

1 **ASSESS OF STABILITY FOR RELIABILITY THEORY IN CONSIDERATION OF**
2 **CHANGE OF SHEAR RESISTANCE BY DEPTH**

3 **Tran Vu DOAN¹, Trung Viet TRAN¹, Van Thao LE^{1*}, Thuy Kim Phuong DOAN¹**

4 ¹ University of Science and Technology –The University of Danang

5 54 Nguyen Luong Bang Street, Lien Chieu District, Danang city

6

7 **Corresponding Author:**

8 **Dr. Van Thao LE**

9 The University of Danang-Danang University of Science and technology

10 54 Nguyen Luong Bang Street, Danang city, Vietnam

11 Email: lvthao@dut.udn.vn

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14 **Abstract**

15 Slope stability problem is one of the most common problems in construction design. The
16 application of tools often follows a pattern, using only fixed input parameters and resulting in
17 a factor of safety according to the parameters themselves. This calculation model is only correct
18 during the time when the shear resistance parameter (c , φ) of the ground does not change and
19 is no accurate after the structure put into use, leading to slope instability, causing landslides and
20 damage to the slope after a period of exploitation and use. The experimental studies have shown
21 that the shear resistance parameter (c , φ) of the soil ground changes randomly with depth. As a
22 result, current mechanical computational models are no accurate. This paper proposes a new
23 model to analyze stability based on reliability theory with the change of shear resistance
24 parameters by depth. Firstly, by using Karhunen – Loeve series, the result of slope stability
25 coefficient of proposed model is smaller than these when not considering the change of shear
26 resistance (c , φ) by depth. Then, by using Monte - Carlo simulations ($n=1000$) combined

27 Karhunen – Loeve series, the forecast results are different from those only considering the static
28 problem and the problem of random quantities, the probability of failure increase significantly.

29 **Key words:** reliability, slope stability, shear resistance, simulation,

30 **Introduction**

31 The problem of slope stability is one of the most common problems in construction
32 design. The years of the twentieth century have researchers and methods proposed by Bishop
33 (1955) [1] , Janbu (1954) [2]. From limit equilibrium theory to complex methods with high
34 accuracy of Morgenstern - Price (1965) [3], Spencer (1973) [4], Janbu (1973) [5], the
35 application of the above methods through the commercial software such as: Slope/W, Plaxis-
36 2D or Plaxis-3D...However the input is static parameters. According to the method of dividing
37 soil columns, the sliding mass above each hypothetical sliding surface divided into vertical soil
38 columns, then analyze the force and moment balance conditions for the force system acting on
39 the earth column to find the slope stability coefficient (FoS). Stability coefficient defined as the
40 ratio of the total shear resistance moment to the total shearing moment acting on the sliding
41 surface. After that, the researchers improved, supplemented, and proposed new calculation
42 methods suitable to the real situation such as Janbu method (1954), Bishop method (1955),
43 Spencer method (1973). The Janbu method does not completely satisfy the force and moment
44 balance equations. Characteristic for the line of action method, in cases it is difficult to
45 converge. At the same time, Janbu gave the coefficient f_0 without any specific basis, so it does
46 not use in practice. The Bishop method is a popular method today, however, this method does
47 not fully consider the vertical forces on both sides of the soil mass, and at the same time it is
48 necessary to find out which sliding arc (sliding center) is the most dangerous, having the lowest
49 factor of safety to evaluate the instability of the slope, so further research is needed. The Spencer
50 method is a method that fully considers the force components of a soil element, strictly satisfies
51 the static equilibrium, fully considers the force and moment balance equations, and may be use

52 for circular sliding surfaces and not round. However, Spencer only calculates the moment
53 equilibrium equation at the bottom of the soil column, thereby not simulating the sliding center
54 and dangerous sliding arc of the slope. This method is quite complicated when the unknowns
55 and the number of equations is large. Zhang and Zhou (2018) [6] use Monte - Carlo (MC)
56 simulation, the LEM limit balance method finds the FoS factor of safety and the failure
57 probability P_f , then compares their method with the classical methods of Bishop and Janbu. The
58 obtained FoS factor of the UD-LASSO method is lower than that of Bishop and Janbu. The
59 simple Bishop and Janbu methods of slope stability analysis have widely used since their
60 presentation in the 1950s. Although Bishop's method does not satisfy the lateral force balance
61 and the method of Janbu does not satisfy moment equilibrium, but FoS factor can easily
62 calculated for most slope types. However, FoS values can differ by up to $\pm 15\%$ from results
63 calculated by methods that satisfy force and moment balance such as Spencer's method or the
64 Morgenstern-Price method. Although a direct comparison between different methods is not
65 always possible, the FoS value determined using Bishop's simplified method for the expected
66 circular sliding surface may differ slightly 5% more than the Spencer or Morgenstem - Price
67 methods. The simple Janbu method used for non-circular surfaces, often giving FoS values up
68 to 30% lower for the more rigorous methods. In contrast, the simplified Janbu method can give
69 up to 5% higher FoS values for slopes and uncommon sliding surface shapes (Fredlund and
70 Krahn, 1977) [7]. Currently, in the world, the researchers more fully evaluate the factors when
71 calculating slope stability, considering the random change of shear resistance (c , γ , φ) of the
72 soil as well as determining the probability of failure, however, very few studies have considered
73 the random variation in depth. Therefore, the article proposes to simulate and evaluate the
74 behavior of random factors in depth. These models usually adopt fixed input parameters for one
75 or more separate locations, then through calculation methods to make general conclusions about
76 the ability to ensure the overall stability of the slope.

77 Soil is a natural material and is sensitive to its surroundings, so its physical properties
78 change from one location to another. This variation can be as part of a heterogeneous soil state.
79 The random change of the shear resistance parameters of the soil is one of the most important
80 problems in the analysis of geotechnical works. The field experiments with different soils have
81 shown that the shear strength of the soil can view as a random quantity and simulated by the
82 Normal distribution function (Lumb, 1966 [8]; Tan et al., 1993 [9]). This random variation
83 characterized by the coefficient of variation (COV).

$$84 \quad COV = \frac{\sigma}{\mu} \quad (1)$$

85 As in slope stability calculations, resistance parameters, soil bearing capacity are the most
86 important indicators in geotechnical design. The calculation methods are based on the input
87 parameters of the soil. Therefore, it is important to identify certainly these parameters.

88 Through the research results on the change index of the soil's physical parameters in the
89 calculation of geotechnical works, it is necessary to consider the physical properties of the soil
90 as a random variable and accurately reflect the working condition of the soil. Phoon and
91 Kulhawy (1999) [10] confirmed that the use of the Normal distribution model to simulate the
92 random changes of mechanical properties is consistent with the experimental results.

93 **Building a model to calculate the stability coefficient**

94 Within the scope of the paper, the Bishop method to determine the slope stability
95 coefficient is used and the problem considers the following parameters: weight density γ ; unit
96 cohesion c ; internal friction angle ϕ ; soil element width b ; soil element self-weight W ; inclined
97 angle of the soil element to the horizontal θ ; frictional force T ; elemental force U ; reaction N
98 to give the factor of safety FoS without considering the shear force S between the soil elements,
99 passive and active soil pressures $E1$, $E2$, and the presence of water in the slope.

100 Consider the slope ABCD as shown in Figure 1, with a dangerous sliding arc EF. To
101 determine the FoS coefficient by the Bishop method, the sliding arc region will be divided into

102 n different pieces and the stability coefficient FoS is determined. The Bishop method does not
 103 take into account the variation of shear resistance with depth. However, when considering the
 104 change with depth of the soil shear resistance parameter in the same i^{th} piece, according to the
 105 depth, the soil shear resistance parameter (c , φ) is different (Fig 1). This change is simulated by
 106 Karhunen-Loeve series and simulated by Matlab software. At this time, the stability coefficient
 107 FoS will be redefined as follow :

108 Considering the i^{th} element fragment is divided into m subdivisions according to the depth
 109 (Δy), then the weight of the soil block i (W_i) is calculated :

$$110 \quad W_i = \sum_{j=1}^m \Delta x \times \Delta y \times \gamma_{ij}$$

111 Where: Δx , Δy - width and thickness of element ij ,

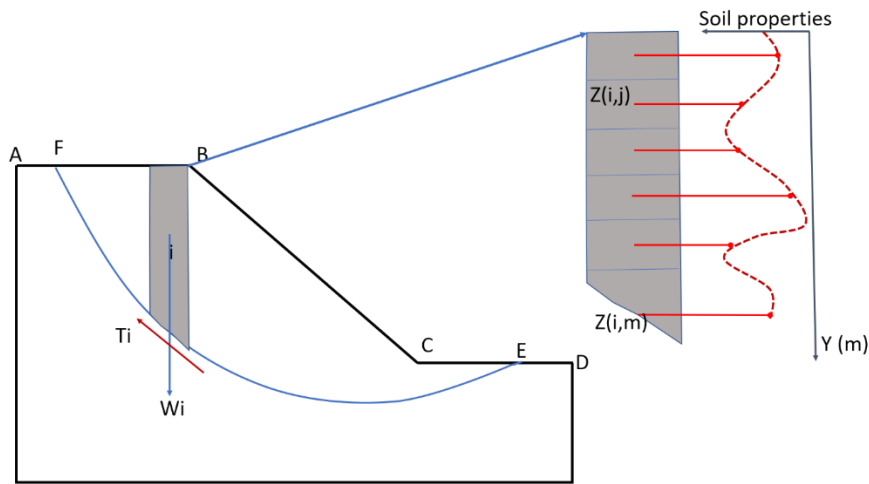
112 γ_{ij} - density of the ij^{th} soil element corresponding to the depth Y_j

113 At the position of the sliding surface of the i^{th} soil element, we will have the values of the
 114 cohesion force C_i and the internal friction angle φ_i which are different according to the depth
 115 Y_i of the i^{th} fragment sliding arc. At this time, the shear resistance of the i^{th} soil element is
 116 determined as follow :

$$117 \quad T_i = C_i \times \Delta x \times \text{Sec}\theta_i + W_i \times \text{Cos}\theta_i \times \tan\varphi_i$$

118 The stability factor FoS is determined in this case :

$$119 \quad \text{FoS} = \frac{\sum_{i=1}^n [C_i \times \Delta x \times \text{Sec}\theta_i + W_i \times \text{Cos}\theta_i \times \tan\varphi_i]}{\sum_{i=1}^n W_i \times \text{Sin}\theta_i}$$

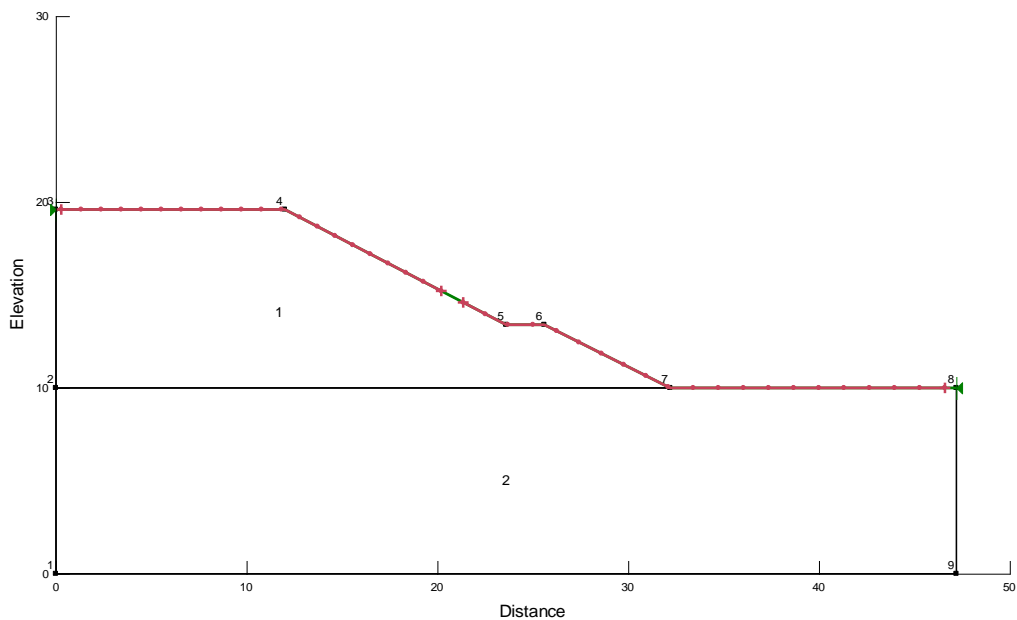


120 Fig 1. Schematic of determining the stability coefficient FoS by Bishop's method
 121 considering the depth variation of the soil's physical properties

122 **Results and conclusions**

123 **Analyze stability coefficient using Kahunen -Loeve serives**

124 A numerical example of slope is considered in Fig 2. The avaraged values of weight of density,
 125 cohension, internal friction angle are hypothesized in Table 1. The value of CoV and width b is
 126 also selected in Table 1.



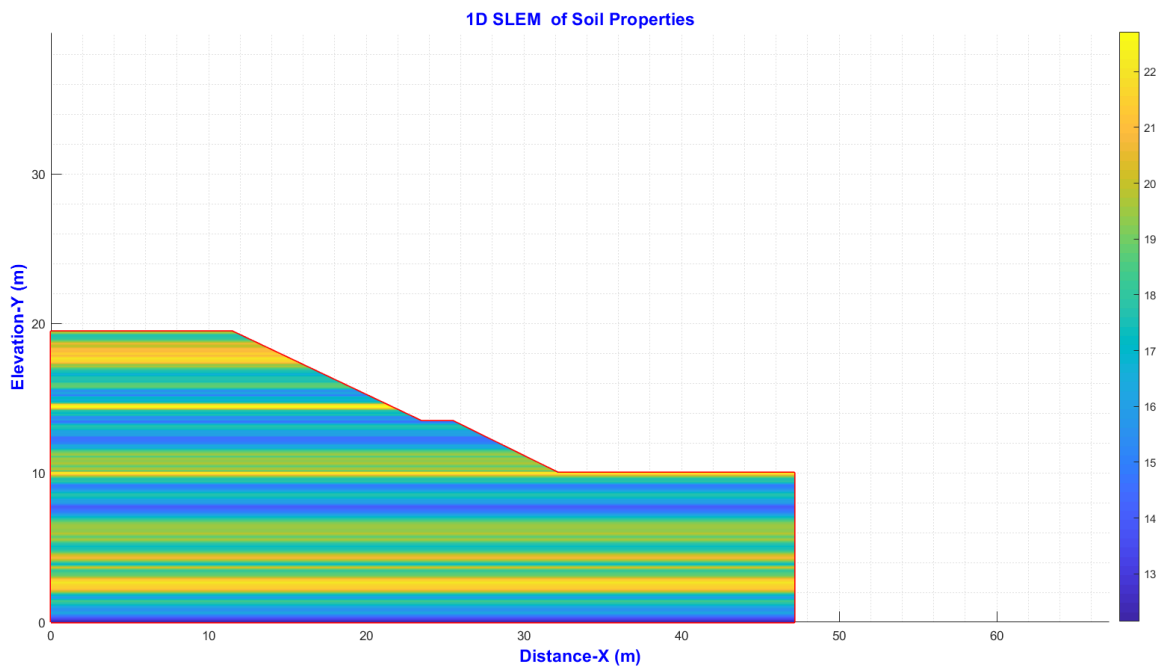
127 Fig 2. Analysis slope diagram

128

Table 1. Soil's physical properties

Physical properties	Average	CoV	b (m)
Weight Density (KN/m ³)	19.5	0.1	1
Cohension (KN/cm ²)	18.4	0.1	1
Internal friction angle (°)	18.4	0.1	1

130 Simulation results of the change with depth of cohesion C by Kahunen -Loeve series are
 131 showed in Fig 3. The changed value of cohesion from 13 to 22 kN/m³. The blue area show
 132 cohesion is smaller than the average value.



133 Fig 3. Simulation results of the change with depth of cohesion C by Karhunen – Loeve
 134 series

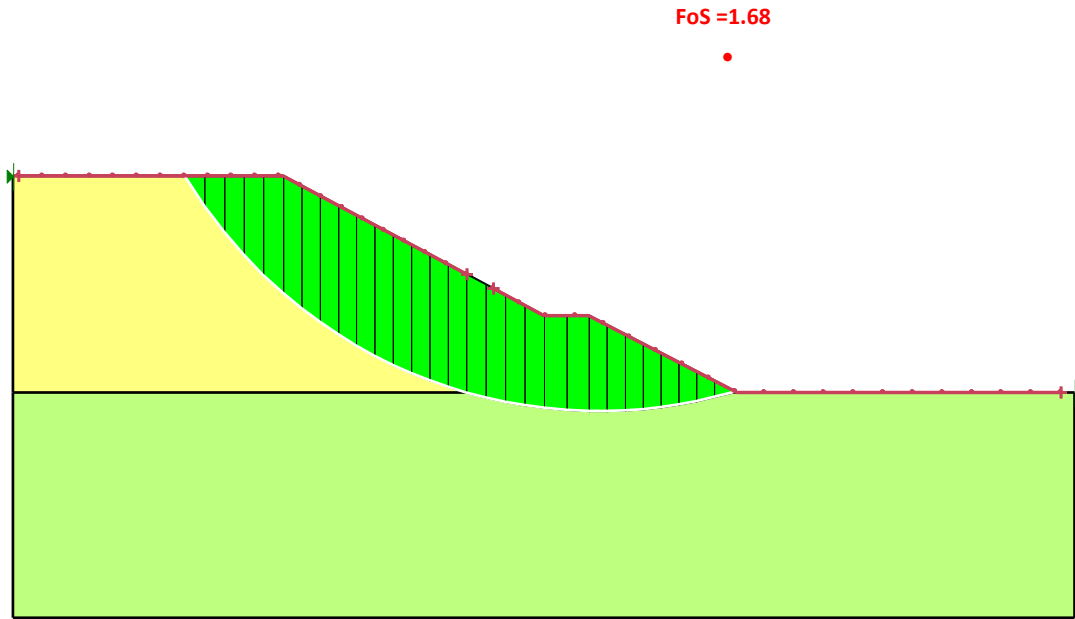
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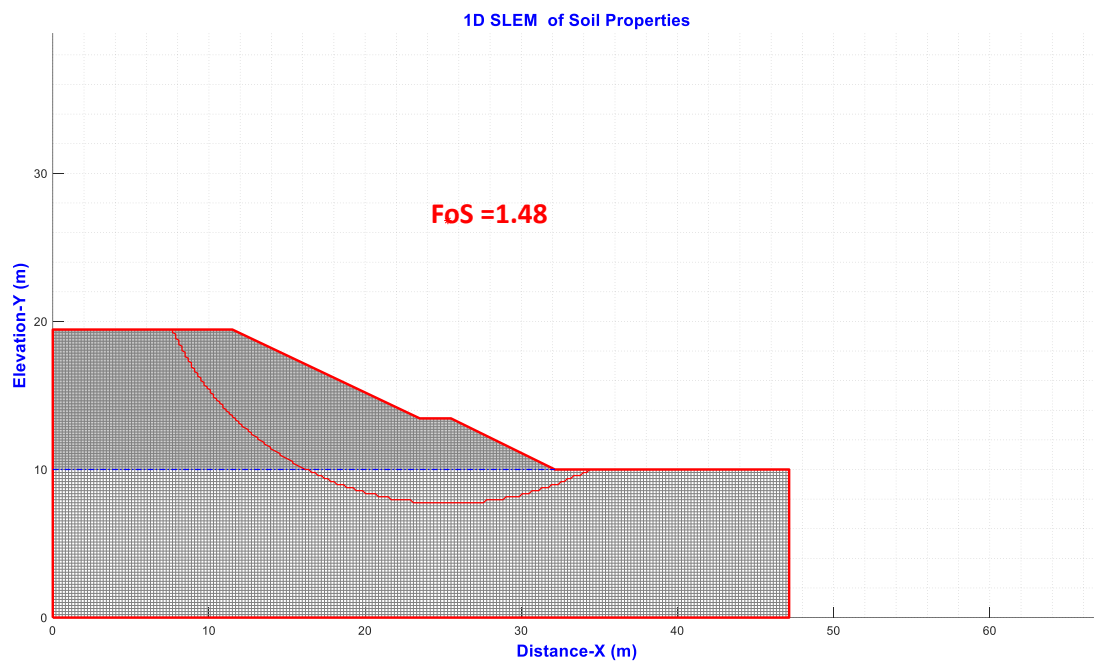
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(4a)



(4b)

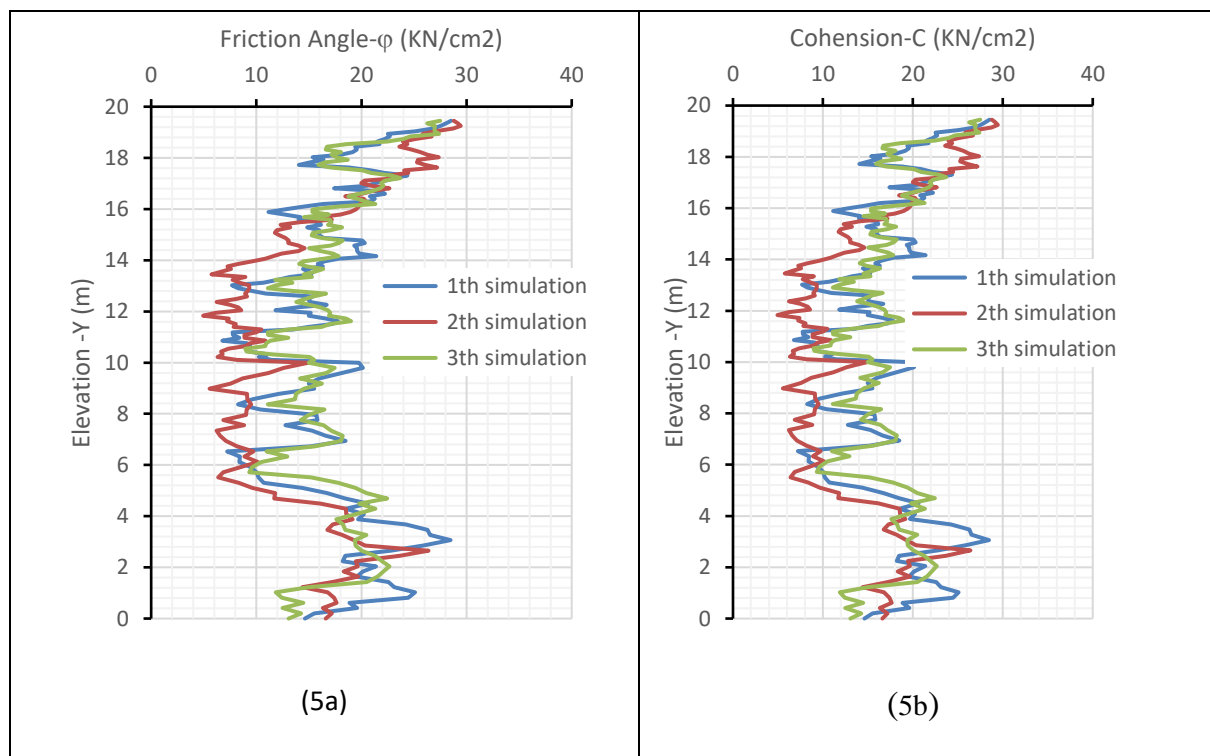
141 Fig 4. Comparison of results with consideration (proposed model) and without
 142 considering the change in soil shear resistance parameter with depth (GeoStudio 2020)

143 After using the Karhunen-Loeve series to simulate the change of parameters (γ , c , ϕ), we have
 144 the coefficient FoS is 1.48. This value is smaller than results (1.68) run using software

145 GeoStudio 2020. These results showed in Fig 4a and Fig 4b). The results compared with the
146 case that do not consider this change show that there is a difference in the sliding arc and the
147 stability coefficient. This is explained because in the area of sliding arc, there are many values
148 of density larger than the average value, while in the area of sliding arc, the load capacity is
149 smaller than the average value (blue area in Fig 3) which make decrease in the load capacity of
150 the slope.

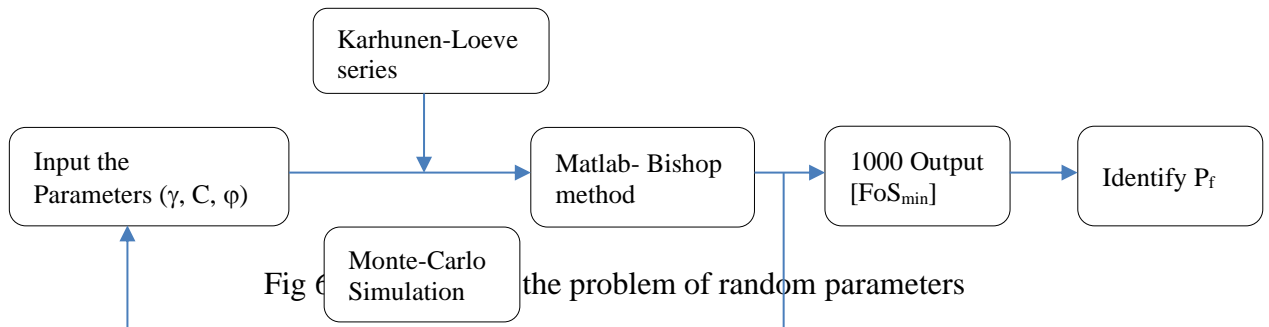
151 **Reliability analysis of slope stability considering the random change by depth of shear**
152 **resistance**

153 To analyze the reliability of the problem, using Karhunen - Loeve series to simulate the
154 change with depth of the soil shear resistance. After using the Karhunen - Loeve series, we get
155 the following results as Fig 5a and Fig 5b. The results of simulation times for shear resistance
156 parameters are different. From this result, it is necessary to perform simulations for accurate
157 analysis results, according to previous studies, the number of simulations from 1000 to 10000
158 will give reliable results.



159 Fig 5. Simulation of change c , ϕ of soil with depth

160 After simulation, the process of determining the stability coefficient (FOS) performed.
 161 However, because the properties of the soil change randomly (with constraints), for analysis
 162 and evaluation, it is necessary to use Monte - Carlo simulation to predict all possible cases, in
 163 the article, we use 1000 times of Monte - Carlo simulation to change the quantity $\xi_i(\theta)$ in the
 164 Karhunen - Loeve simulation and the result is 1000 FoS values. The analysis process shows in
 165 Fig 6.



170
 171 To determine the reliability, from the results of 1000 FOS values, the failure probability
 172 value P_f is determined as in the following formula:

$$173 \quad P_f = P[FOS \leq 1,4] = \int_{-\infty}^{1,4} \frac{1}{\sigma_{FOS} \times \sqrt{2\pi}} \times e^{-\frac{(FOS - \mu_{FOS})^2}{2\sigma_{FOS}^2}} dx$$

174 In which: mean μ_{FOS} and standard deviation σ_{FOS} are determined from 1000 analyzed
 175 FOS values based on the formula:

$$176 \quad \mu_x = \frac{1}{n} \sum_{i=1}^n x_i$$

$$177 \quad \sigma_x = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \mu_x)^2}$$

178 The steps to analyze the reliability in the above problem are as follows:

179 Step 1: Input the parameter values from geological data.

180 Step 2: Use the Karhunen - Loeve series to simulate a random field X that varies with the
 181 depth of the roadbed with the following characteristics: mean (μ), standard deviation (σ) and
 182 correlation characteristic (b).

183 Step 3: From the combination of variables $\{c, \varphi, \gamma\}$ randomly generated in step 2, using
 184 the Bishop problem and Matlab software to calculate the factor of safety FOS_{min}

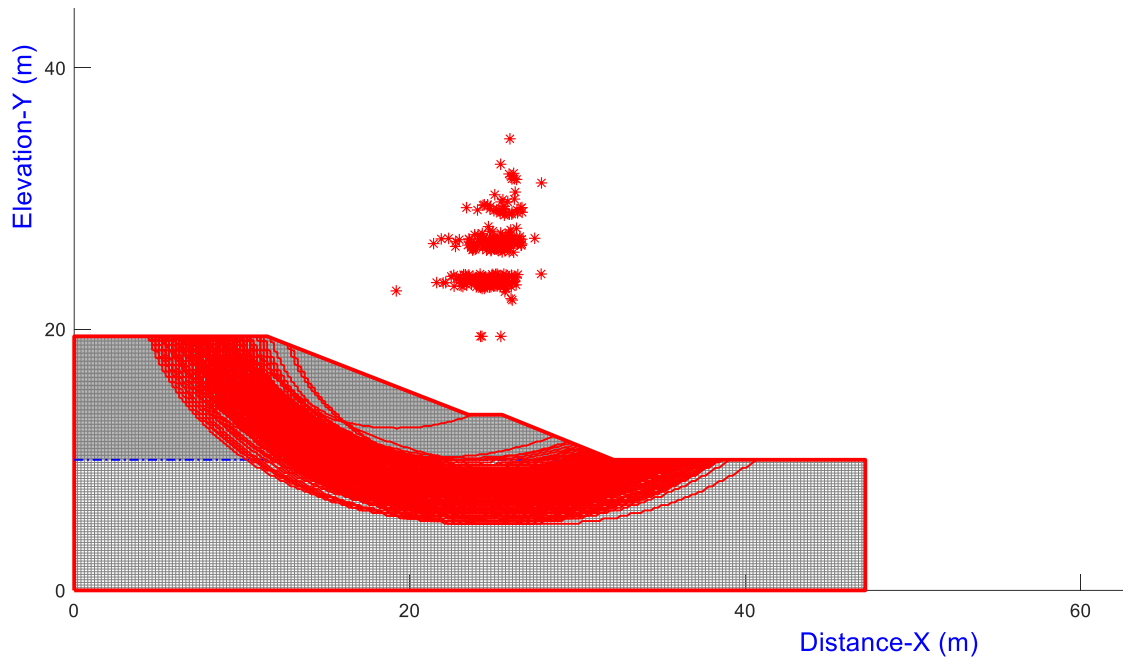
185 Step 4: Use the Monte - Carlo simulation to repeat steps 2 and 3 with the number of
 186 simulations $n=1000$, the result will be a set of 1000 FOS_{min} values.

187 Step 5: Apply the theory of reliability and failure probability to evaluate the influence of
 188 shear resistance on slope stability.

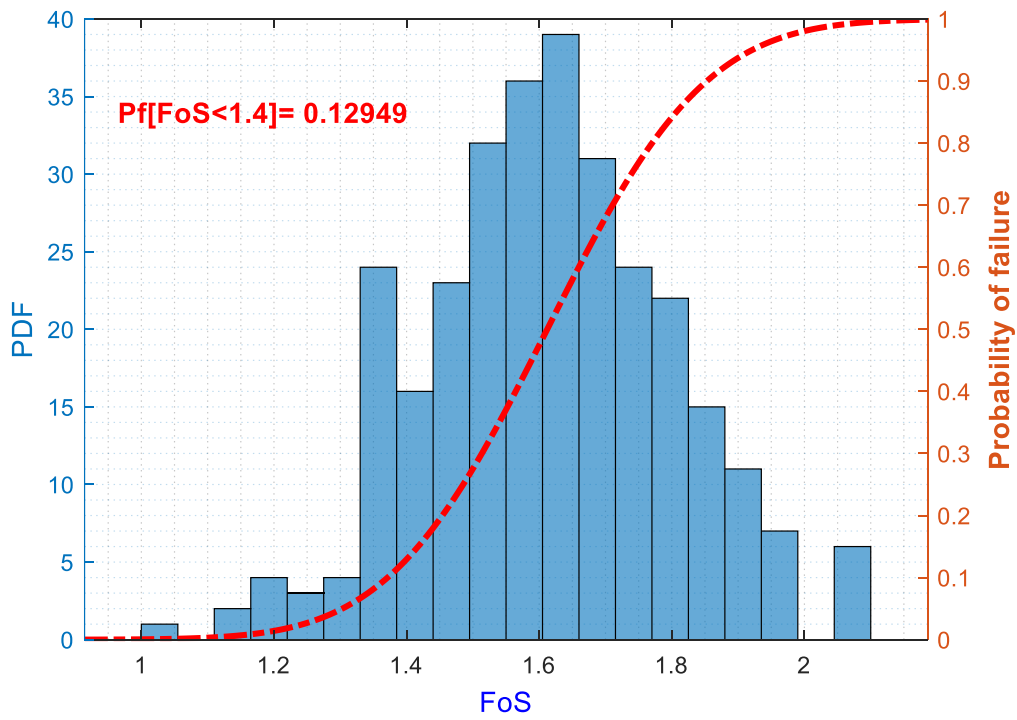
189 Table 2. Data needed to simulate random parameters

Parameters	μ	$\sigma=0.1\mu$	$\sigma=0.2\mu$	$\sigma=0.3\mu$	μ	$\sigma=0.1\mu$	$\sigma=0.2\mu$	$\sigma=0.3\mu$
Density γ (kN/m ³)	19.58	1.958	3.916	5.874	19.54	1.954	3.908	5.862
Cohesion c (kPa)	18.4	1.84	3.68	5.52	18.4	1.84	3.68	5.52
Internal friction angle φ (độ)	18.4	1.84	3.68	5.52	18.4	1.84	3.68	5.52

190
 191 With data in table 2, after performing the five above steps, we get the result as shown in
 192 Fig 7a and Fig 7b. From the analysis results, we find that, when not considering random change,
 193 value $FoS=1.68 > 1.4$, slope ensures stability, however, when considering random change of
 194 basic properties, we found up to 12.95% ability of failure of the slope. This shows that when
 195 considering the random change with the depth, the slope is unstable, and the distribution is
 196 stable when considering the random change with the depth of the shear resistance parameter (c ,
 197 φ) of the soil is need.



(7a)



(7b)

200 **Conclusions**

201 The article has built a model to analyze the slope stability when considering the change
202 of shear resistance with depth based on the Bishop's equation using Karhunen – Loeve series.
203 With a proposed numerical model, the results compared with the case that do not consider this
204 change (results run using software GeoStudio 2020) show that there is a difference in the sliding
205 arc and the stability coefficient. Specifically, the results show that the value of the stability
206 coefficient of the proposed model is 1.48, which is smaller than 1.68 of the models run with
207 GeoStudio 2020 software. By using Monte - Carlo simulations combined with Karhunen -
208 Loeve series, when considering the random change in depth of soil shear resistance, the forecast
209 results are different from those only when considering the static problem and random quantity
210 problems, the probability of failure will increase significantly (12.95%). The research results
211 open a new direction in the calculation and quality control of slope stability.

212

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216

217 **Conflict of interest**

218 All authors certify that they have participated sufficiently in the work to take public
219 responsibility for the content, including participation in the concept, design, analysis, writing,
220 or revision of the manuscript. Furthermore, all authors also certify that this material or similar
221 material has not been and will not be submitted to or published in any other publication before
222 its appearance in the Indian Geotechnical Journal.

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