

## PERMEABILITY AND COMPRESSIBILITY OF SLURRIES FROM SEEPAGE-INDUCED CONSOLIDATION

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**ABSTRACT:** A one-dimensional mathematical model based on finite-strain theory is developed to solve the problem of seepage-induced consolidation in sedimented slurries or very soft clays. The direct solution employs known or assumed material property relationships to determine the final thickness of a soft sediment subjected to a constant piezometric head. It is useful for predicting the capacity of a disposal area and the time-dependent improvement in material properties. Alternatively, the inverse solution utilizes final settlement and steady-state flow data from laboratory or field tests to deduce permeability and compressibility relationships for soft sediments. This approach is especially helpful in the case of permeability determinations because it avoids some of the major problems associated with permeability testing of such materials. The resulting model shows that the coefficient of permeability influences both the time to reach the steady-state condition and the nature of the steady-state condition itself. An illustrative example is presented wherein data from a series of tests on a kaolinite slurry are used to establish material property relationships that are then used to predict the response of other tests on the same soil under different conditions.

### INTRODUCTION

The disposal of waste slurries from mining operations and various industrial processes presents an ever-increasing number of challenging problems to the geotechnical engineering profession. The primary issues center around predicting the time-dependent capacity of a given disposal area and the time-rate of improvement of material properties for reclamation purposes. Intrinsic to both issues is the process of consolidation and the associated material changes (e.g., decrease in volume and increase in shear strength) that result. Presented herein is a one-dimensional mathematical model that may be solved: (1) Directly to obtain the final or steady-state thickness of a soft sediment subjected to a constant piezometric head; or (2) in an inverse fashion to deduce permeability and compressibility relationships for a soft sediment. This latter capability is extremely useful because it offers a relatively simple method for determining material relationships that have previously been rather elusive.

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## PHYSICAL PERSPECTIVE

Briefly stated, the physical problem involves pumping a waste slurry (solids content is usually on the order of 10–20%) into a diked containment area, where the solids are allowed to settle, and the unevaporated water is either decanted or allowed to seep downward into an underlying drainage blanket. The process of sedimentation prevails at a given point in the deposit until the solid particles come into contact with each other, at which time the process of consolidation is initiated and governs the subsequent behavior of the deposit. It is clear, of course, that these processes (sedimentation and consolidation) are in different stages of development throughout the deposit at any point in time, and the deposition is therefore heterogeneous in the vertical direction. However, for mathematical tractability, it is often assumed that there is some point in time when sedimentation is complete everywhere in the deposit and self-weight and/or seepage-induced consolidation has not yet started; this assumption will be made in this study.

Due to the loose nature of the sedimented deposit, the ensuing vertical settlements are usually extremely large and beyond the range that can be handled by classical small-strain consolidation theories; accordingly, finite-deformation models must be developed. The next section contains a brief background of finite-strain theory and a derivation of the applicable steady-state seepage-induced consolidation equations, together with pertinent boundary conditions (constant piezometric head and fully drained case). This is followed by a statement of the direct problem and a description of a numerical algorithm that is used to compute the solution to the governing field equation, given the initial and boundary conditions and the material property relationships. Next is an explanation of the inverse problem, whereby the solution to the direct problem and appropriate experimental data (final height of deposit and seepage velocity) are utilized to deduce the constants in the compressibility and permeability relationships.

## THEORETICAL CONSIDERATIONS

When the results of seepage-induced consolidation are analyzed, the complete time-dependent problem is usually solved for every boundary condition. This requires the numerical solution of several nonlinear partial differential equations which takes a considerable amount of computer time and introduces a number of truncation errors. It is suggested here that in many cases it is sufficient to study only the steady-state consolidation conditions. The theory presented here is patterned after the early work by Gibson et al. (1967), adaptations and modifications of which have been described by Monte and Krizek (1976), Somogyi (1979), Schiffman (1980), Gibson et al. (1981), Krizek and Somogyi (1984), and others.

### One-Dimensional Initial Boundary Value Problem

Two standard formulations for the consolidation equation are employed in nonlinear finite-strain theory, depending upon whether the void ratio or the pore pressure is selected as the unknown in the problem. The approach adopted here is based on the large-strain formulation originally developed by Gibson et al. (1967), and excess (i.e., greater than hydrostatic)

pore-water pressure is used as the dependent variable. Appropriate manipulation of the basic equations that need to be satisfied (namely, continuity of particle flow, continuity of fluid flow, equilibrium, and Darcy's law) yield the following expression describing the one-dimensional consolidation process:

$$\frac{\partial e}{\partial t} + \frac{\partial}{\partial z} \frac{-k}{\gamma_w(1+e)} \frac{\partial u}{\partial z} = 0, \quad 0 \leq z \leq z_0 \dots \dots \dots (1)$$

in which the material coordinate (height of solids)  $z$  is given by

$$z(a) = \int_0^a \frac{d\bar{a}}{1+e_0(\bar{a})} \dots \dots \dots (2)$$

where  $a$  = the initial vertical (Lagrangian) coordinate;  $e$  = the void ratio;  $e_0$  = the initial void ratio;  $t$  = time;  $k$  = the coefficient of permeability;  $\gamma_w$  = the unit weight of water; and  $u$  = the excess pore-water pressure; the surface of the solids corresponds to  $z = 0$ .

The solution of Eq. 1 requires the specification of appropriate boundary conditions and relationships between the void ratio and the effective stress and between the coefficient of permeability and the void ratio. Several specific types of equations have been proposed for these two relationships [see review by Krizek and Somogyi (1984)], but the method presented here is independent of their explicit form. For illustrative purposes, the following power relationships employed by Somogyi (1979) are used:

$$e = A\bar{\sigma}^B \dots \dots \dots (3)$$

$$k = Ce^D \dots \dots \dots (4)$$

where  $\bar{\sigma}$  = the effective stress; and  $A$ ,  $B$ ,  $C$ , and  $D$  = empirical constants, typical values for which have been suggested by Carrier et al. (1983). It should be noted that  $B$  is negative, but usually greater than  $-0.5$ , and  $D$  is greater than 2.

If the soil deposit remains submerged, the effective stress is related to the excess pore water pressure by

$$\bar{\sigma}(z) = (G_s - 1)\gamma_w z - u(z) + \bar{\sigma}(0) \dots \dots \dots (5)$$

where  $G_s$  = the specific gravity of the solids; and  $(G_s - 1)\gamma_w z$  = the buoyant stress at any depth. Although the effective stress at the surface of the solids is usually assumed to be zero, this assumption, when utilized in Eq. 3, implies that the void ratio at the surface is infinity. To avoid this problem without changing the compressibility relationship,  $\bar{\sigma}(0)$  is chosen equal to  $(e_0/A)^{1/B}$ ; the validity of this assumption will be discussed later. Note, however, that Eq. 5 indicates an upper bound for the excess pore-water pressure, namely, the effective stress at the surface of the solids plus both the effective and buoyant stress at the depth considered.

Two problems will be studied, both of which have the following common boundary condition at the surface of the solids:

$$u(0) = 0 \dots \dots \dots (6)$$

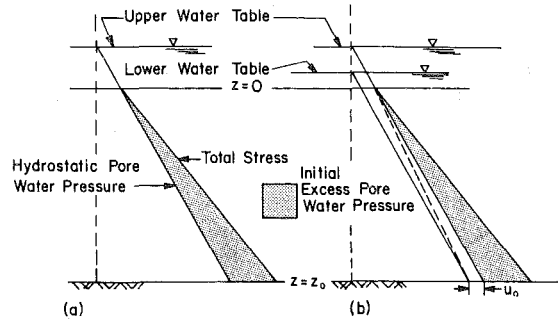


FIG. 1. Initial Conditions: (a) Fully Drained Case; (b) Piezometric Head Case

The problems are distinguished by their boundary condition at the bottom (i.e.,  $z = z_0$ , where  $z_0 =$  the depth of the deposit in material coordinates), which are respectively given by

$$u(z_0) = 0 \quad (\text{fully drained}) \quad \dots \dots \dots (7a)$$

$$u(z_0) = u_0 \quad (\text{piezometric head}) \quad \dots \dots \dots (7b)$$

where  $u_0 =$  the difference between the lower and upper water tables. Fig. 1 shows these two cases at  $t = 0$ . Thus, the two boundary value problems considered here are defined by Eqs. 1-7, with either Eq. 7a or Eq. 7b designating the condition at the lower boundary.

**Scaled Steady-State Problems**

The dimensionless variables used in this analysis are defined by

$$Z = \frac{z}{z_0} \quad \dots \dots \dots (8a)$$

$$U = \frac{u}{u_b} \quad \dots \dots \dots (8b)$$

$$F = Z - U \quad \dots \dots \dots (8c)$$

$$\mu = Au_b^B \quad \dots \dots \dots (8d)$$

where  $u_b = (\gamma_s - \gamma_w) z_0$  (i.e., the buoyant stress at the bottom of the deposit); and  $F_0 =$  the effective stress at the surface  $\bar{\sigma}(0)$  divided by  $u_b$ ; note that  $F + F_0$  is the dimensionless effective stress and that Eq. 8c comes from Eq. 5. The incorporation of these dimensionless variables into Eqs. 1-7 gives

$$T(F + F_0)^{B-1} \frac{\partial U}{\partial t} = \frac{\partial}{\partial Z} \left[ \frac{(F + F_0)^{BD}}{1 + \mu(F + F_0)^B} \right] \frac{\partial U}{\partial Z}, \quad 0 \leq Z \leq 1 \quad \dots \dots \dots (9)$$

$$U(0) = 0 \quad \dots \dots \dots (10)$$

and

$$U(1) = 0 \quad \dots \dots \dots (11a)$$

$$U(1) = U_0 \dots \dots \dots (11b)$$

where  $U_0 = u_0/u_b$  and  $T = \mu^{1-D} \tau_w z_0^2 B / C u_b$ . The steady-state problem is obtained by setting the right-hand side of Eq. 9 equal to zero and integrating with respect to  $Z$  from zero to one; the result thus obtained can be simplified to

$$\frac{U_Z}{H(F)} = c_0, \quad 0 \leq Z \leq 1 \dots \dots \dots (12)$$

where

$$H(F) = \frac{[1 + \mu(F + F_0)^B]}{(F + F_0)^{BD}} \dots \dots \dots (13)$$

$U_Z = \partial U / \partial Z$ , and  $c_0$  = an unknown constant of integration. Based on the limits for  $B$  and  $D$ , the limit of  $H(F)$  is zero as  $F$  goes to  $-F_0$ .

The two different problems corresponding to one or the other of Eqs. 11 will now be considered. The fully drained case (Eq. 11a) has only the trivial solution; this is seen by first combining Eqs. 8c and 12 to obtain

$$F_Z + c_0 H(F) = 1 \dots \dots \dots (14a)$$

The boundary conditions

$$F(0) = 0 \dots \dots \dots (14b)$$

and

$$F(1) = 1 \dots \dots \dots (14c)$$

are obtained by inserting Eqs. 10 and 11a into Eq. 8c. Integrating Eq. 14a yields

$$F(1) - F(0) + c_0 \int_0^1 H(F) dZ = 1 \dots \dots \dots (14d)$$

Inserting the boundary conditions given by Eq. 14b into Eq. 14d and noting that  $H(F)$  is positive between zero and one proves that  $c_0$  is zero. Referring to Eq. 12, it follows that  $U_Z$  is zero and  $U$  is a constant, which must be zero according to Eq. 11a. Even if the bottom boundary were undrained [i.e.,  $U_Z(1) = 0$ ], the same trivial solution would be found. Both of these situations correspond to self-weight consolidation (with single and double drainage), and this solution will be used to compute the final height of the deposit.

The second and most interesting case, which corresponds to the seepage problem, requires the solution of Eq. 14a with the boundary conditions

$$F(0) = 0 \dots \dots \dots (15a)$$

and

$$F(1) = 1 - U_0 \dots \dots \dots (15b)$$

which are obtained by inserting Eqs. 10 and 11b into Eq. 8c. Since Eq. 14a is a first-order differential equation and Eq. 15 gives two boundary conditions, it may seem that the existence of a solution is not guaranteed; however, the problem is not "ill-posed" because  $c_0$  is unknown. This problem is solved in the following sequence. First, Eq. 14 is solved with the first condition of Eq. 15 to formulate a new problem:

$$\phi_Z + c_0 H(\phi) = 1 \dots\dots\dots (16a)$$

$$\phi(0, c_0) = 0 \dots\dots\dots (16b)$$

For every value of  $c_0$ , a solution  $\phi$  to Eqs. 16 may be found. However, there is only one value,  $c_0^*$ , that will imply  $F(Z) = \phi(Z, c_0^*)$ , where  $F(Z)$  is the solution sought. Requiring  $\phi(Z, c_0)$  to satisfy the second part of Eq. 15 yields the following highly nonlinear equation for determining  $c_0^*$ :

$$f(c_0) \equiv \phi(1, c_0) + U_0 - 1 = 0 \dots\dots\dots (17)$$

If  $c_0^*$  is the root of this equation, then  $F(Z) = \Phi(Z, c_0^*)$  is the solution to Eq. 14a. One standard technique for finding the root of an equation such as Eq. 17 is Newton's method:

$$c_0^{n+1} = c_0^n - \frac{f(c_0^n)}{f'(c_0^n)} \dots\dots\dots (18)$$

where  $c_0^0$  = the initial estimate; and  $f'(c_0)$  = the derivative of  $f$  with respect to  $c_0$ . From Eq. 17 we get

$$\phi^* \equiv f'(c_0) = \frac{\partial \phi(1, c_0)}{\partial c_0} \dots\dots\dots (19)$$

and, by differentiating Eqs. 16 with respect to  $c_0$ , it follows that  $\phi^*$  satisfies

$$\Phi_Z^* + c_0 H'(\Phi) \Phi^* = -H(\Phi) \dots\dots\dots (20a)$$

$$\Phi^*(0, c_0) = 0 \dots\dots\dots (20b)$$

Every iteration in  $c_0$  requires the solution of Eqs. 16 and 20, but both problems can be easily handled by using the Runge-Kutta method in a simple and efficient numerical code.

### DIRECT PROBLEM

Solving the direct problem for steady-state seepage consolidation requires as input the initial height of the homogeneous slurry, the initial void ratio, the unit weights of water and solids, the compressibility and permeability constants, and the piezometric head. For Newton's method to converge, a "good" initial estimate for  $c_0$  is needed. Since it was proven that  $c = 0$  for the fully drained case, this known solution is used as a starting point, and at every new increment for the boundary condition  $U_0$ , the initial estimate for  $c_0^0$  is approximated by the solution found in the preceding increment. In the examples studied, increments of one in  $U_0$  required less than four Newton iterations to converge with  $10^{-4}$  accuracy. This incremental procedure naturally follows that employed in a typical

laboratory seepage consolidation test, wherein the sample is first consolidated under its own weight ( $U_0 = 0$ ), and then incrementally increasing water heads are applied.

After Eq. 18 converges for the desired boundary condition,  $U(Z)$ ,  $F(Z)$ , and  $c_0$  are computed. Thus, the variations with depth of excess pore-water pressure, effective stress, void ratio, and permeability are known, and the final height of the solids is computed from

$$\varepsilon_0 = \int_0^{z_0} (1 + e) dz \dots\dots\dots (21)$$

which, by substituting the compressibility relation given by Eq. 3 and the dimensionless variables defined by Eqs. 8, becomes

$$\varepsilon_0 = z_0 \int_0^1 [1 + \mu(F + F_0)^B] dZ \dots\dots\dots (22)$$

The flow rate through the deposit is related to  $c_0$  and may be deduced by combining Darcy's law

$$q = \frac{k}{\gamma_w(1 + e)} \frac{\partial u}{\partial z} \dots\dots\dots (23)$$

and the dimensionless variables given by Eqs. 8 to obtain

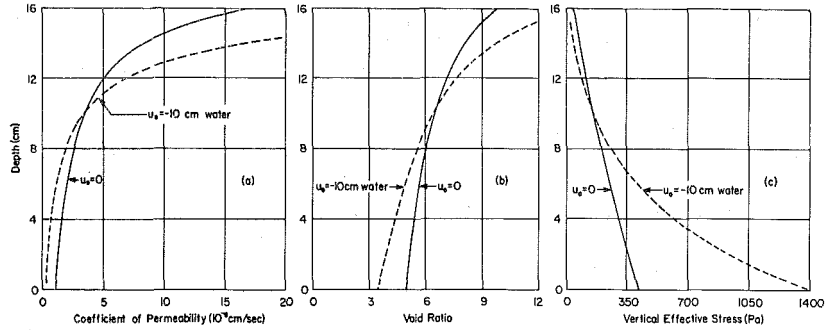
$$q = C\mu^D \frac{u_b}{\gamma_w z_0} \frac{U_Z}{H(F)} \dots\dots\dots (24)$$

Finally, with the use of Eq. 12 and the definition of  $u_b$ , Eq. 23 becomes

$$q = c_0(G_s - 1) C\mu^D \dots\dots\dots (25)$$

This method was used to analyze the behavior of a kaolinite slurry with an initial height of 31.5 cm, an initial void ratio of 12.35, and compressibility and permeability parameters as follows: compressibility  $A = 27$  ( $\bar{\sigma}$  in Pa) and  $B = -0.29$ ; permeability  $C = 2 \times 10^{-9}$  cm/s and  $D = 4$ . The results plotted in Fig. 2 show the void ratio, permeability, and effective stress as a function of Eulerian depth for the case of no induced seepage and the case of a constant piezometric head loss of  $-10$  cm of water across the sample. At the surface of the solids, neither the permeability constant nor the void ratio vary with  $u_0$  because of the effective stress boundary condition. At the bottom of the sample, however, both parameters decrease with decreasing piezometric head loss. Fig. 2(c) shows, as expected, that the variation of effective stress with depth is not linear for  $u_0$  different from zero.

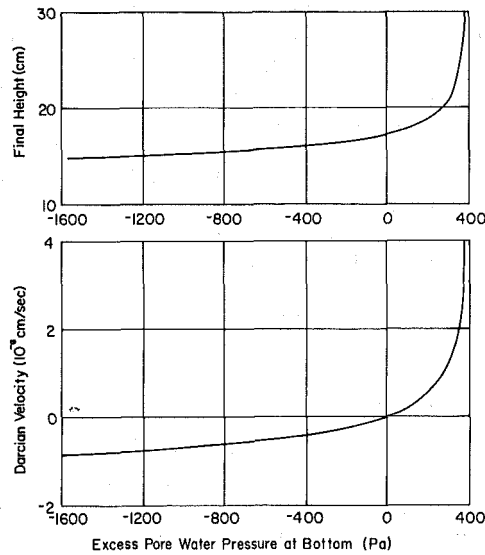
To check the validity of the hypothesis that  $\bar{\sigma}$  at the surface of the solids,  $\bar{\sigma}(0)$ , can be approximated by  $(e_0/A)^{1/B}$ ,  $\bar{\sigma}(0)$  was divided by an arbitrary factor as high as five. Variations of less than 5% were determined for the final height, flow rate, and distributions of effective stress and void ratio with depth. The influence on the coefficient of permeability was also less than 5% in most of the sample. However, due to the extremely high



**FIG. 2. Variations of Permeability, Void Ratio, and Vertical Effective Stress with Depth for Two Piezometric Heads**

variations of permeability with depth in the upper zone of the deposit [see Fig. 2(a)], the relative error was greater, even though the distance between the respective response curves is small.

Fig. 3 shows the variations of the final height and Darcy's velocity with the piezometric head. The classical cases of a singly and doubly drained deposit correspond to  $u_0 = 0$ . When water flows downward (i.e.,  $u_0$  is negative), the induced seepage stresses are added to the buoyant self-weight stresses to decrease the final height. However, when the flow is upward, the seepage stresses decrease the buoyant self-weight stress, which, in turn, increases the final height. Note that, if  $u_0$  is equal to the buoyant self-weight stress at the bottom (i.e.,  $U_0 = 1$ ), the final height will



**FIG. 3. Final Height and Darcian Velocity as Functions of Excess Pore-Water Pressure at Bottom**



be equal to the initial height because the upward seepage stresses will equal the downward buoyant self-weight stresses throughout the deposit. Thus, the effective stress everywhere will be equal to the effective stress at the surface, and no seepage-induced consolidation will occur. If  $u_0$  is increased beyond this value, a "quick" condition will result, the deposit will loosen (i.e., decrease in density) physically, and an asymptotically unbounded mathematical solution will result (i.e., the height of the deposit will go to infinity as  $u_0$  approaches the limit of  $u_{0max}$ ). This limiting value for  $u_0$  is defined by Eq. 5 as the value that makes the effective stress equal to zero. It is given by

$$u_{0max} = (G_s - 1)\gamma_w z_0 + \left(\frac{e_0}{A}\right)^{1/B} \dots \dots \dots (26)$$

or

$$U_{0max} = 1 + F_0 \dots \dots \dots (27)$$

There are admittedly reasonable doubts regarding the accuracy of this model for values of  $u_0$  approaching  $u_{0max}$ , since the concept of effective stress becomes less and less valid. No solution exists at this limit because of the inconsistency associated with the fact that  $U_z = 0$  from Eqs. 12 and 13 and the boundary conditions are  $U(0) = 0$  [see Eq. 10] and  $U(1) = U_{0max}$  (see Eqs. 11b and 27) with  $U_{0max}$  different from zero.

### INVERSE PROBLEM

Of all the material properties affecting the consolidation process, permeability is perhaps the most important; unfortunately, it is also one of the most difficult properties to quantify (Pane et al. 1983; Carrier et al. 1983) for soft or sedimented clays due in large measure to the consolidation induced by seepage forces. The technique described here offers a procedure for utilizing experimental data and steady-state results to determine the permeability relationship for a soft deposit undergoing seepage-induced consolidation.

Eqs. 8c, 13, 14, and 22 show that the final height of the deposit is a function of the compressibility parameters  $A$  and  $B$ , the permeability parameter  $D$ , and other material and initial conditions, but it is not dependent on  $C$ ; this may be expressed as

$$\varepsilon_0 = f(A, B, D, \dots) \dots \dots \dots (28)$$

and Eq. 25 can be rewritten as

$$q = Cg(A, B, D, \dots) \dots \dots \dots (29)$$

where the dots represent initial and boundary conditions or material properties not related to the permeability or compressibility equations.

The dependence of the final height on the permeability parameter  $D$  is suggested mathematically by the influence of  $D$  on  $F$  (associated with the effective stress) through Eq. 13 and the definition of  $H(F)$ . Physically, this dependence can be explained by the following argument. Although the final void ratio throughout the deposit is usually considered to be indepen-

dent of the permeability and a function of only the excess pore-water pressure at the bottom, this situation does not exist in the case of seepage-induced consolidation, except at the bottom of the deposit, because an increase in  $D$  enhances the tendency for a "cake" to form at the bottom and, consequently, for the hydraulic gradient in this zone to increase. This, in turn, changes the hydraulic gradient and therefore the resulting void ratio in the rest of the deposit. Thus, sediments with a large value of  $D$  will have, at a given material coordinate (except very near the bottom where the cake is formed), a higher void ratio than materials with a small value of  $D$ ; this is due to the lower gradient and therefore lower seepage forces throughout most of the deposit. The opposite situation would prevail for upward flow (i.e., the final thickness decreases as  $D$  increases).

In the case of self-weight consolidation, the integration of Eq. 22 is straightforward, because, as shown earlier,  $F$  is equal to  $Z$  and independent of  $D$ . Replacing  $F$  by  $Z$  in Eq. 21 and integrating gives

$$\epsilon_0 = z_0 \left\{ 1 + \mu \left[ \frac{(1 + F_0)^{B+1} - F_0^{B+1}}{B+1} \right] \right\} \dots \dots \dots (30)$$

which shows that the final height is independent of the permeability. Once self-weight consolidation is attained, Eq. 30 gives an approximate relation between the compressibility parameters  $A$  and  $B$ . The dimensionless effective stress at the surface of the solids  $F_0$  may be taken equal to zero or between 2–6% if the approximation that  $\bar{\sigma}(0)$  is a function of the initial void ratio is used. Eqs. 28 and 29 suggest that an inverse problem can be posed, whereby seepage-induced consolidation data,  $\epsilon_0$  and  $q$ , can be used to back-calculate the coefficients in the permeability and compressibility relations;  $C$  can be deduced from Eq. 29 once  $A$ ,  $B$ , and  $D$  are known.

The proposed procedure for deducing the material coefficients is as follows. Let  $U_{0i}^*$  be the given piezometric head (or excess pore-water pressure) at the bottom of the sample in the  $i$ th experiment, let  $\epsilon_{0i}^*$  be the measured final height, and let  $\epsilon_{0i}$  be the final height obtained from Eq. 22 and the solution to the steady-state problem in the preceding section using  $U_{0i}^*$  as the boundary condition and assumed values of  $A$ ,  $B$ , and  $D$ . If the error function is defined as

$$E = \sum_{i=1}^N (\epsilon_{0i}^* - \epsilon_{0i})^2 \dots \dots \dots (31)$$

where  $N$  = the number of tests, we have

$$E = E(A, B, D, \dots) \dots \dots \dots (32)$$

and the scheme is to minimize  $E$  by choosing approximate values for  $A$ ,  $B$ , and  $D$ . However, as clearly shown in Fig. 4, the final height of the deposit is not a unique function of the compressibility parameters  $A$  and  $B$ . Therefore, any mathematical minimization procedure for  $E$  is ill-conditioned unless more information is given to distinguish between the different solutions. Knowing the void ratio at the bottom of the sample when steady-state conditions are attained would resolve this problem, since the

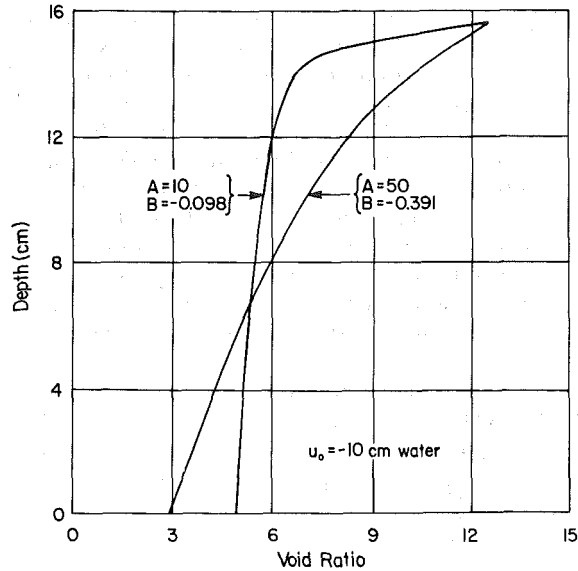
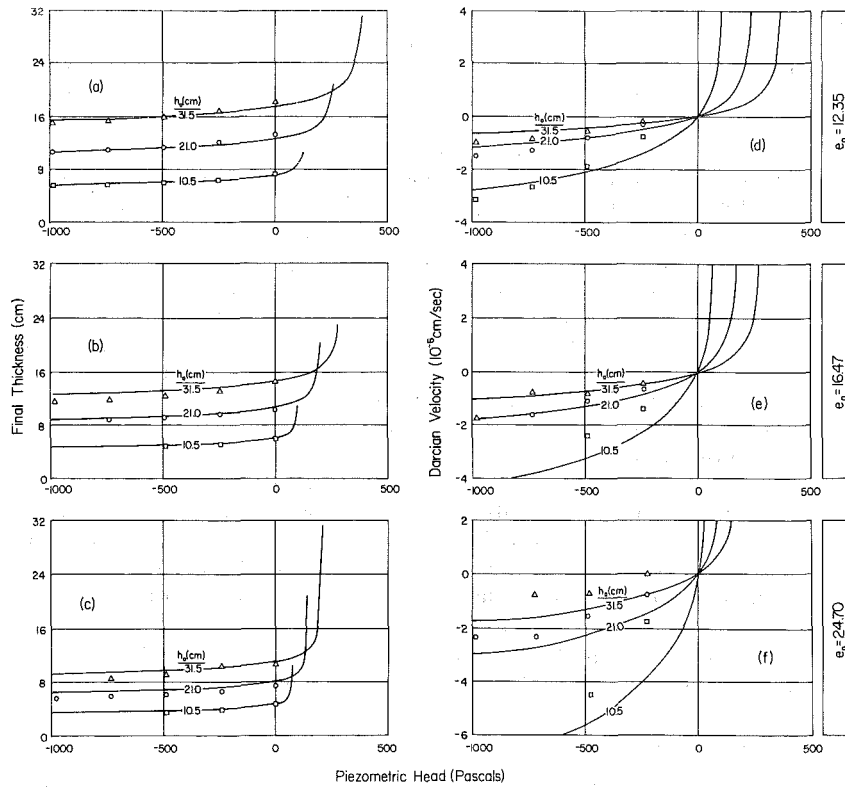


FIG. 4. Nonuniqueness of Final Thickness for Different Values of Compressibility Parameters

effective stress at the bottom is defined by the pore-water pressure boundary condition, and Eq. 3 gives another relationship between  $A$  and  $B$ .

The primary concern here is to enhance the accuracy in determining the permeability constants  $C$  and  $D$ , and this can best be accomplished by fixing one or both of the compressibility constants  $A$  and  $B$ , since they are better known. If reported values for both  $A$  and  $B$  are assumed (Carrier et al. 1983), the error function depends only on the permeability constant  $D$ . Once  $D$  is chosen to minimize  $E$ ,  $C$  can be obtained from Eq. 29. The scheme is then to use the  $i$ th experimental value for  $\epsilon_{0i}^*$  and estimate a value for  $D$  to compute the final height  $\epsilon_{0i}$  from Eq. 21 and the solution to the time-independent problem. Once this is done for each test, and the data incorporated into Eq. 31,  $D$  is incremented by a standard optimization routine to minimize  $E$ . This technique can also be used with no computational difficulty to obtain values for  $A$ ,  $D$ , and  $C$  if only  $B$  is fixed, and this approach is physically appealing because  $B$  is almost constant for clays sedimented from a slurry (Carrier et al. 1983; Carrier and Beckman 1984).

To illustrate an application of this method, data reported by Belhomme (1985) were used to determine  $A$ ,  $C$ , and  $D$ ;  $B$  was chosen equal to  $-0.286$ , as recommended by Carrier et al. (1983). Belhomme (1985) conducted nine seepage-induced consolidation tests on kaolinite slurries, using all possible combinations of three initial void ratios (12.35, 16.47, and 24.7) and three initial thicknesses of slurry (31.5 cm, 21.0 cm, and 10.5 cm). Each sample was consolidated, first under its self-weight, and then under an incrementally increasing piezometric head. After steady-state conditions were essentially reached under a given piezometric head, the head was in-



**FIG. 5. Comparison between Predicted and Observed Final Thickness and Flow Rate**

created by 2.5 cm of water and held constant until steady-state was again attained; this procedure was repeated until the head was 10 cm of water or until side flow was obvious. The reported experimental data for the final height and flow rate are plotted in Fig. 5.

Only data from the most extreme test (initial void ratio at 12.35 and initial thickness of 31.5 cm) were used as input for the inverse problem, and  $A$ ,  $C$ , and  $D$  were computed. These parameters, as listed previously, were then used in the direct scheme to predict the response for the other eight tests. The results are shown as solid curves in Fig. 5. The relative and absolute errors in the final height are 4% and 0.41 cm, respectively, whereas the experimental data are good to within  $\pm 0.10$  cm. The relative and absolute errors for the flow rate are 28% and  $0.48 \times 10^{-6}$  cm/s, respectively. In a normal engineering situation, the inverse problem would have been solved for all of the tests, and the average values for  $A$ ,  $C$ , and  $D$  would be used. The values obtained here for  $A$ ,  $C$ , and  $D$  are similar to those determined by Belhomme (1985); in the latter case the settlement-time plots were curve-fit by eye, and the incomplete time-dependent problem was solved numerically for each trial.

## CONCLUSIONS

Based on the results described in this study of seepage-induced consolidation of a sedimented slurry, the following conclusions can be advanced:

1. The steady-state solution is most useful when the final conditions are of primary interest; the proposed numerical computation scheme is more efficient than the time-step analysis normally used in such cases.

2. In seepage-induced consolidation, the coefficient of permeability influences not only the required time for reaching a steady-state condition, but also the steady-state itself (i.e., the final height of the consolidated deposit depends on the variation of permeability with void ratio).

3. The compressibility and permeability parameters (for the empirical relations studied here) can be back-calculated from the results of seepage-induced consolidation tests and a measure of the final void ratio at the bottom of the solids; this provides a very important means for quantifying material property relationships that have been elusive to date.

4. From the point of view of permeability testing, this method accounts for both seepage and consolidation; high gradients can be avoided, thereby reducing deviations from Darcy's law and some potential testing problems (such as side flow). Since measurements of the height and flow rate are required only during the final stages of consolidation, the test procedure is simple, and sophisticated measuring equipment is not needed. Furthermore, the problem of preparing "identical" specimens is avoided because a single sample can be tested under increasing piezometric heads.

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This work is dedicated to the memory of Frank Somogyi: teacher, colleague, and friend.

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