Comparative assessment of the vertical scale of fluctuation from CPTU and DMT testing: a case study in central Italy

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ABSTRACT

The quantification of the spatial variability of soil properties allows the enhanced engineering modelling, analysis, and design of geotechnical systems. Evolutionary design codes such as Eurocode 7 are awarding spatial variability an increasing central role in geotechnical design. The spatial variability of geotechnical properties is often investigated using a random field approach. Among the defining parameters of a random field is the scale of fluctuation, which describes the extent of significant spatial correlation in a specific spatial direction. The scale of fluctuation can be estimated quantitatively using a variety of methods relying on statistical approaches. The scale of fluctuation is not an inherent property of a soil. Existing studies demonstrate its dependency from numerous factors including the spatial direction, measurement interval, and user-defined modelling options. This paper illustrates the procedures and main results of the comparative estimation of the vertical scale of fluctuation of undrained shear strength of a layer of silty clay from piezocone (CPTU) and dilatometer (DMT) testing at a rural site in the region of Tuscany in central Italy. Vertical scales of fluctuation were calculated using two methods available in the geotechnical literature. Quantitative estimates are compared and analysed critically.

Keywords:.spatial variability, random field modelling, scale of fluctuation, cone penetration testing, dilatometer testing.

1. Introduction

Soils are natural materials whose compositional and mechanical properties vary spatially in all directions due to the complex genetic and modification process brought by the physical environment and by human activity. Myriad publications have demonstrated that duly accounting for spatial variability is beneficial because it increases the level of reliability of geotechnical design by reducing the probability of under-conservatism and limiting over-conservatism. While the reduction of under-conservatism has been the leading criterion underlying engineering design, the importance of limiting overconservatism is taking an increasingly central role in the light of global paradigms of sustainability and cost-performance optimization. The quantitative modelling of the vertical and horizontal spatial variability of soils is de facto required by many evolutionary geotechnical design codes. In Eurocode 7, for instance, the representative value of any soil property must be estimated by accounting for the relevance of spatial averaging. The spatial averaging effect results in a reduction of the effect of spatial variability on the computed performance because the variability is averaged over a volume, and only the averaged contribution to the uncertainty is of interest as it is representative of the "real" physical behavior of a geotechnical system with respect to a specific limit state. In geotechnical design codes, the spatial averaging effect is parameterized by the relative magnitude of the spatial extension of the limit state mechanism and the spatial variability of that property in the context of that limit state.

The spatial variability of geotechnical properties is often investigated using a random field approach. Actually, a random field is described (in the second-moment sense) by its mean, standard deviation (or coefficient of variation), scale of fluctuation, and a functional form for the autocorrelation function. The scale of fluctuation describes the extent of significant spatial correlation in a specific spatial direction. Within separation distances smaller than the scale of fluctuation, the deviations from a spatial trend function representing physical phenomena induced by in-situ effects such as overburden stress, are expected to show relatively strong correlation. When the separation distance between two sample points exceeds the scale of fluctuation, it can be assumed that little correlation exists between the fluctuations in the measurements.

The scale of fluctuation is also useful to quantify spatial averaging. In the upcoming second-generation Eurocode 7, the scale of fluctuation serves as spatial variability parameter in defining the representative value of a soil property. More specifically, the scale of fluctuation θ appears in the variance reduction coefficient Γ^2, which can be approximated as

$$\Gamma^2 \approx \theta/L \tag{1}$$

where L is the extension of the limit state mechanism in the spatial direction considered. The value of Γ^2

determines which approach is to be used for quantifying the characteristic value of a design parameter.

As detailed in another section of this paper, the scale of fluctuation is not an inherent property of a soil as it depends on the estimation process and source data. It is thus important to explicitly report the modelling process as well as the characteristics of the dataset of in-situ and/or laboratory testing data available for the estimation. This paper aims to provide a case study of the quantitative estimation of the scale of fluctuation of undrained strength of a layer of silty clay at “I Bandi”, a rural site in the Civitella-Paganico Municipality in the Grosseto province in southern Tuscany (central Italy). The investigated layer spans over a depth interval L=6.5m, ranging from 20.5m to 27m below ground level. The estimation relies on data from piezocone (CPTU) and dilatometer (DMT) testing. These in-situ testing methods are known for their high repeatability and low measurement error in comparison to other testing methods such as standard penetration tests (SPT) and dynamic superheavy penetration tests (DPSH). However, due to their instrumentation and measurement processes, CPTU and DMT differ significantly in terms of measurement spacing, typically providing data at 2cm and 10cm spacings, respectively. Moreover, the transformation models which are required to estimate geotechnical parameters such as undrained strength are testing method-specific and are likely to lead to differing outputs.

This paper aims to provide a case study example for the estimation of the scale of fluctuation from a real-world, small-scale geotechnical project. More specifically, it aims to address a set of relevant aspects in the estimation process and to highlight the importance of conducting such estimation in a structured manner by demonstrating the dependency of the scale of fluctuation from source data (by comparing CPTU- and DMT-based estimates) and modelling choices.

Geotechnical research has led to the development of ingenious approaches to the modelling of spatial variability of soil properties relying on both frequentist and Bayesian statistical methods. This paper deliberately adopts conceptually simple and computationally inexpensive approaches to convey that spatial variability modelling can and should be applied routinely in geotechnical practice. The level of sophistication of the methods adopted in this paper should be attainable by present-day and future geotechnical engineers, given the ongoing transition of design formats towards non-deterministic paradigms.

2. Description of the site

The geotechnical site characterization at the “I Bandi” site was conducted preliminarily in the context of a structural renovation and seismic retrofitting system for a privately owned rural building. The characterization process relied on a small-scale but rationally planned testing campaign involving a borehole, a variety of in-situ (seismic dilatometer, piezocone, dynamic penetrometer super heavy, plate load) and laboratory (index properties, direct shear, triaxial compression, oedometer, resonant column, cyclic torsional shear) geotechnical testing as well as geophysical tests (MASW, down-hole seismic refraction, electric tomography). The borehole revealed a stratigraphic profile including a surficial, cemented gravelly conglomerate underlain by silty sands and, at greater depths, interbedded layers of silty clays and clays. Among the in-situ tests comprising the site investigation, a seismic dilatometer test (SDMT) and a piezocone test (CPTU) were conducted on two spatially proximal verticals to minimize the likelihood of the existence of significant horizontal spatial variability of the stratigraphic profile, thus allowing a more direct comparative estimation of geotechnical uncertainty from distinct testing methods.

3. Assessment of geotechnical homogeneity

The physical homogeneity of soil units is a fundamental prerequisite for variability analyses. Performing variability analyses on soils which are not homogeneous in terms of the property of interest can result in incorrect and non-meaningful estimates.

The assessment of homogeneity assessment can be performed subjectively or objectively. A purely subjective assessment relies uniquely on expert judgment. The over-reliance on subjectivity, which is overly diffused among geotechnical practitioners, has widely proven to hinder the optmization of geotechnical design and contravenes evolutionary design codes. On the other extreme, a solely date-driven assessment (i.e. based exclusively on numerical criteria) is not warranted, as at least the check on data quality and the definition of relevant parameters requires geotechnical background on the part of the user. Hence, the assessment of homogeneity should rely on both data and a critical, expert-based evaluation of results.

In operational terms, homogeneity can be assessed in terms of soil composition or soil behaviour. Research has widely shown that these do not display a one-to-one correspondence: soils which may be homogeneous in terms of composition may not be so in terms of mechanical behaviour. This paper focuses on the processing of data from geotechnical in-situ testing methods, which provide direct information regarding the mechanical response of soil to penetration, and not regarding composition. This section details the process adopted for the assessment of the homogeneity of soil behavior within the investigated layer.

Soil behavior classification can be conducted from CPTU testing data through the soil behavior classification index (e.g., Robertson 2009)

$$I_c = [(3.47 - \log_{10} Q_{tn})^2 + (\log_{10} F_r + 1.22)^2]^{0.5} \quad (2)$$

In Eq. (2), the stress-normalized friction ration is defined (in %) as

$$F_r = [f_s / q_{net}] \cdot 100 \quad (3)$$

where f_s is the field-measured sleeve friction and

$$q_{net} = q_t - \sigma_{v0} \quad (4)$$

is the net cone resistance, calculated from the corrected cone resistance

$$q_t = q_c + u_2(1 - a_c) \tag{5}$$

In Eq. (5), q_c is the measured cone resistance, u_2 is the measured pore pressure, and a_c is the equipment-specific cone factor (in the case under investigation, a_c=0.80). The stress-normalized cone resistance can be calculated as

$$Q_{tn} = [q_{net}/p_a](p_a/\sigma'_{v0})^n \tag{6}$$

where σ'_{v0} is the vertical effective stress, p_a is the atmospheric pressure, and n is a variable stress exponent which can be calculated iteratively from I_c (and Q_{tn}) as

$$n = 0.381(I_c) + 0.05(\sigma'_{v0}/p_a) - 0.15 \tag{7}$$

as suggested in Robertson (2009). The approximate boundary between sand-like and clay-like behavior is around I_c=2.60. Drained behavior can be expected for I_c<2.60. Partially drained behavior can be expected in the range 2.05≤I_c≤2.60, while I_c>2.60 likely corresponds to undrained behavior.

In DMT testing, soil behavior classification can be pursued from through the material index I_D, calculated from the corrected readings p_0 and p_1 and the hydrostatic pore pressure u_0 as

$$I_D = \frac{p_1 - p_0}{p_0 - u_0} \tag{8}$$

Marchetti & Crapps (1981) proposed a classification system by which soils having a dilatometer modulus E_D>1.2, I_D<0.6 can be associated with cohesive-behavior, 0.6≤I_D≤1.8 can be associated with intermediate-behavior, and I_D>1.8 corresponds to cohesionless behavior.
Depth-wise profiles of I_c and I_D for the investigated layer are shown in Fig. 1a and Fig. 1b, respectively. Visual assessment suggests that; (1) in accordance with the contents of the borehole log, the soil layer can be classified as a cohesive-intermediate behavior soil from both testing methods; and (2) the layer is homogeneous in terms of mechanical behavior. This subjective assessment was supported by quantitative statistical procedures. A second-moment descriptive statistical analysis was conducted on the samples of I_c and I_D, yielding sample means, standard deviations, and coefficients of variation (COVs), the latter being the ratio of the standard deviation to the sample mean. In addition, the non-parametric Kendall‘s tau test was applied to assess the statistical independence (in this case, the absence of a significant depth-wise trend) of the samples.

Kendall's test involves the calculation of the test statistic, τ_{ken}, which measures the probability of concordance minus the probability of discordance between measurements in a data set:

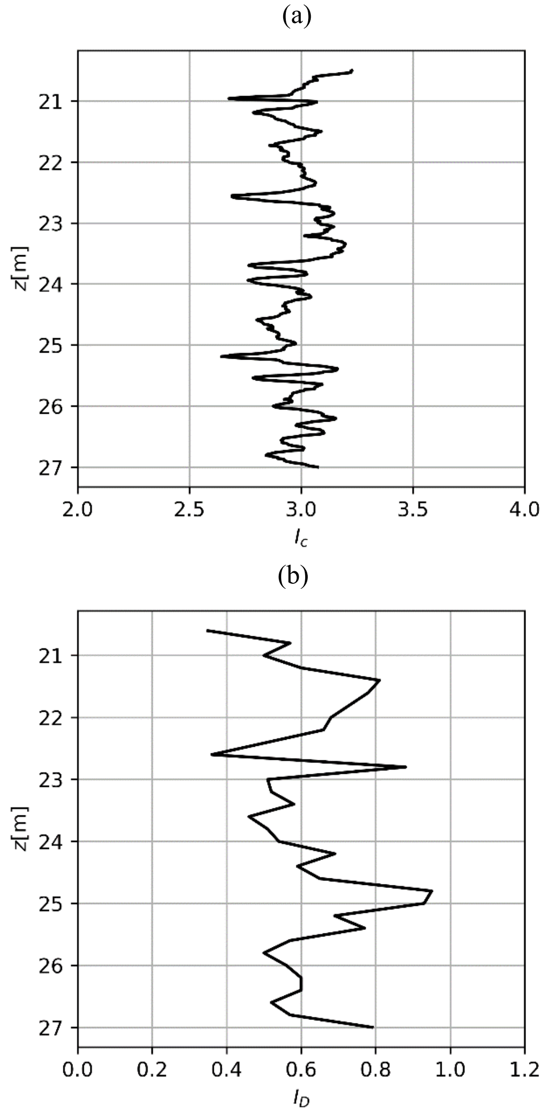

Figure 1. Depth-wise profiles of: (a) I_c from CPTU; and (b) I_D from DMT

$$\tau_{ken} = \frac{n_{con} - n_{dis}}{\frac{1}{2}n_d(n_d - 1)} \tag{9}$$

where n_d is the number of pairs of observations in a data set; n_{con} is the number of concordant pairs of observations and n_{dis} is the number of discordant pairs of observations. To calculate τ_{ken}, a total of (1/2)n_d(n_d-1) comparisons are made between all possible pairs of observations. A pair of observations (z_i, x_i), (z_j, x_j) of a generic parameter x with respect to depth z is said to be concordant if, when z_i>z_j, it is true that x_i>x_j, while it is said to be discordant if, when z_i>z_j, it is true that x_i<x_j. The values of τ_{ken} range from -1 to +1, indicating, respectively, perfect negative and positive correlation. A value close to zero indicates low correlation. For n_d≤40, critical values of τ_{ken} for rejecting the null hypothesis of statistical independence are available in tabulated form (e.g. Daniel 1990). For n_d>40, the statistic $z_{\tau,ken}$ is calculated from τ_{ken} as

$$z_{\tau,ken} = \frac{3\tau_{ken}\sqrt{n_d(n_d - 1)}}{\sqrt{2(2n_d + 5)}} \tag{10}$$

This statistic is normally distributed with zero mean and unit standard deviation. The p-value p_{val} related to the testing of the null hypothesis of statistical independence (at a user-defined significance level corresponding to p_{val}) can be obtained as the inverse standard normal distribution of $z_{\tau,ken}$. Outputs of the descriptive statistical analysis and Kendall's tau test are given in Table 1.

Several relevant inferences can be made. First, sample means confirm that the central tendencies of the samples are located in the cohesive-to-intermediate behavior zones for both CPTU and DMT classification systems. Second, the magnitude of sample COVs attest to the low degree of relative scatter of sample values around the respective means. Third, Kendall's tau test confirms the statistical independence of I_c and I_D from depth, i.e., the absence of significant depth-wise trends at the 0.05 significance level.

4. Estimation of undrained strength

In-situ testing methods do not allow the direct measurement of undrained strength. Transformation models are required to transform field measurements. The procedures adopted in this study are detailed in the following.

4.1. Estimation from CPTU

Undrained shear strength was estimated from CPTU using the cone factor model (e.g., Lunne et al. 1997):

$$s_u = \frac{q_n}{N_{kt}} \tag{11}$$

The cone factor N_{kt} is influenced by stress and strength anisotropy, rigidity index, strain softening and rate effects. It is a markedly site-specific parameter which, in principle, requires calibration from in-situ testing measurements of cone resistance, duly corrected for pore pressure and in-situ stress and selective laboratory measurements of s_u performed on high-quality samples is the best approach. However, laboratory tests were not available from the geotechnical testing campaign. Based on the critical analysis of the outputs of geological and geotechnical investigations conducted at the "I Bandi" site, a constant value of N_{kt}=20, which can be seen as plausible value for normally consolidated silty clays, was selected.

Undrained strength was estimated from DMT testing outputs using the transformation model provided by Marchetti (1980)

$$s_u = \sigma'_{v0}[0.22(0.5K_D)^{1.25}] \tag{12}$$

where σ'_{v0} is the vertical effective stress and

Table 1. Statistical assessment of geotechnical homogeneity

Par.	mean	st.dev.	COV	τ_{ken}	p_{val}	Ind.
I_c	2.98	0.12	0.04	-0.052	0.05	Y
I_D	0.62	0.15	0.24	0.096	0.44	Y

$$K_D = \frac{p_0 - u_0}{\sigma'_{v0}} \tag{13}$$

is the horizontal stress index. CPTU- and DMT-calculated profiles of s_u are plotted in Fig. 2.

5. Estimation of the scale of fluctuation

The estimation of the scale of fluctuation is conducted using statistical approaches which make several fundamental assumptions, and which require specific attributes on the part of the data. This section describes the stepwise process which leads to the formally correct estimation of the scale of fluctuation.

5.1. Decomposition

Statistical modelling of spatial variability relies heavily on the hypothesis of data stationarity. In broad terms, stationarity denotes the invariance of a data set's statistics to spatial location. More specifically, weak stationarity is required to apply numerous techniques including random field theory. If a data set of interest is not at least weakly stationary, the results of statistical analyses may be erroneous or biased. In geotechnical applications, it is often necessary to transform the data set because source data is not stationarity due to multiple causes including in-situ effects (overburden stress, stress history, progressive variation in soil composition, etc.).

"Data transformation" is a general term referring to several techniques (mostly from time series analysis) which purpose is the transformation of a non-stationary data set to a stationary set. Decomposition is the most widely adopted data transformation technique in the geotechnical engineering literature. By decomposition, the spatial variability of a set of m spatially ordered measured geotechnical properties [$\psi(z_1...z_m)$] in a sufficiently physically homogeneous soil unit may be is broken down into a trend function [$t(z_1...z_m)$] and set of residuals about the trend [$\xi(z_1...z_m)$]. In the one-dimensional case, for instance, taking depth (z) as the single spatial coordinate, decomposition is expressed by the following additive relation:

$$\psi(z) = t(z) + \xi(z) \tag{14}$$

Eq. (14) neglects measurement error. This is justified by the hypothesis that the aleatory uncertainty resulting from inherent soil variability and the epistemic uncertainty due to measurement error are uncorrelated and can be addressed separately. The presence of a spatial trend in soil properties, even for extremely homogeneous soils, is very common in geotechnical engineering due to the aforementioned in-situ effects.

In the decomposition procedure, the trend is described deterministically by an analytical model; the residuals are characterized statistically as a variable, with (usually) zero-mean and (always) non-zero variance. As visible in Fig.2, CPTU- and DMT-calculated profiles of undrained strength are satisfactorily superimposed in broad terms but very different in terms of erratic

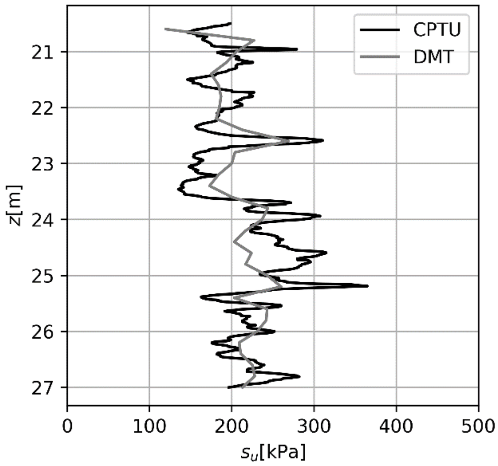

Figure 2. CPTU- and DMT-calculated profiles of undrained strength

variations, with the first displaying a much more regular behavior than the second. This difference can be ascribed to some extent to the difference in measurement interval, but primarily to the respective transformation models. As discussed in a subsequent section, it is of interest to investigate this difference quantitatively through the calculation of the standard deviation of the residuals.

The separation into a deterministic trend and random variation is an artefact of the analysis. There is no univocally "correct" trend to be selected. The choice of the trend function must be consistent with the requirements of the mathematical techniques adopted and must not conflict with geotechnical theory and evidence.

5.1.1. Trend analysis

Regarding point (b), research has shown that the scale of fluctuation depends on the measurement interval. More specifically, it tends to increase with increasing measurement interval. Trend removal generally results in a decrease in the estimated scale of fluctuation, because, as seen previously, the trend accounts for some spatial correlation. Moreover, for a given estimation methodology, the scale of fluctuation decreases with increasing complexity of the trend.

To assess quantitatively the dependency of the scale of fluctuation from the trend, polynomial trends from degree 1 to degree 4 are considered in this study. The most general formulation, i.e., pertaining to degree-4 trends, is given in Eq. (15). Such formulation is adapted to lower-degree trends by eliminating terms as necessary. Table 2 reports the coefficients of the polynomial trends (from degree-1 to degree-4) obtained by decomposition performed using least squares regression for CPTU- and DMT-calculated profiles of undrained strength.

$$t(z) = p_0 z^4 + p_1 z^3 + p_2 z^2 + p_3 z + p_4 \tag{15}$$

Visual examples of decomposition for degree-1 and degree-4 trends are provided in Fig. 3.

5.1.2. Statistical modelling of residuals

As discussed in a previous paragraph, it is of interest to parameterize the fluctuating component of undrained strength estimates, it is useful to calculate the standard deviation of the residuals

$$s_\xi = \sqrt{\frac{1}{m-1}\sum_{i=1}^{m}[\xi(z_i)]^2} \tag{16}$$

This statistic can be used to calculate the coefficient of variation of inherent variability COV_ξ (Phoon & Kulhawy 1999), which provides another fundamental parameter for random field modelling along with the scale of fluctuation. The coefficient of variation of inherent variability is not discussed further in this paper.

5.1.3. Assessment of weak stationarity

The assessment of weak stationarity is a fundamental step in the estimation process because it is required for the implementation of random field theory. Since statistical independence implies weak stationarity (the converse is not true), Kendall's tau test can be used to this purpose. Values of Kendall's tau and the associated p-values are reported in Table 2 for varying degrees of polynomial trends and for both CPT and DMT, along with the assessment of weak stationarity at the 0.05 significance level.

5.2. Estimation of the scale of fluctuation

The scale of fluctuation is not an inherent property of a soil parameter. Estimated values of the scale of fluctuation are closely bound to the estimation methodology, as they depend at least on: (a) the spatial direction [e.g. horizontal, vertical]; (b) the measurement interval in the source data; (c) the type of trend which is removed during decomposition; (d) the method of

Table 2. Polynomial trend coefficients and outputs of stationarity assessment using Kendall's tau test

Trend degree	Test	p_0	p_1	p_2	p_3	p_4	s_ξ [kPa]	τ_{ken}	p_{val}	Stationarity
1	CPT	10.0	-24.1	-	-	-	41.7	-0.027	0.098	Y
	DMT	7.2	40.6	-	-	-	19.6	-0.008	0.693	Y
2	CPT	-2.0	104.4	-1138	-	-	41.2	-0.027	0.106	Y
	DMT	-1.3	69.8	-699	-	-	19.2	0.017	0.408	Y
3	CPT	-2.0	139.9	-3253	25240	-	39.9	-0.009	0.578	Y
	DMT	-0.9	59.6	-1375	10682	-	18.8	0.012	0.563	Y
4	CPT	0.2	-20.2	786	-13432	85194	39.9	-0.009	0.587	Y
	DMT	-0.1	11.2	-370	5400	-29316	18.7	0.018	0.390	Y

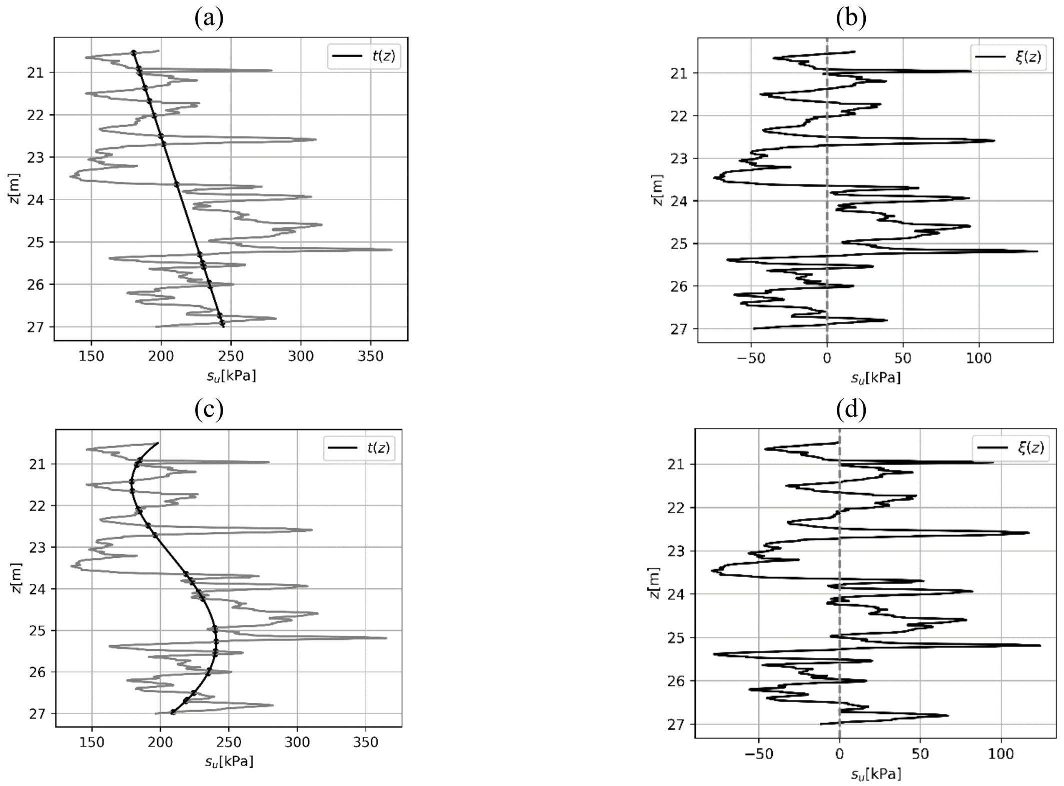

Figure 3. Example outputs of spatial decomposition of the undrained strength profile: (a) data and degree-1 polynomial trend; (b) residuals of degree-1 polynomial trend; (c) data and degree-4 polynomial trend; (d) residuals of degree-4 polynomial trend.

estimation of the scale of fluctuation from residuals; and (e) modelling options from the specific estimation method (e.g., choice of best-fit autocorrelation model).

With respect to point (a), intuitive reasoning suggests that geological and geomorphological formation and modification processes, as well as factors contributing to the definition of the in-situ state would result in a greater heterogeneity (and, thus, in weaker spatial correlation) of soil properties in the vertical direction. The anisotropy of spatial variability has been amply confirmed by geotechnical research. This study addresses only vertical spatial variability.

Two approaches to the estimation of the scale of fluctuation are presented and implemented comparatively as discussed in the following.

5.2.1. Zero-crossings method (ZCM)

Vanmarcke (1977) proposed the following approximate relationship, here termed "zero-crossings method" for evaluating the scale of fluctuation:

$$\theta \approx \sqrt{\frac{2}{\pi}}\bar{\Delta} \tag{17}$$

where

$$\bar{\Delta} = \frac{1}{n_c}\sum_{i=1}^{n_c} \Delta_i \tag{18}$$

is the average distance between the intersections of the fluctuating component and the trend of a given profile (see Fig. 4). The zero-crossings method was derived under the assumption of a Gaussian random field governed by the squared exponential autocorrelation function (e.g., Zhu et al. 2019). The method is operationally convenient due to its simplicity. However, its outputs are most reliable under the assumptions of Gaussian marginal distribution and squared exponential autocorrelation function) are valid, and sufficiently long length of the measurement profile (Cami et al. 2020).

5.2.2. Method of moments (MOM)

Available finite-scale approaches for quantifying autocorrelation essentially include Bayesian and frequentist approaches. Common frequentist approaches include maximum likelihood estimation and moment estimation. Moment estimators use the statistical moments of a set of data (e.g. means, variances, correlation) as estimators of the corresponding populations which are being sampled, and whose real moments are not known. Moment estimators are operationally simple and have the advantage of being non-parametric (i.e., they do not require knowledge or assumption regarding population distributions, but only require that the statistical moments of the distributions may be estimated).

The autocorrelation function of a stationary stochastic process describes the variation of the strength of spatial correlation as a function of the spatial separation distance between two spatial locations at which data are available. In practice, it is not possible to calculate the real autocorrelation function of a stochastic process (and, thus, to investigate its true spatial correlation structure) because data sets are always limited

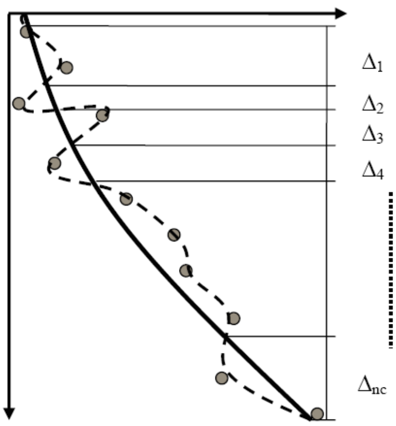

Figure 4. Explicative scheme of the zero-crossings method

in size. Hence, it is necessary to refer to the sample autocorrelation function, i.e., to an approximation of the real autocorrelation function, calculated from an available set of data which is deemed representative of the stochastic process. The (empirical) sample autocorrelation function is given by

$$\hat{\rho}(\tau_j) = \frac{1}{\hat{\sigma}_k^2} \sum_{i=1}^{k-j} (X_i - \hat{\mu})(X_{i+j} - \hat{\mu}), \; j=0,...,k\text{-}1 \tag{19}$$

where $\hat{\rho}(\tau_j)$ is the sample correlation coefficient between two points separated by a spatial lag distance τ_j; $\hat{\sigma}_k^2$ is the sample variance; $\hat{\mu}$ is the sample mean; k is the sample numerosity; and i is the total number of pairs of points separated by lag distance τ_j. The scale of fluctuation is estimated by fitting theoretical models to the sample autocorrelation function computed at discrete lags. This procedure has been widely used in geotechnical engineering (e.g., Fenton 1999; Uzielli et al. 2005; Cami et al. 2020 among others).

An autocorrelation model quantifies autocorrelation as a function of separation distance and is described by a function and a characteristic model parameter. Five autocorrelation models are considered in this study; namely:

First-order Markov (FOM)

$$\rho(\tau) = \exp\left(-2\frac{|\tau|}{\theta}\right) \tag{20}$$

Second-order Markov (SOM)

$$\rho(\tau) = \left(1 + 4\frac{|\tau|}{\theta}\right) \exp\left(-4\frac{|\tau|}{\theta}\right) \tag{21}$$

Third-order Markov (SOM)

$$\rho(\tau) = \left[1 + \frac{16}{3}\frac{|\tau|}{\theta} + \frac{256}{27}\left(\frac{|\tau|}{\theta}\right)^2\right] \exp\left(-\frac{16}{3}\frac{|\tau|}{\theta}\right) \tag{22}$$

Squared exponential (Gaussian) (SPX)

$$\rho(\tau) = \exp\left[-\pi\left(\frac{|\tau|}{\theta}\right)^2\right] \tag{23}$$

Cosine exponential (CSX)

$$\rho(\tau) = \exp\left(-\frac{|\tau|}{\theta}\right) \cos\left(\frac{|\tau|}{\theta}\right) \tag{24}$$

According to Spry et al. (1988) among others, no autocorrelation model is univocally preferable over others on the basis of physical motivations. The choice of the correlation structure for a given data set can be based on the comparative assessment of goodness-of-fit of one or more theoretical autocorrelation models to the empirical sample autocorrelation function. The procedure entails the optimization of the characteristic model parameter for each autocorrelation model (e.g., by least-squares regression or other numerical optimization procedures). Optimization can be assessed through the calculation of one or more goodness-of-fit parameters. The autocorrelation model yielding the best goodness-of-fit parameter can be selected as best-fit model.

In this case study, the best-fit autocorrelation model and the corresponding scale of fluctuation were identified through a numerical optimization procedure involving the minimization of the root mean square error between the theoretical and experimental functions for varying values of the scale of fluctuation θ:

$$RMSE = \sqrt{\frac{1}{k}\sum_{j=1}^{k}\left[\hat{\rho}(\tau_j) - \rho(\tau_j)\right]^2} \tag{25}$$

Caution must be exercised in applying the MOM approach. In principle, minimization of least squares assumes that all estimates from Eq. (19) are equally accurate. However, the statistical uncertainty associated with the correlation between two data values increases with lag distance, because the number of data pairs i of sampled points decreases with lag distance τ_j. To account for the increase in statistical uncertainty with lag distance and ensure a more meaningful fitting of theoretical autocorrelation models, the empirical ACF is calculated only to lag distances shorter than $L/4$ as suggested by Lumb (1975). An example of the fitting of theoretical models to the empirical ACF is shown in Fig. 5.

6. Presentation and discussion of results

The estimated scales of fluctuation are shown in Table 3 by estimation approach (ZCM, MOM), testing method (CPTU/DMT) and degree of polynomial trend. In the MOM outputs, RMSE values are provided along

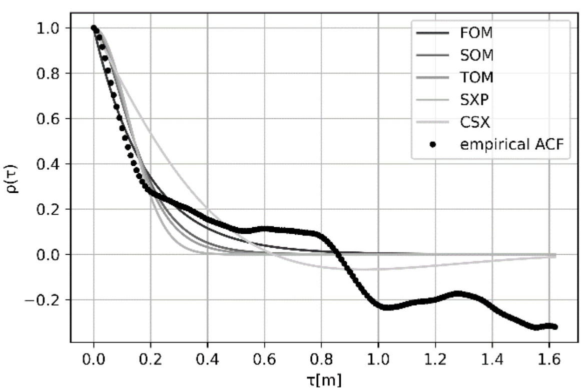

Figure 5. Estimation of the scale of fluctuation by the method of moments: example fit of theoretical autocorrelation models to sample ACF

Table 3. Estimated scales of fluctuation

Trend degree	Test	ZCM	MOM									
			FOM		SOM		TOM		SXP		CSX	
		θ	θ	RMSE	θ	RMSE	θ	RMSE	θ	RMSE	θ	RMSE
1	CPTU	0.32	0.37	0.1618	0.34	0.1684	0.33	0.1771	0.30	0.1775	0.40	0.1657
	DMT	0.39	0.17	0.1143	0.21	0.1131	0.03	0.1109	0.25	0.1123	0.19	0.0969
2	CPTU	0.25	0.35	0.1631	0.32	0.1684	0.31	0.1710	0.29	0.1761	0.38	0.1680
	DMT	0.38	0.18	0.1417	0.23	0.1401	0.24	0.1396	0.26	0.1389	0.20	0.1213
3	CPTU	0.22	0.27	0.1692	0.26	0.1682	0.26	0.1691	0.25	0.1715	0.23	0.1723
	DMT	0.45	0.17	0.1456	0.21	0.1445	0.23	0.1442	0.25	0.1437	0.19	0.1274
4	CPTU	0.18	0.27	0.1691	0.26	0.1680	0.26	0.1689	0.25	0.1713	0.23	0.1722
	DMT	0.47	0.19	0.1637	0.24	0.1617	0.25	0.1611	0.04	0.1712	0.21	0.1411

with the scale of fluctuation for each of the theoretical autocorrelation models considered in the study. The table allows the comparative estimation of the estimates and the appreciation of the scatter in outputs.

A first notable result is the full compatibility of the estimates with literature values (see, e.g., Uzielli et al. 2006, Cami et al. 2020, Ching & Schweckendiek 2021). For all degrees of polynomial trends, the ZCM estimate is lower than all MOM estimates for CPTU, but higher than all MOM estimates for DMT. With respect to ZCM estimates, this result stems from the larger vertical distance between zero-crossings. Such difference can arguably be ascribed to: (1) the considerable difference in measurement interval (2cm and 10cm for CPTU and DMT, respectively); and (2) the transformation models, which result in more erratic profiles for CPTU-estimated undrained strength and, thus, a greater likelihood of intersecting the trend. In the MOM approach, the degree of scatter in estimates of θ varies according to the degree of the removed polynomial trend even though RMSE values are generally quite uniform for all trend degrees.

In the perspective of geotechnical design, using the ZCM approach with DMT-based estimates for the assignment of representative design values leads to more conservative parameterization of the spatial averaging effect with respect to CPTU, because larger values of θ result in larger values of the variance reduction coefficient Γ^2. If the estimation of θ is conducted from CPTU, however, the opposite is true: more conservative estimates are obtained by using the MOM approach.

7. Concluding remarks

This paper illustrated the comparative estimation of the scale of fluctuation of undrained shear strength from CPT and DMT testing using two statistical methods and four polynomial trends to a silty clay layer at a site in central Italy. The quantitative critical analysis of outputs confirmed the dependence of the scale of fluctuation on the testing method (through the measurement interval), the transformation model used to obtain undrained shear strength from testing data, and the degree of polynomial trend removed during the spatial decomposition process. Results obtained herein are case-specific and must not be generalized and exported to other cases. It is paramount to conduct case-specific estimation and to refer to literature data critically. Given the variety of factors influencing the estimates, it is warranted to report explicitly the details of the modelling process and, ideally, to use more than one approach and modelling options (e.g., in terms of spatial decomposition) to assess the sensitivity of estimates to the estimation process.

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