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HOMOGENIZATION OF FIBER REINFORCED ELASTOMER LAMINATES

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Summary: *The effective hyperelastic anisotropic response of a cross-ply laminate built by unidirectional fiber reinforced elastomer plies is studied. The matrix is described by Ogden type hyperelasticity and the fibers are taken to be linear elastic with a volume fraction of 40%. Two homogenization strategies are employed in the context of the Finite Element Method. First, a periodic unit cell of the [90/0]*$_s$ *laminate is set up whereby the matrix–fiber microstructure is resolved over the entire thickness. For the second approach, the laminate unit cell is simplified with respect to the ply representation. Here, the effective behavior of the plies is modeled by the anisotropic hyperelastic Holzapfel-Gasser-Ogden constitutive law. A number of in-plane load cases are studies and the non-linear responses of the different modeling schemes are compared. Good agreement is found up to moderate applied strains of some 40%.*

1. INTRODUCTION

Continuous fiber reinforced elastomers (FRE) allow for the design of composites and structures with particular property profiles. These materials exhibit high stiffness and load carrying capability in specific directions and loading scenarios. Under other loading configurations they are highly flexible and enable large deformations of the structure. Experimental characterization of FRE composites and structural set-ups can be very difficult, time consuming, and costly. Therefore, fast and efficient computational tools are desired which help in tailoring the response of FRE composites and structures to meet the required demands. The Finite Element Method (FEM) provides an established framework for such tasks and can incorporate the predictions of FREs across different length scales. Thereby, FREs can be evaluated in the complete chain from the material to the structural application.

In this work, the effective response of a FRE cross-ply laminate with four plies in a [90/0]$_s$ layup is studied. Two approaches are used to predict the effective response of the laminate. First, a periodic multi-fiber unit cell is employed to model the entire thickness of the laminate. The modeling and the numerical solution is demanding, however, the resolution of the microstructure leads to more realistic results and is used as reference model. Second, a much less demanding approach is employed in which the plies are modeled as homogeneous material by an anisotropic hyperelastic constitutive law. Various load cases are studied and the predicted responses of the two approaches are compared. All simulations are carried out by the commercial FEM package `Abaqus/Standard 2020` *(Dassault Systèmes Simulia Corp., Providence, RI, USA).*

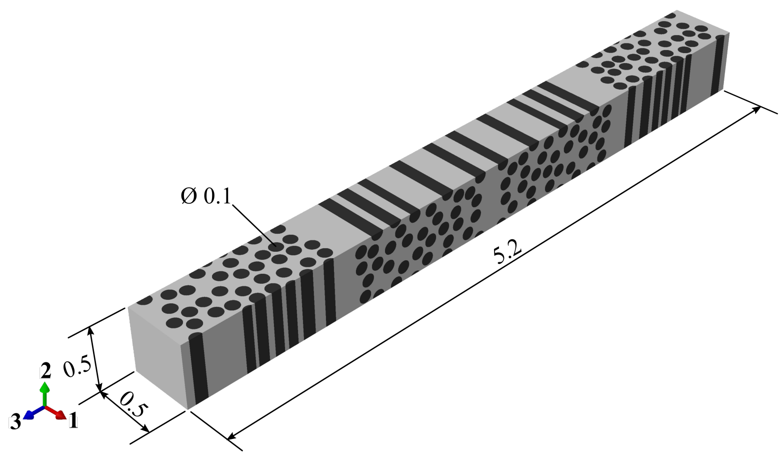

Figure 1. Periodic unit cell of the $[90/0]_s$ laminate; dimensions in millimeters.

2. MODELING

2.1 Laminate unit cell model

The four layer laminate with symmetric layup is modeled by a multi-fiber unit cell which features periodicity, Fig. 1. The modeling starts at ply level by the set-up of two unit cells with aligned continuous fibers generated by `Digimat-FE` *(e-Xstream engineering SA., Hautcharage, Luxembourg).* Since laminate stacking is aimed at, cut fibers are not allowed in the plane parallel to the surface of the laminate. In each ply, the fibers account for 40,91% volume fraction. The plies are assembled to form a [90/0] sub-laminate, which itself is mirrored with respect to the laminate's mid-plane to build the symmetric layup, i.e. $[90/0]_s$. Between the plies, layers of pure matrix are placed with the width of a fiber. This reduces the need for a very fine discretization in the region between the plies, thus, reducing the computational effort in solving. The overall fiber volume fraction of the laminate amounts to 38.88%.
Eight node and six node continuum elements with linear interpolation functions and full integration are employed. Hybrid formulation is selected throughout the model, to account for the nearly incompressible behavior of the matrix. The average element size is 0.01 mm, which results in approximately six million variables in FEM equation. The interface between the fibers and the matrix is assumed to be perfect. The model is equipped with periodic boundary conditions by controlling the nodal displacements. The effective applied load and response is handled via master nodes which sit at the corners of the unit cell.

The matrix material is modeled by an Ogden type strain energy density function as being available in `Abaqus`,

$$U = \frac{2\mu_1}{\alpha_1^2}\left(\bar{\lambda}_1^{\alpha_1} + \bar{\lambda}_2^{\alpha_1} + \bar{\lambda}_3^{\alpha_1} - 3\right) + \frac{1}{D_1}\left(J^{\mathrm{el}} - 1\right)^2 \quad , \tag{1}$$

where $\bar{\lambda}_i$ are the deviatoric principal stretches and μ_1, α_1, and D_1 are material parameters.

Table 1. Material parameters of fiber and matrix in the FRE composite and for the homogenized plies based on two methods.

	Material parameters
Matrix – Ogden type	$\mu_1^{(\mathrm{m})} = 0.3490$ MPa
	$\alpha_1^{(\mathrm{m})} = 2.163$
	$D_1^{(\mathrm{m})} = 9.961 \cdot 10^{-9}$ 1/MPa
Fibers - linear elastic	$E^{(\mathrm{f})} = 8 \cdot 10^4$ MPa
	$\nu^{(\mathrm{f})} = 0.2$
Homogenized HGO–MFUC	$C_{10} = 0.4985$ MPa
	$k_1 = 1.5 \cdot 10^4$ MPa
	$D = 1.0 \cdot 10^{-4}$1/MPa
Homogenized HGO–MTM	$C_{10} = 0.4070$ MPa
	$k_1 = 1.5 \cdot 10^4$ MPa
	$D = 1.0 \cdot 10^{-4}$1/MPa

They are listed in Table 1 together with the properties of the glass fibers which are taken to be isotropic linear elastic.

2.2 Laminate with homogenized plies

In the second modeling approach, the plies are homogenized and represented by an anisotropic hyperelastic constitutive law. A Holzapfel-Gasser-Ogden (HGO) type strain energy density function [1] is used in a simplified form and for unidirectionally aligned fibers,

$$U = C_{10}(\bar{I}_1 - 3) + \frac{1}{D}\left(\frac{(J^{\mathrm{el}})^2 - 1}{2} - \ln J^{\mathrm{el}}\right) + \frac{k_1}{2}\left(\exp\left[\langle \bar{I}_{4(11)} - 1\rangle^2\right] - 1\right) \quad . \qquad (2)$$

The $\bar{I}_i$ are (pseudo) invariants of the deviatoric part of the right Cauchy-Green stretch tensor and J^{el} is the elastic volume ratio, for details see [1, 2]. The material parameters, C_{10}, D, and k_1 have been calibrated in [3] based on a multi-fiber unit cell with unidirectional reinforcements (MFUC) and, alternatively, based on an analytical Mori–Tanaka scheme (MTM). Corresponding values are listed in Table 1.
For simplicity, easy post-processing, and consistent work-flow, a periodic unit cell is set up as in Section 2.1. A much coarser regular mesh of eight noded continuum elements with reduced integration is set up with material assignments corresponding to the laminate layup. Note that the hybrid element formulation cannot be used for anisotropic hyperelastic materials [2].

3. LOAD CASES, RESULTS, AND DISCUSSION

The investigated load cases prescribe applied strain states by controlling the displacements of the unit cell's master nodes. Note that stress controlled loading for nonlinear unit cell analyses is not straightforward for general stress states. The nonlinear homogenization procedure and the evaluation of the effective stress deviators is conducted with `Medtool` *(Dr. Pahr Ingenieurs e.U., Vienna, Austria).*

All load cases show a pronounced in-plane shear component as being the dominant mode of deformation. Even though the unit cell does not resemble traction free laminate faces, the simulations are expected to give a good approximation of the laminates response for the studied loads.

3.1 Pure shear

Pure shear loading in the 1-2 plane is applied by the isochoric macroscopic deformation gradient

$$\mathbf{F} = \begin{bmatrix} 1.25 & 0.75 & 0 \\ 0.75 & 1.25 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad , \tag{3}$$

in the final state. During the gradual load application the volume must be preserved which requires non-linear master node displacement–time histories. The effective strain energy density is evaluated both for the multi-fiber unit cell laminate and the homogenized model, see Fig. 2 (top). Up to a shear stretch of $\lambda_{12} = 0.4$ both models are in good agreement. Upon further loading, the homogenized model exhibits a marked increase in the effective strain energy density. The deformation states of the unit cell for $\lambda_{12} = 0.4$ and $\lambda_{12} \approx 0.75$ is shown in Fig. 2 (middle and bottom), respectively. The fibers "rotate" in the microstructure and align themselves parallel to unit cell faces. In contrast, the homogenized model cannot resemble such a micro-deformation. There, the fibers remain aligned with the global coordinate system, since pure shear is not accompanied by any rotation of the material axes.

The response is also evaluated with respect to the effective deviatoric PK2 stresses. The shear and normal components with respect to the applied shear stretch are shown in Figs. 3 and 4, respectively. The excessive stiffening of the response for the homogenized model is obvious, again.

3.2 Simple shear

The second case is simple shear loading in the 1-2 plane. The displacements of the master nodes are derived from the macroscopic deformation gradient at the final state,

$$\mathbf{F} = \begin{bmatrix} 1 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad . \tag{4}$$

Note that simple shear is accompanied by a rotation of the material axes.
Figure 5 compares the effective strain energy density of the laminate unit cell model to the homogenized models with different material parameter calibration. For this load case, the homogenized models give good predictions of the strain energy density for the entire range of the

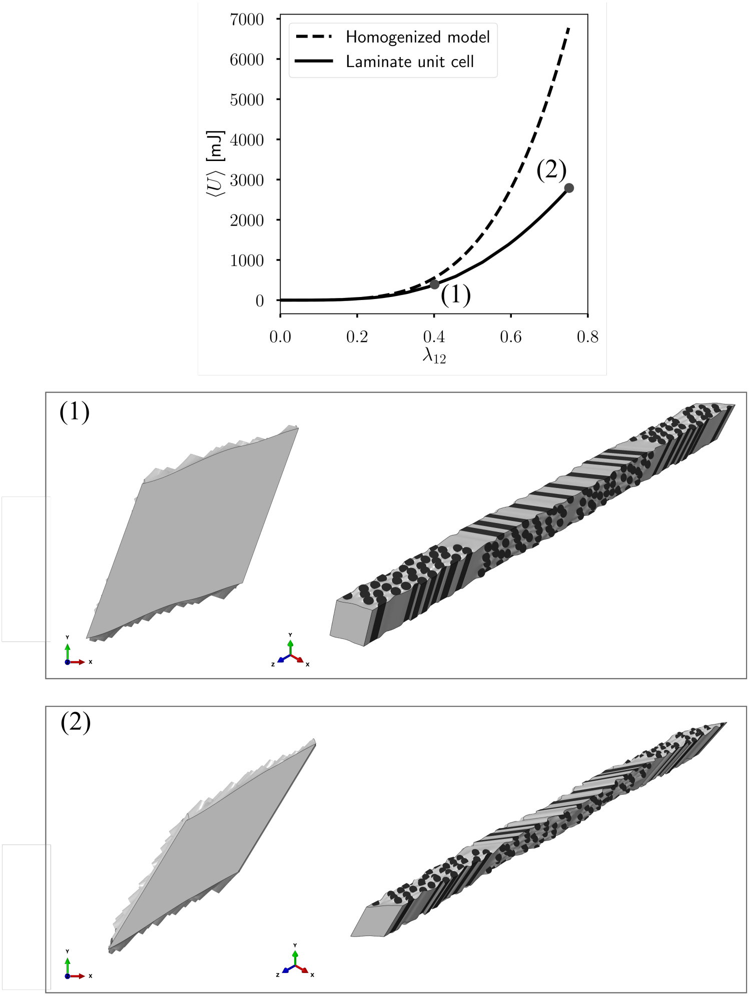

Figure 2. Predicted effective strain energy density under pure shear loading for the laminate unit cell and the homogenized HGO-MFUC model (top) and corresponding unit cell deformations for selected load levels (middle and bottom).

applied strain. The deformation of the homogenized model at the final loading state is shown in Fig. 6 (left) together with the orientation of the local material axes. The corresponding deformation of the laminate unit cell can be seen in Fig. 6 (right) in which also the orientation of the fibers is indicated.

4. CONCLUSIONS

The effective response of a cross-ply fiber reinforced elastomer laminate is studied by non-linear homogenization of a periodic unit cell solved by the Finite Element Method. Two approaches are utilized with different degree of resolution of the micro-geometry. First, the entire laminate thickness is modeled where the fibers in the plies are resolved. Second, the plies are

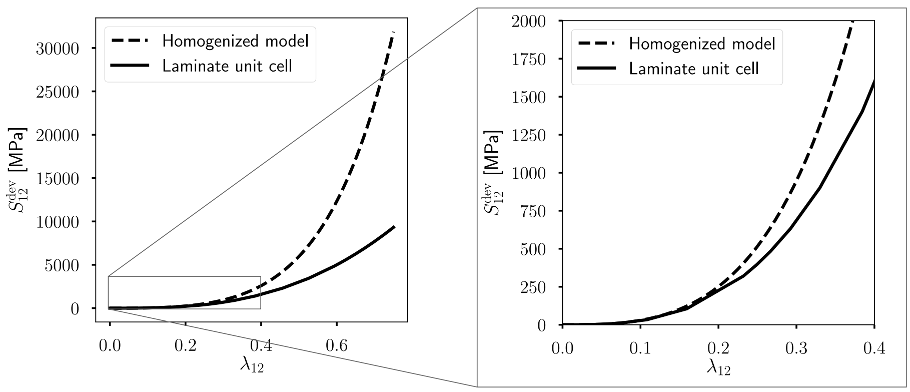

Figure 3. Predicted effective deviatoric PK2 shear stress component under pure shear loading for the laminate unit cell and the homogenized HGO-MFUC model.

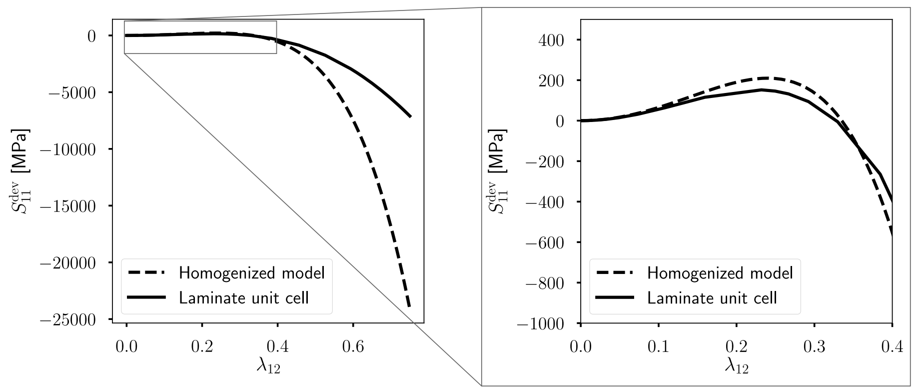

Figure 4. Predicted effective deviatoric PK2 normal stress component under pure shear loading for the laminate unit cell and the homogenized HGO-MFUC model.

described by a hyperelastic anisotropic constitutive law to represent the homogenized behavior.

Strain controlled load cases with dominant shear deformation are investigated and the prediction form the two approaches are compared. Up to moderate stretches both models give similar results. At higher strains the model with the homogenized plies tends to exhibit too stiff results for one load case.

The much simpler approach based on the homogenized material law is suited, in principle, for structural analyses where computation time is an issue, provided that the local laminate strain states remain limited. However, in the anisotropic hyperelastic Holzapfel-Gasser-Ogden type constitutive law the fibers cannot carry compressive loads, which may be unrealistic for technical composites.

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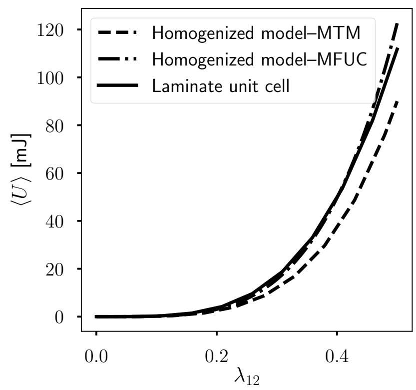

Figure 5. Predicted effective strain energy density under simple shear loading for the laminate unit cell as well as the homogenized HGO-MFUC and HGO-MTM models.

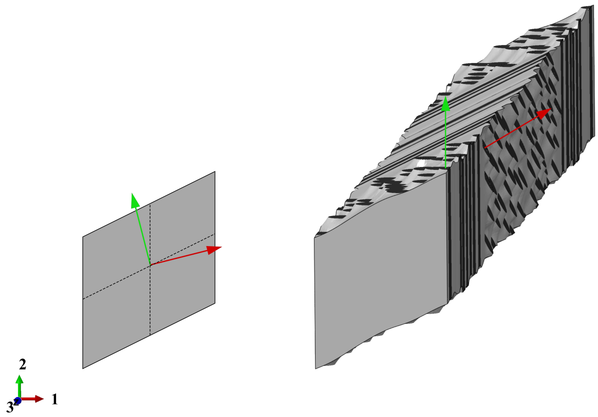

Figure 6. Predicted unit cell deformations under simple shear loading for the homogenized HGO-MFUC model with the local material orientation (left) and the laminate unit cell with the fiber orientations (right).

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