

## POST-BUCKLING BEHAVIOUR AND DELAMINATION GROWTH IN DEFECTED VARIABLE ANGLE TOW COMPOSITE LAMINATES

DANIELE GAETANO<sup>\*</sup>, FABRIZIO GRECO<sup>\*</sup>, LORENZO LEONETTI<sup>\*</sup>, ANDREA PRANNO<sup>\*</sup> AND GIOVANNI ZUCCO<sup>†</sup>

<sup>\*</sup> Department of Civil Engineering  
University of Calabria  
Via P. Bucci Cubo 39B, 87036 Rende, Italy  
e-mail: [daniele.gaetano@unical.it](mailto:daniele.gaetano@unical.it); [fabrizio.greco@unical.it](mailto:fabrizio.greco@unical.it); [lorenzo.leonetti@unical.it](mailto:lorenzo.leonetti@unical.it);  
[andrea.pranno@unical.it](mailto:andrea.pranno@unical.it)

<sup>†</sup> Bernal Institute and School of Engineering  
University of Limerick  
VP94 T9PX Limerick, Ireland  
email: [giovanni.zucco@ul.ie](mailto:giovanni.zucco@ul.ie)

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**Summary.** *Due to their specific strength and stiffness properties, composite materials are largely used in lightweight structural applications in aerospace, automotive and mechanical engineering. Understanding how these materials fail under service loads is a challenging aspect of designing advanced composite structures. In fact, the failure of composite laminated structures is often governed by complex interactions of multiple interlaminar failure and damage mechanisms. Among them, delamination is one of the damage modes requiring large attention due to the low interlaminar resistance between the different layers comprised in a composite laminate. In addition, this phenomenon may be triggered by defects introduced in the construction phase or by the presence of connections leading to stress concentrations. When coupled with buckling phenomena, delamination inevitably decreases the load-carrying capacity of lightweight composite structures. Variable Angle Tow (VAT) laminates have been proven to improve the buckling and post-buckling response of those structures significantly. However, little is known about the geometrically nonlinear behaviour of VAT composite laminates with delaminations.*

*This work applies the cohesive finite element method to model delamination growth in VAT composite laminates containing initial defects under compressive loading conditions. Numerical simulations investigate the effects of the fibre angle variation on the geometrically nonlinear static response of VAT composite laminates compared to that of their classical straight fibre counterparts.*

## 1 INTRODUCTION

The study of damage phenomena in composite materials is still an important topic due to the use of composite laminates in a wide range of engineering applications (i.e. aerospace, civil, and mechanical engineering). Delamination is a significant issue in laminated composites because the layers have low interlaminar resistance, leading to potential separation under stress. This weak point can compromise the material's structural integrity, making it important to address delamination risks during design, manufacturing, and use. All of these phenomena can also occur when composite laminated plates are used as strengthened systems [1,2].

Several models have been proposed in the past decades to study the behaviour of composite laminates subject to delamination phenomena. Initially, analytical models using one-dimensional assumptions and a perturbative approach were proposed, such as those proposed by Chai et al. [3] and Smitses et al. [4], who analysed the buckling of composite laminate beams and plates with delamination along the thickness for different sizes and positions of the delamination. However, in [1] and [2], the effects of shear deformability were not considered, which, instead, play an important role when dealing with thick plates [5,6]. To this end, the work of Smitses et al. [4] was extended by Kardomateas and Schmueser [7] who investigated how shear deformability affects composites' buckling and post-buckling response with through-the-width delamination. Alongside the one-dimensional model based on classical beam theories, other numerical and analytical models based on plate theory have been proposed over the years [8–10]. In addition, a study about delamination buckling and growth behaviour in composite laminate plates using the global energy approach and the local J-integral approach can be found in [11]. The common strategy in some of those models was to consider the composite laminate as made of two different plates, one in the delaminated area and another in the intact region. All these different types of modelling, however, highlight the importance of a proper strategy to model the delamination in composite laminates in order to better understand the interaction mechanisms between the individual layers, when delamination evolves during the loading phase. Finding a correct approach to model the delamination is still of fundamental issue nowadays [12,13], in order to prevent one of the most common and risky failure phenomena in composites, especially when modern and new materials are employed. Indeed, advances in production technologies, especially in the manufacturing processes, allow new materials and new shapes to be obtained.

Among these novel procedures, most recently, new technologies in the two-steering process have made it possible to manufacture composite materials with variable stiffness. They are characterized by a variation in the fibre angle along the length of the composite panel [14]. Several experimental and numerical works have shown new approaches and methodologies for the manufacturing process of these kinds of materials [15–18], also showing their benefits in different applications [19,20]. In particular, it has been proved that the load-carrying capacity of a lightweight composite could be improved using variable angle tow (VAT) composite laminates [21,22].

Among all these works, much literature studies the Variable Angle Tow (VAT) composite laminates buckling and post-buckling response assuming an elastic material behaviour [23]. However, to the author's best knowledge, no work exists studying their geometrically nonlinear structural response in the presence of delamination. To this end, the aim of the present work is to investigate how delaminations affect the post-buckling behaviour of VAT composite

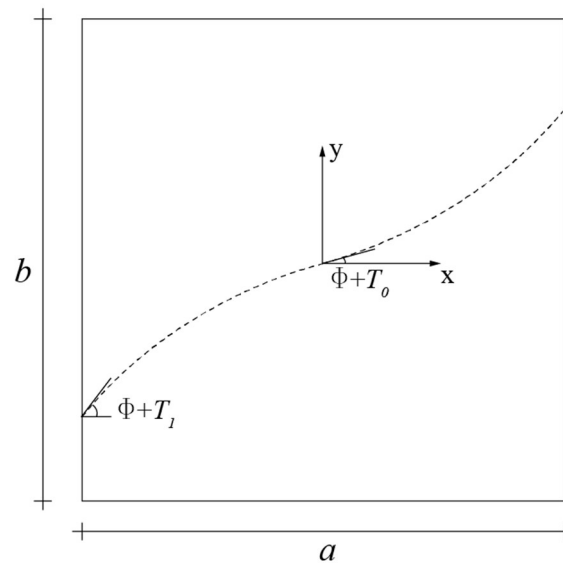
laminates and how they behave, considering the growth of delamination in the post-buckling regime. The numerical results have been provided for both through-the-width and embedded delamination for composite laminates subjected to different boundary conditions.

## 2 POST-BUCKLING BEHAVIOUR AND DELAMINATION GROWTH IN COMPOSITE LAMINATES: THEORETICAL BACKGROUND

Figure 1 shows the fibre angle variation over one layer of a plate. In particular, the stiffness properties of the individual layer vary over its domain by changing the orientation of the fibre angle with respect to the panel coordinates  $x$  and  $y$  as described in Eq. 1. Several works [14,24–26] showed that such variation of the fibre angle can improve the overall behaviour of lightweight VAT composite laminate structures, especially enhancing their performance in the post-buckling regime and increasing the linear buckling load. In particular, a linear fibre angle variation as in [11] is used:

$$\vartheta(x) = \phi + \frac{2(T_1 - T_0)}{a}|x| + T_0 \quad (1)$$

in which,  $\phi + T_0$  and  $\phi + T_1$  represent the fibre angle at the centre and the end of the panel, respectively (see Fig. 1).



**Figure 1:** Linear fibre angle variation over a layer of a VAT composite laminate panel.

Therefore, differently from their straight-fibre composite laminate counterparts, the coefficients of the constitutive equations are not constant anymore, but they are functions of the plate coordinate and are evaluated as follows:

$$A_{ij} = \sum_{k=1}^K Q_{ij}^k (z_{k+1} - z_k), B_{ij} = \frac{1}{2} \sum_{k=1}^K Q_{ij}^k (z_{k+1}^2 - z_k^2), D_{ij} = \frac{1}{3} \sum_{k=1}^K Q_{ij}^k (z_{k+1}^3 - z_k^3), \quad (2)$$

in which  $k$  and  $z_k$  are the number of plies and the location of the single ply with reference to the mid-plane, respectively. The terms  $Q_{ij}^k$  represent the reduced transformation stiffness of the single ply, and as introduced previously, they are functions of the plate coordinates and can be evaluated according to the classical laminated plate theory [27]. Based on the previous assumptions, a numerical tool has been developed into the finite element software COMSOL Multiphysics to generate composite laminates with a linear variation of the fibre angle along the panel length. A layerwise laminate theory has been employed in the present model, in order to account for the effects of thickness and make the proposed approach as general as possible [5,28,29].

The delamination growth is considered via a cohesive zone model [30–32] based on a damage propagation criterion of the Benzeggagh-Kenane type [33]. Moreover, a contact algorithm is needed to prevent penetration between the upper and sub-laminates during the analysis.

The unstable growth of the delamination could lead to snapback phenomena in the softening branch of the global structural response. Classical path-following approaches, such as the displacement control scheme, are unable to trace the equilibrium path in the presence of snapbacks. To overcome this issue, the hybrid continuation approach proposed in [34] has been adopted in the present work. Such an approach incorporates a new path-following scheme for unstable equilibrium branches, after introducing the following average crack separation as continuation parameter:

$$\gamma = \frac{1}{|\Gamma_d|} \int_{\Gamma_d} \left( \sqrt{\langle \delta_I \rangle^2 + \delta_{II}^2 + \delta_{III}^2} \right) \quad (3)$$

where  $\delta_I$ ,  $\delta_{II}$  and  $\delta_{III}$  are the mode I, mode II and mode III displacement jumps, respectively, while  $\Gamma_d$  indicates the embedded cohesive interfaces. In other words, the analysis is driven by the average value of the current mixed-mode displacement jumps over all the embedded cohesive interfaces  $\Gamma_d$ , being computed at each analysis step starting from the nodal displacements. In practice, the adopted hybrid scheme acts in the following manner. At the beginning, the analysis is performed by using a classical displacement control scheme. When the delamination propagation becomes unstable, the solver switches to the average crack separation scheme. This crack separation (or displacement jump) parameter is closely related to the energy dissipated by the cohesive interfaces when delamination occurs. The adopted control scheme is based on the assumption that such a dissipated energy has monotonically increasing values during delamination propagation. From a computational point of view, the switching criterion between the two adopted control schemes relies on the number of Newton-Raphson iterations. Specifically, once a pre-defined threshold value of such iterations has been reached, suggesting that the loss of stability is close to occurring, the solver automatically switches from the displacement control to the average crack separation control. As expected, such an approach allows the unstable branch of the equilibrium path to be followed.

### 3 NUMERICAL RESULTS

This section presents the numerical results obtained using the model described in the previous Section. First, the buckling-induced delamination is studied for a composite plate with

a through-the-width delamination to validate the proposed hybrid path following continuation scheme. Subsequently, the post-buckling behaviour of VAT composite panels with embedded circular delamination has been studied, and a comparison between the numerical results for a straight-fibre composite plate and its VAT counterpart is presented.

### 3.1 Post-buckling behaviour of composite laminates with through-the-width delamination

A straight-fibre composite laminate plate with through-the-width delamination is considered. The plate is rectangular, with dimensions  $a = 50.8$  mm and  $b = 25$  mm (see Fig. 2). This laminate has 20 unidirectional layers, each with a thickness  $t = 0.1295$  mm. The plate is clamped at the loading edges and free on the other two. An in-plane uniaxial compression along the  $x$ -axis is considered. The delamination has a size  $c = 19.05$  mm and a depth  $h_1 = 0.518$  mm, measured from the top surface of the plate, equal to 4 layers. The lamina properties are:  $E_{11} = 139.3$  GPa,  $E_{22} = 9.72$  GPa,  $G_{12} = 5.58$  GPa, and  $\nu_{12} = 0.29$ . A cohesive model, characterized by a mixed-mode traction-separation law, has been employed in the numerical simulation using the parameters shown in Table 1. Due to the modelling employed for the plate, the upper and lower parts of the composite laminates may exhibit different transverse displacements and, therefore, may come into contact during the analysis. A penalty contact algorithm has been implemented into the finite element model to prevent penetration between the elements, with a penalty stiffness value 100 times higher than the one used for the cohesive interface. In order to capture the post-buckling response, an initial imperfection is needed proportional to the first mode shape of the plate. It has been applied to the composite with an amplitude factor equal to 1% of the panel height.

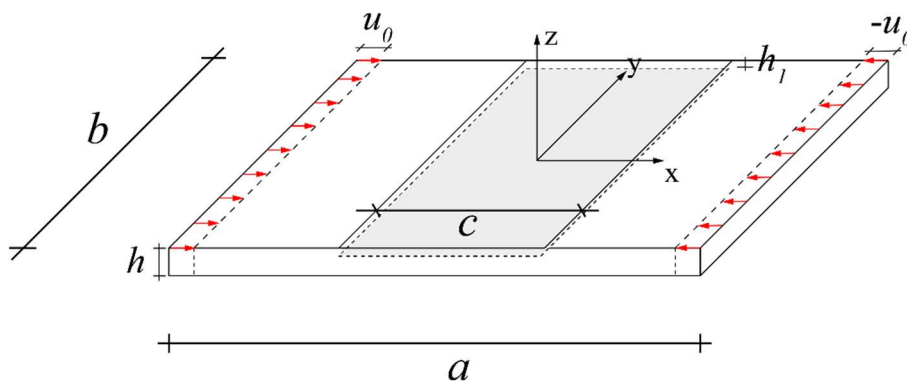
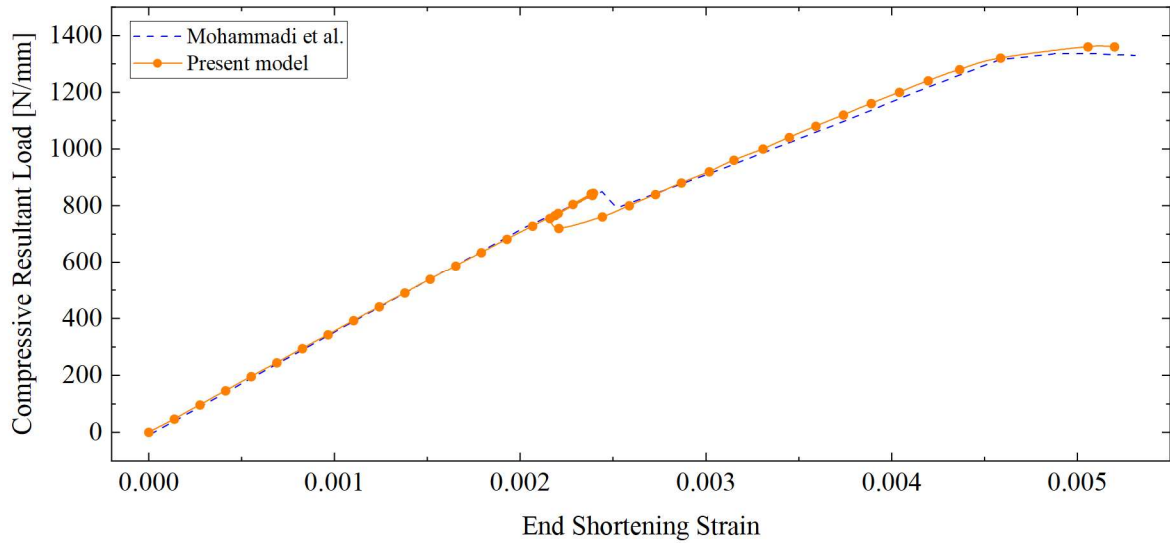


Figure 2: Geometry and boundary conditions of a composite plate with a through-the-width delamination.

Table 1: Cohesive parameters.

$G_{IC}$ [N / mm]	$G_{IIC}$ [N / mm]	$\sigma_n^0$ [MPa]	$\tau^0$ [MPa]
0.0876	0.3152	44.54	106.9

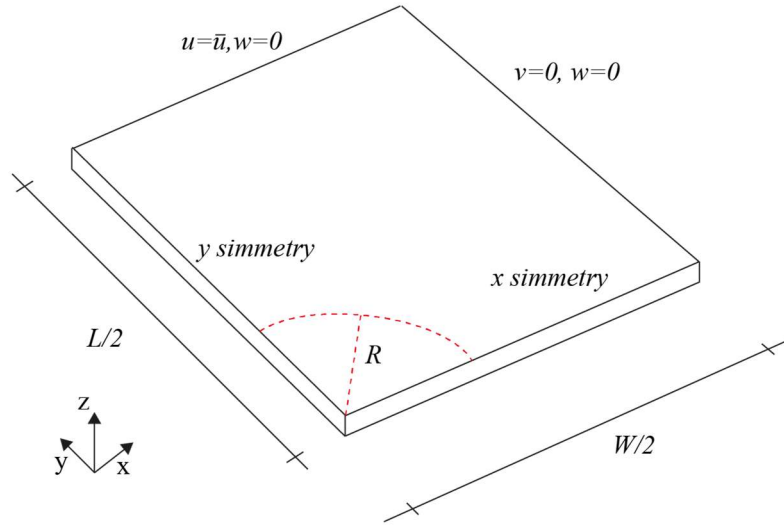
Figure 3 shows the results of the post-buckling behaviour of the plate. Such a test has been used to validate the hybrid path-following scheme because it is characterized by an unstable delamination growth. The average value of the displacement jump along the cohesive interfaces was monitored during the first part of the analysis and was driven by a classical displacement control procedure. The solver switches to the proposed hybrid scheme when the delamination becomes unstable. Figure 3 shows that the method can reproduce the unstable growth of the delamination, capturing the pre- and post-onset growth stiffnesses and the global buckling mode. The obtained results agree with those in [35], highlighting the reliability of the proposed approach.



**Figure 3:** Axial load versus end axial strain.

### 3.1 Post-buckling behaviour and delamination growth of defected VAT composite laminate panels

The post-buckling behaviour of a defected VAT composite laminate panel, including the growth of the delamination, is studied. The in-plane dimensions of the panel are  $L = 150$  mm and  $W = 100$  mm. The laminate has 24 layers, and its thickness is  $H = 3$  mm. The panel has an embedded circular delamination with a radius  $R = 25$  mm. Due to the symmetry of the problem, only a quarter of the panel has been modelled. The geometry, as well as the boundary conditions of the panel, are shown in Fig. 4. The mechanical properties of the single lamina are:  $E_{11} = 128$  GPa,  $E_{22} = 8.76$  GPa,  $G_{12} = 4.27$  GPa and  $\nu_{12} = 0.28$ , whereas the cohesive parameters are shown in Table 2. Even in this case, an out-of-plane displacement proportional to the first mode shape, evaluate using a linear buckling analysis, with an amplitude factor of 0.01 times the height of the composite, has been applied as initial imperfection for the non-linear analysis.



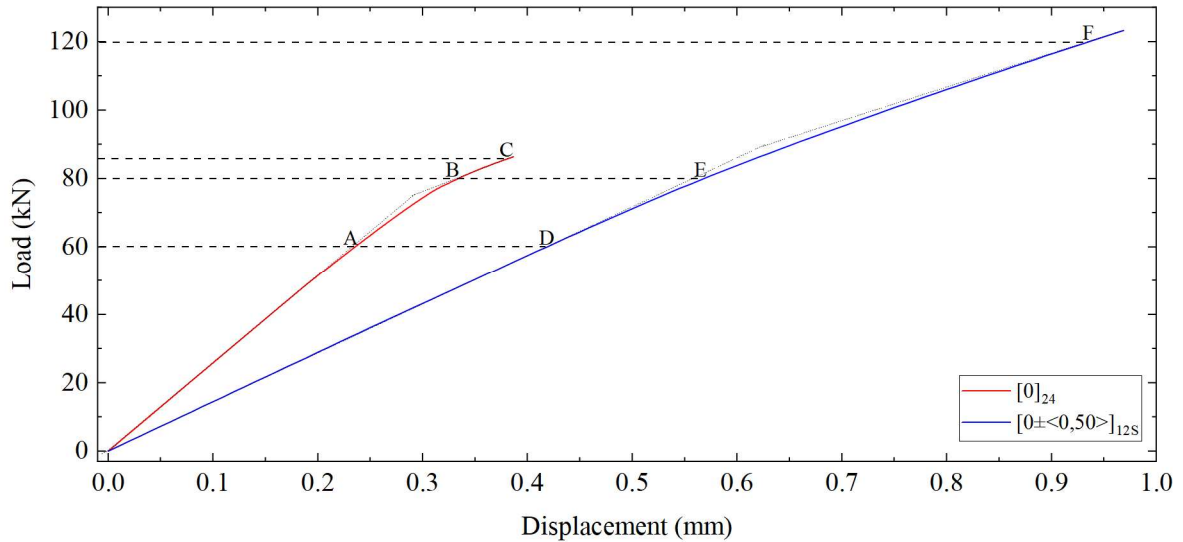
**Figure 4:** Geometry and boundary conditions of a composite plate with an embedded circular delamination.

**Table 2:** Cohesive parameters

$G_{IC}$ [N / mm]	$G_{IIC}$ [N / mm]	$\sigma_n^0$ [MPa]	$\tau^0$ [MPa]
0.3	0.9	30	60

The structural response of a constant stiffness plate with stacking sequence  $[0]_{24}$  has been compared with that of a VAT plate with  $[0\pm <0,50>]_{12S}$  layup and reported in Figure 5. For the VAT plate stacking sequence, 0 represents the angle  $\phi$ , while  $<0,50>$  represent the angles  $T_0$  and  $T_1$ , respectively (see Eq. 1); the  $\pm$ -sign, instead, indicates the presence of two layers having fibre variation angles opposite to each other [36].

Regarding the numerical results, the VAT plate has a reduced initial stiffness due to the variation of the fibre angles along its length, but its post-buckling response is improved with respect to its constant stiffness counterpart as shown in Fig. 5. Furthermore, the VAT plate exhibits a stiffer response in post-buckling regime than that with straight fibres also in the presence of embedded delamination. This behaviour is in line with what has been observed for cases without delamination, thus showing how variable stiffness composites have beneficial effects even when buckling-induced delamination is present.

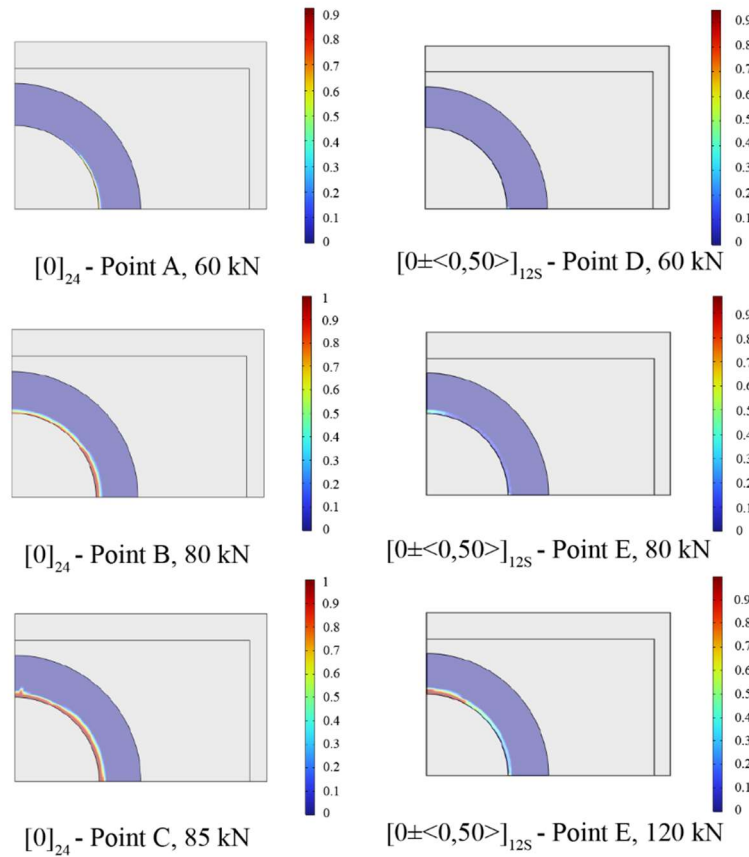


**Figure 5:** Load-displacement curves for a composite plate with embedded circular delamination: comparison between straight-fibre and VAT composite plates.

The behaviours of those plates at different load levels were investigated in more detail. In particular, a study on the evolution of the damage variable associated with the introduced cohesive model was conducted (see Fig. 6). Three different load levels for each curve were selected, indicated with points A-F, as shown in Fig. 5. Specifically, two equal levels between the two curves have been chosen, associated with the 60 kN and 80 kN load, and a final level corresponding to the maximum load at which the analysis of the straight fibre plate was terminated manually to avoid unnecessary computations. Indeed, it is worth noting that, for the purpose of the present results designed to make a comparison between the two types of composites, the numerical analysis has not been pushed all the way to collapse, but has been stopped in the early stages of crack propagation since the purpose of the present work is to analyse the effects of VAT on the onset of fracture related to the growth of delamination in the post-buckling regime.

As shown in Fig. 6, the two composites have different behaviour in the pre- and post-buckling regimes regarding the onset and growth of damage. In particular, for a load level equal to 60 kN, the unidirectional composite shows an initial growth of the delamination, while for the VAT plate, the onset of damage is barely detected. Furthermore, for a load value of 80 kN, the straight fibre plate exhibits the classical circular crown shape. In contrast, the damage starts in a different location and shape for the variable stiffness plate, for which the growth is still in its early stage of propagation. More precisely, at that load level, the damage in the VAT plate is still barely detected, and the damage variable has not yet reached its upper limit value of 1, as instead observed for the straight-fibre counterparts. The same behaviour was observed for the final load level, where the damage has a uniform contour for the unidirectional composites and a non-uniform one, with a preferential damage area, for the VAT plate. Such differences are related to the different stiffness provided by the variable fibre angles, which creates a distribution of the damage that ultimately leads to a delay in the onset of the crack.





**Figure 6:** Damage evolution for the unidirectional and the variable angle tow composite laminates.

#### 4 CONCLUSIONS

This work presented the post-buckling behaviour and the growth of delamination in defected variable angle tow composite laminates through a numerical approach that uses the layer-wise theory to model the composite laminate plate, a cohesive zone model for describing the delamination growth as the load level increases, and a hybrid path-following continuation strategy to capture the unstable growth of delamination. The main goal of the present work is to study the post-buckling behaviour of VAT composites in the presence of delamination growth and how their performance compares to that of their traditional straight-fibre counterparts.

To validate the proposed approach, the post-buckling behaviour of a straight fibre unidirectional composite plate with through-the-width delamination was initially studied, and its results were compared with those found in the literature. In particular, the comparison with numerical and experimental results available in the literature has shown the capability of the proposed hybrid-path following scheme to reproduce the unstable growth of the delamination, accurately capturing the pre- and post-onset of growth stiffnesses and the final global buckling mode. Subsequently, the post-buckling behaviour of a variable angle tow composite plate with embedded circular delamination was studied. Numerical results have shown the beneficial effects of the stiffness variation in the VAT plate on damage initiation and propagation. Future

work will investigate the effects of the delamination size, shape and depth, and also include an advanced contact algorithm to evaluate the effects of the local and global contact on the structural behaviour of composite laminate plates with constant and variable stiffnesses. Moreover, an improved strategy will be developed to follow the behavior of the structural system after the onset of delamination and until the global failure of the plate.

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