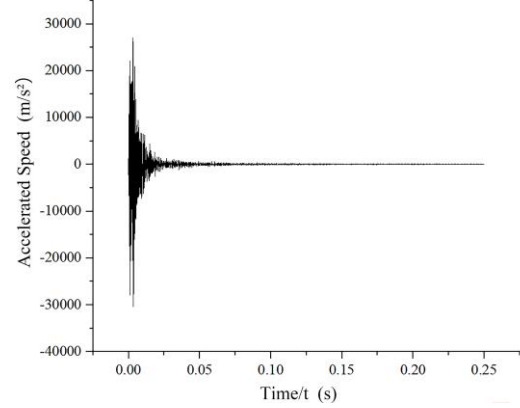
(c) Prototype acceleration curve



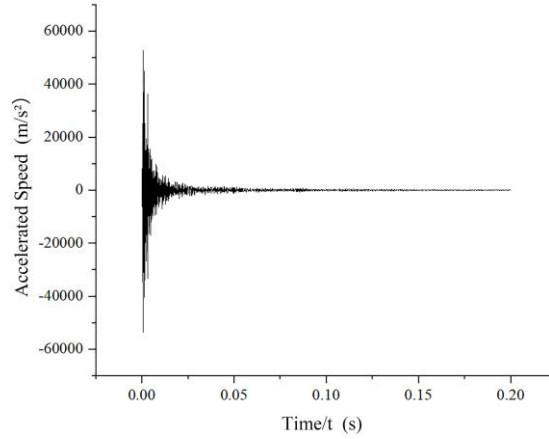
(d) 1/2 scaled acceleration curve



(e) 1/3 scaled acceleration curve



(f) 1/4 scaled acceleration curve



(g) 1/5 scaled acceleration curve

Figure 4: Calculation result curve of different scaling ratio within the impact pulse width

3 NONLINEAR MOTION SIMILARITY TRANSFORMATION BASED ON HURST INDEX

3.1 Derivation of nonlinear motion similar transformation

If only the vertical impact response of the ship's deck frame is considered, the ship's impact environment can be simplified as a non-linear and non-stationary Brownian motion[41]. This means that the prototype deck frame structure's vertical motion is represented by $X(t)$, and the model deck frame structure's vertical motion is represented by $x(t)$, both following a random walk model. If the prototype and model fully satisfy geometric similarity, under the condition that the explosion distance and charge quantity satisfy traditional similarity, the working conditions meet the conditions of similar excitation load and similar boundary conditions (This means that the boundary motion conditions are similar and the displacement and velocity satisfy the similarity law.) Therefore, we should have:

$$x(t) = \lambda X(T) \quad (6)$$

Assuming $x_i = x(t_i)$, $X_i = X(T_i)$ ($i = 0, 1, 2, \dots, N$), we can obtain from equation(6):

$$x_i = \lambda X_i \quad (7)$$

Equation (7) is derived from the motion similarity law based on geometric similarity, i.e., scaling ratio. However, in practical engineering, due to the nonlinear characteristics of the dynamic response of the plate frame, the excitation load and boundary conditions of the prototype and model of a certain position show uncertainty. Therefore, for the complex plate-frame structure of a ship, the motion similarity between equations (6) and (7) is difficult to guarantee consistency.

According to the scale invariance (self-similarity) of fractional Brownian motion[42], we can obtain:

$$y_i = \lambda^H x_i \quad (8)$$

$$t = \lambda^H T \quad (9)$$

The mean square response $\langle x^2 \rangle$ of the model motion is:

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 p(x) dx \quad (10)$$

In the formula, $p(x) = p(x, \tau_x)$, y is the step size of the model's random walk, τ_y is the time used for N steps and assumed to be constant. When τ_y takes a certain value, the corresponding mean square response of the prototype $\langle Y^2 \rangle$ is:

$$\begin{aligned} \langle Y^2 \rangle &= \int_{-\infty}^{\infty} y^2 p(y) dy = \int_{-\infty}^{\infty} (\lambda^H x)^2 p(\lambda^H x) \lambda^H dx \\ &= \int_{-\infty}^{\infty} \lambda^{2H} x^2 \lambda^{-H} p(x) dx \\ &= \lambda^H \int_{-\infty}^{\infty} x^2 p(x) dx \\ &= \lambda^H \langle X^2 \rangle \end{aligned} \quad (11)$$

In the derivation of the above equation, the self-similarity of fractional Brownian motion is used.

$$p(\lambda^H x) = \lambda^{-H} p(x) \quad (12)$$

We can obtain from equation (11):

$$\sqrt{\langle Y^2 \rangle} = \lambda^H \sqrt{\langle X^2 \rangle} \quad (13)$$

Comparing equations (7) and (13), we can notice that in the classical similarity law, $y = \lambda x$, while in the fractional Brownian motion similarity law, $y \sim \sqrt{\langle Y^2 \rangle}$, $x \sim \sqrt{\langle X^2 \rangle}$, $y = \lambda^H x$, there are significant differences between the two. Substituting $\lambda = l / L$ into both similarity laws, for the classical similarity law:

$$x = \frac{1}{\lambda} y = \frac{L}{l} y \quad (14)$$

For the fractional Brownian motion similarity law:

$$\sqrt{\langle X^2 \rangle} = \frac{1}{\lambda^H} \sqrt{\langle Y^2 \rangle} = \left(\frac{l}{L} \right)^H \sqrt{\langle Y^2 \rangle} \quad (15)$$

Taking the derivative of the discrete displacement response with respect to time, we obtain:

$$u = \frac{dy}{dt} = \frac{d(\lambda^H x)}{d(\lambda^H T)} = \frac{dx}{dT} = v \quad (16)$$

In the formula, u is the velocity response of the model, v is the velocity response of the prototype; similarly:

$$a = \frac{1}{\lambda^H} A \quad (17)$$

In the formula, a is the acceleration response of the model, and A is the acceleration response of the prototype.

3.2 Transformation of displacement response

The nonlinear motion similarity transformation relationship derived in this section is verified by using the similar model conditions discussed in the previous section. The central measuring point of the structure is selected, and the mean square response of the displacement is calculated as shown in Figure 5. The statistical results of the mean square displacement response of both the prototype and the model are shown in Table 4. From Figure 6 and Table 4, it can be seen that the use of fractional Brownian motion similarity law has a good similarity transformation for mean square response of displacement. The error between the prototype and model is less than 10% when the last data point of the mean square response of the impulse load in the prototype and model is used for calculation.

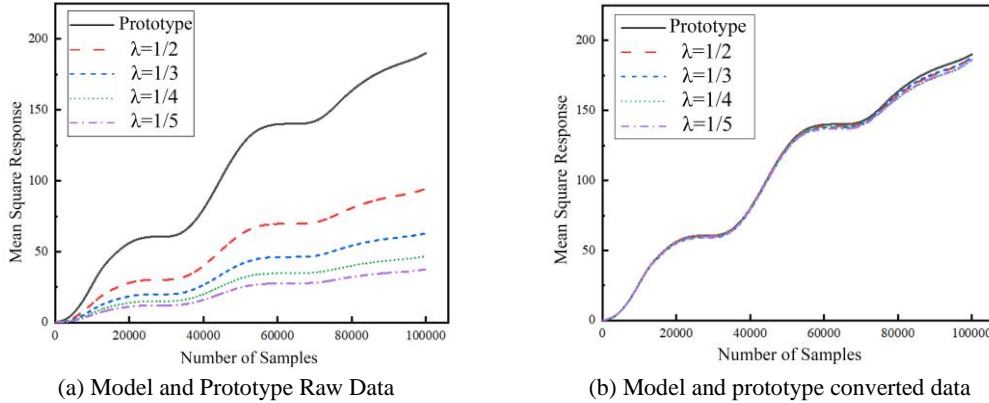


Figure 5: Calculation result curves of different scales of hull plate frame after similar treatment

To ensure the correspondence of time between the model and prototype, and to exclude the influence of excessively small mean square response values, we take the last data point of the mean square response of the prototype and model to calculate the error between them.

Table 4: Prototype and Model Displacement Mean Square Response Results Statistics

Scale ratio	Prototype mm	The original values of the model	$1/\lambda^H$	The transformed values of the model	Error%
$\lambda = 1/2$	0.005618	0.004540	1.612458	0.005386	-4.32%
$\lambda = 1/3$	0.005618	0.002596	2.155164	0.005207	-7.90%
$\lambda = 1/4$	0.005618	0.002112	2.59242	0.006175	9.02%
$\lambda = 1/5$	0.005618	0.001817	3.069514	0.006191	9.26%

3.3 Transformation of Velocity and Acceleration Response

According to the similarity formula mentioned earlier, it can be seen that the velocity of the scaled model is equal to the velocity of the prototype. The statistical results of mean square response of velocity and acceleration of the prototype and model are shown in Tables 5 and 6, respectively.

Table 5: Prototype and Model velocity Mean Square Response Results Statistics

Scale ratio	Prototype mm/s	Model	Error %
$\lambda = 1/2$	67.46801	67.46801	-2.31%
$\lambda = 1/3$	67.46801	65.90623	-7.10%
$\lambda = 1/4$	67.46801	62.68012	-12.41%
$\lambda = 1/5$	67.46801	59.09498	-14.14%

Table 6: Prototype and Model Acceleration Mean Square Response Results Statistics

Scale ratio	Prototype mm/s ²	The original values of the model	λ^H	The transformed values of the model	Error %
$\lambda = 1/2$	516015.6	864604.3	0.652542	564190.6	9.34%
$\lambda = 1/3$	516015.6	1158146	0.494582	572798.3	11.00%
$\lambda = 1/4$	516015.6	1455217	0.403662	587415.6	13.84%
$\lambda = 1/5$	516015.6	1715446	0.35188	603631.1	16.98%

According to Table 6, the error of mean square response of velocity between prototype and model is less than 15%. From Table 6, it can be seen that the use of fractional Brownian motion similarity law has a very good similarity transformation for mean square response of acceleration when the scale ratio is 1/2. The error between the prototype and model is less than 10%, and under other scale ratios, the error exceeds 10% but is still less than 20%. Comparing the mean square response errors of each parameter, it can be seen that as the scale ratio increases, the error also increases accordingly. This is because in the similarity transformation, the approximate relationship ignores some factors between the prototype and the model, and the larger the difference in size between the prototype and the model, the more distorted the approximate relationship becomes, which is also in line with the general rules of similarity model experiments. Through the analysis of this nonlinear similarity transformation law, it can be known that when designing model tests, setting the scale ratio within 1/5 can achieve relatively good similarity transformation results.

4 SUMMARY

4.1 Main headings

In order to solve the problem that the nonlinear characteristics of the response of the plat frame structure were not considered in the design of ship structural damage model test under traditional explosion load, a similar transformation relationship between the model and prototype was established by means of nonlinear non-stationary Brownian motion analysis combined with Hurst index. Based on Hearst's empirical formula, the equality relationship between the model and the original Hearst exponent is verified, and the transformation relationship of $y_i = \lambda^H x_i$ is obtained according to the scale invariance (self-similarity) of fractional Brownian motion. The simulation and prediction test of shrinkage of a real ship under waterline frame was carried out. The results show that the Hurst index of displacement, velocity and acceleration of the similar model under different shrinkage scales is about 0.7, and the error

of the Hurst index of each parameter response between the model and prototype is less than 5%. According to the conversion relationship between the prototype and the model displacement mean square response results, the mean square response values of the model results after conversion to the prototype are obtained, and the errors between the prototype and the model are less than 10%. Meanwhile, the mean square response error of prototype and model velocity is less than 15%, and the mean square response error of prototype and model acceleration is less than 20%.

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