

UNSUPERVISED DATA-DRIVEN TOOL FOR TRACK SUPPORT CONDITIONS ASSESSMENT – FIRST RESULTS

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Summary. Taking advantage of recent developments in machine learning procedures, railway researchers have been creating new tools and methodologies to improve upon structural health monitoring solutions that were developed over the last century. These new tools are now being used to assess a wide range of railway related scenarios.

However, current vehicle-based railway monitoring methodologies cannot provide reliable data on track subgrade conditions. This is a key issue since subgrade condition significantly influences track dynamic response and overall track support conditions.

An alternative and novel methodology to assess railway track support conditions is now under development and validation, which is based on modal analysis of the characteristic frequencies of the multi-element system composed by an instrumented railway vehicle and the railway infrastructure under assessment.

Furthermore, an unsupervised data-driven procedure is currently being developed to enhance the capabilities of the proposed track monitoring methodology to automatically extract adequate results from collected data. This tool is also expected to improve the overall reliability of the proposed methodology to work with more complex data sets.

To ensure the intended continuous assessment of the railway track, the developed tool is formed by a sequential combination of four steps, applied in a sliding window process over the collected input data, to reach the intended results. The four steps are, in order of application, a feature extraction step, a feature modelling step, data fusion and a feature discrimination step. This paper provides an overall description of this tool and on the obtained preliminary results, which are based on numeric simulations performed using the Simpack® software.

1 INTRODUCTION

The assessment of railway track support conditions is a naturally important aspect of modern railway companies, since it is related with several relevant social and economic aspects. These include maintenance management, operational safety, and general reliability of this transportation service. In the context of vehicle-based railway monitoring methods, there are several well-established parameters for the assessment of railway track support conditions (e.g., vertical track stiffness, track geometry) ^[1, 2]. However, these extensively used parameters cannot provide reliable data on the conditions of the track subgrade ^[3-5]. This is a significant issue since subgrade condition is one of the major aspects that influences track dynamic response and overall support conditions of a railway track ^[1, 6-8].

A novel vehicle-based methodology to evaluate railway track support conditions is currently under development and validation. This methodology is based on modal analysis of the characteristic frequencies of the multi-element system composed by a railway vehicle and a railway infrastructure. It states that by observing the characteristic frequencies of the railway infrastructure, with a focus on the frequencies related with the railway subgrade, it should be possible to collect data on overall track support conditions ^[9, 10]. This assessment can be performed over the entire length of a railway infrastructure, since this methodology is based on instrumented railway vehicles to serve as the data collection points.

The proposed methodology uses a two Degree-of-Freedom (2-DoF) lumped-elements type model to perform preliminary interpretation on the acquired railway related frequency data. The model consists of an assembly of two spring-mass sub-systems in series, to represent the vehicle, per axle, and the modal properties of the railway infrastructure. The outputs of this model are two characteristic frequencies (ω_1 and ω_2), that are related with these 2-DoF ^[11]. Frequency ω_2 is mainly related with the vehicle's modal characteristics and can be attributed to the vertical oscillatory motion of the vehicle's sprung mass ^[12]. For most railway vehicles, this oscillatory motion has a frequency value between 1 Hz and 5 Hz. Frequency ω_1 is mainly influenced by the modal parameters of the railway infrastructure elements, thus represents the natural frequency of interest for the proposed railway monitoring methodology. Even if this ω_1 frequency cannot be directly attributed to a natural frequency of a railway subgrade layer, since the 2-DoF model is a simplification of a real track infrastructure, the actual difference between the modelled and a realistic frequency value can be relatively low if adequate mass and stiffness values are used in the infrastructure related lumped elements on the 2-DoF model. Common values for the first natural frequency from a subgrade layer can be from 20 Hz to 50 Hz ^[5, 13], which can be used as an expected range for ω_1 in the aforementioned conditions. Morais et al. ^[13] provides a more in-depth description on the proposed methodology and on the motivations that lead to its development. Numerical simulations tests using the multibody simulation software Simpack® have been performed to validate and assess the developed methodology ^[14]. The obtained results demonstrated its significant potential as an alternative track monitoring solution that could more directly assesses subgrade conditions, although some difficulties were detected in extracting reliable results in the more realistic accurate numerical models and from experimental data.

In an attempt to solve the aforementioned issue and improve upon the overall reliability of the proposed methodology, a new unsupervised data-driven tool is currently under development to apply the methodology in an automatic and more efficient way. For the current application,

a data-driven approach was chosen since their computational simplicity makes them more attractive and efficient to implement in Structural Health Monitoring (SHM) operations. Data-driven approaches rely on data mining techniques to extract meaningful information from the acquired data ^[15], which suits this application since there is a lack of a sufficiently accurate model to allow for a model-updating solution. The statistical nature of data-driven solutions should also improve the odds of the developed tool to provide reliable results when analysing data obtained from the complex system of interest.

This paper presents a brief description on the developed unsupervised data-driven tool. This is followed by two sections describing a numerical model that was used to assess the current version of this tool and the obtained results. Overall, the results point to a very promising solution to solve the aforementioned difficulties in extracting reliable results while using the unenhanced way of applying the proposed track monitoring methodology (i.e., as described in Morais et al. ^[14], that entailed manually analysing frequency patterns in spectrograms), with the added benefit of representing an automatic and autonomous results extraction tool.

2 UNSUPERVISED DATA-DRIVEN TOOL

The unsupervised data-driven tool (UDDT) presented in this paper for the automatic application of the proposed track monitoring methodology (TMM) aims at being a robust and generic enough solution to be applied to any type of plain railway infrastructure. Since it is based on traffic-induced dynamic responses (i.e., acceleration data acquired with an instrumented vehicle), it is very important that the tool can account for and discard the effects of the environmental and operational variations (EOV) that are common in experimental data from real structures. In this context, EOV include vehicle running speed, rail defects and irregularities, temperature changes and other weather-related phenomena. As explained in Morais et al. ^[13], most of the weather-related phenomena should not significantly interfere with the frequencies of interest for the TMM, neither should changes in vehicle running speed. However, it is still recommended to implement generic and self-sufficient steps in a new unsupervised damage detection tool for the mitigation of EOV, since this is meant to be an automatic process without direct influence from the user or the consideration of direct measurements of such phenomena. The current version of the UDDT entails the four steps shown in Figure 1: (i) a feature extraction step applied to the acquired acceleration data to calculate relevant features, (ii) a feature modelling step to remove EOV, (iii) a data fusion step to merge features from each sensor and merge data from multiple sensors to improve the sensitivity to damage cases, and (iv) a feature discrimination step to automatically classify the extracted merged features into two categories: healthy or damaged track section.

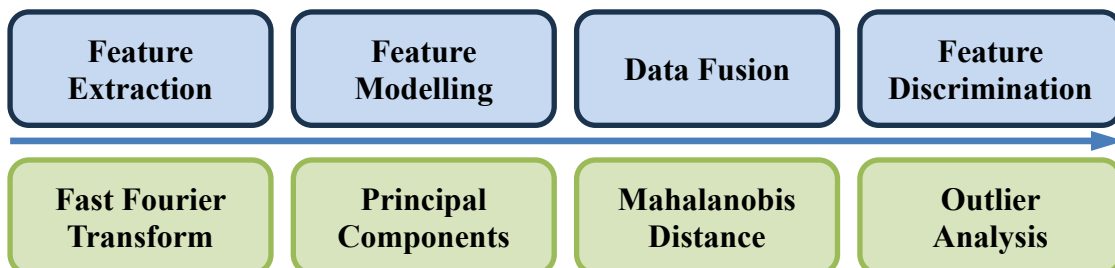


Figure 1: data processing steps of the proposed unsupervised data-driven tool

In generic terms, feature extraction refers to the process of transforming the acquired data in the time domain into an alternative form, where damage-related information can be more readily obtained ^[15]. Modal-based features are the most commonly used solutions in the literature ^[15] due to the advantage of them being directly associated with the modal properties of the structure under assessment, which are expected to change in the presence of damage ^[11]. This is particularly true for modal stiffness, which tends to be more sensitive to the presence of damage in a structure ^[11]. Since the TMM already converts acquired acceleration data from the time domain to the frequency domain to search for frequencies that could be related with the modal properties of the track subgrade ^[13], transforming the input data into the frequency domain was a very natural choice as the first solution for the feature extraction step. The data is converted to the frequency domain by the application of Fast Fourier Transforms (FFT), in overlapping time windows, thus making the calculated frequency components the extracted features to be further processed. Thus, the concept is that the UDDT will theoretically be looking to detect changes in a specific set of frequency components that could be more sensitive to a damaged subgrade layer, which should potentially correspond to the natural frequencies of interest identified and analysed in the unenhanced version of the TMM.

Effective damage detection solutions must always contend with the issue of separating between measurable changes in the structure under analysis caused by EOv from those triggered by the presence or propagation of damage in said structure. To accomplish this, most of the current damage detection solutions used in SHM resort to feature modelling tools as a means to solve this issue ^[15]. This step is crucial in the railway context to prevent false positive results since environmental effects (e.g., temperature variations) or operational change (e.g., vehicles running at different speeds) may impose greater variations in the collected data than those caused by damage, thus triggering possible false positive results if countermeasures are not applied. The current version of the UDDT applies the Principal Components Analysis in this step, that represents a statistical solution to remove the effects of most EOv from the collected data. In brief, the concept behind this tool in the feature modelling context is that most of the energy content from a signal is associated with effects caused by EOv ^[15], from a statistical standpoint. Thus, if the signal is discretised into several components based on their respective energy contributions, the main contributors can be identified, and their effects mitigated in the original signal. The signal can then be reconstructed with the effects caused by the EOv properly mitigated, in statistical terms, thus highlighting any effects caused by the eventual presence of damage in the structure under analysis.

In general, the purpose of a data fusion step is to reduce the data volume while assuring a greater or at least similar capabilities to analyse the object of interest, when compared to what could be achieved when using the original data format ^[15]. Usually, the fusion process may combine features from a single sensor, features from multiple sensors referencing the same object or even features from different sensor types (e.g., features obtained from acceleration data and displacement data). The Mahalanobis distance is one of the more common solutions used in the SHM context for this application due to its capacity to describe the variability in multivariate data sets ^[15]. The method basically calculates a “distance” between two data sets, magnifying the obtained value according to how different they are in terms of the individual values of each comparable feature. This is the characteristic that makes this method a very useful solution if a healthy state scenario is definable for the structure under analysis (i.e., a baseline case). Every new set of features can then be compared with the baseline and any

difference present are magnified, thus enhancing the overall capabilities of the devised tool to detect the presence of an anomaly. If EOVs are properly removed, then any detected anomaly is likely the result of damage in said object or structure. The current version of the UDDT first calculates the Mahalanobis distance with the data from each sensor, thus merging multiple features into a single more sensitive feature per data point. Then it merges the features from each sensor into a final form that should represent a class of features that is even more sensitive to the presence of damage in a track.

Feature discrimination solutions classify each set of features calculated from the acquired data into a healthy or damaged classification. They can be divided into supervised and unsupervised learning algorithms ^[15]. In situations where training data is available from both undamaged and damaged states of a structure, supervised learning algorithms can be used since they usually provide more reliable results. But the inherent need to have data on all the possible states of the structure that are meant to be identified (i.e., both damaged and undamaged states), which can be difficult or even almost impossible to acquire in many situations, usually prevents the application of these solutions to several real cases. These situations are where unsupervised learning algorithms are preferred, since they do not require training data. This aspect justifies the choice of selecting a UDDT for the present application. Due to its simplicity and effectiveness, outlier analysis was the chosen method for the feature discrimination step in the current implementation of the UDDT. It consists of fitting a probability distribution to the baseline condition data of the structure under assessment and then testing whether the new data complies with that distribution. The cases where it does not comply are automatically classified as a damage situation. The current version of the UDDT uses the Gaussian distribution in this step, due to its adequacy for application in most cases related with reality ^[15].

The overall workflow of application of the proposed UDDT to a new track is to first define a baseline profile for the track, which can be set as its current condition defined upon a first few batches of acquired data. Then future monitoring data can be processed and compared against the baseline-related features to check for the appearance of damage on a specific point along the track, or potentially check for the deterioration of a previously detected damaged track section. As such, although this methodology requires a set of baseline features to provide the damage identification results, it is still considered as an unsupervised method. This is so because the data required to build the baseline just considers the state condition of the track infrastructure at the start of the monitoring campaign, which does not necessarily correspond to an undamaged state. There is also no need to have data regarding the specific damage cases that are meant to be identified by it.

3 NUMERICAL MODEL OF THE VEHICLE-TRACK SYSTEM

This section presents the Simpack® model (Figure 2) that was developed to assess the capabilities of the current version of the UDDT. The main elements of this model were a generic railway passenger carriage, with two bogies, and four sets of elements to represent the presence of the railway infrastructure below each vehicle axle. Simpack® is a multi-body simulation software that has been used to validate the proposed TMM, mainly due to how adequate this type of modelling software is to simulate case studies with a focus on performing modal analysis and because of their overall better computational efficiency when there is the need to run simulations with significantly large models. In previous tests using this software,

simulations of a vehicle running over a 2000 m stretch of railway track only took 15 to 20 minutes to simulate, while a similar simulation could require several hours or even days to run in a Finite Elements modelling software. The main objective of this model was to assess the UDDT by synthesizing the data required for a proper implementation of the proposed tool. An initial batch of simulations with a track in adequate support conditions were performed, with several EOVC conditions, to define the baseline. Then additional simulations batches were run on the base Simpack® model, each with different damaged track cases introduced on a specific track section. The goal was to test if the UDDT could reliably identify the baseline cases as undamaged (i.e., negative damage classification) and detect the presence of damage in the latter cases (i.e., positive damage classification).

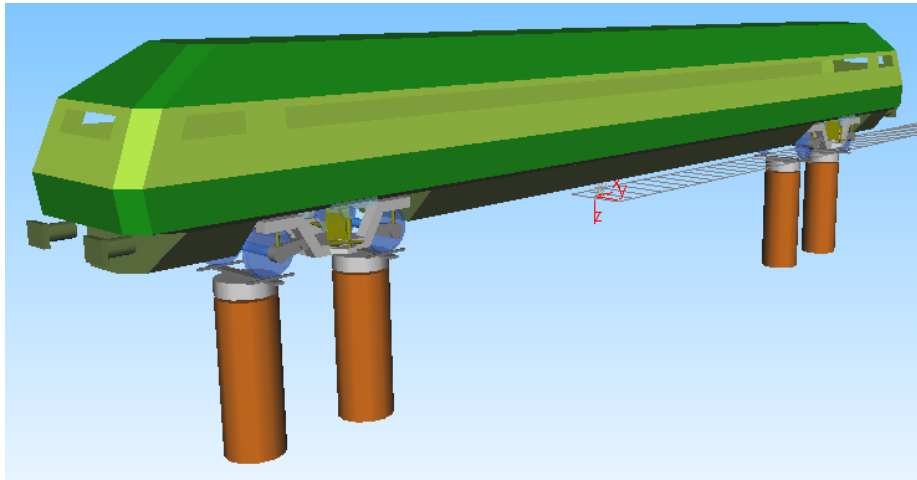


Figure 2: Simpack® model containing a railway passenger carriage and the railway infrastructure (track and the cylindrical elements sets)

3.1 Base model

The generic vehicle model included all the components relevant for these tests, namely: springs and dampers for the two levels of suspension systems, and mass elements to represent both the sprung and unsprung mass components of the vehicle. Table 1 presents the nominal values of the main modal parameters of the vehicle model. These parameter values are in accordance with a common passenger carriage model used in Portuguese lines (carriage BNN from the Alfa Pendular).

Table 1: Main parameters used in the Simpack® vehicle model

Vehicle parameters	Value
Car body mass, with no passengers on-board ($m_{v.c}$)	35,640 kg
Bogie mass ($m_{v.b}$)	5204 kg
Axle mass ($m_{v.a}$)	1538 kg
Primary suspension stiffness, per axle ($k_{v.s1}$)	0.564 kN/mm
Secondary suspension stiffness, per bogie ($k_{v.s2}$)	0.256 kN/mm
Primary suspension damping, per axle ($C_{v.s1}$)	18,000 Ns/m
Secondary suspension damping, per bogie ($C_{v.s2}$)	35,000 Ns/m

The railway infrastructure was described by four equal sets of body elements, each including two rail elements, one sleeper, and two cylinders stacked on top of each other. These cylinders represented a ballast layer (grey cylinder) and a subgrade layer (brown cylinder). This part of the model also contained spring-damper elements to simulate the equivalent vertical stiffness behaviour from each component, including the vertical stiffness of the subgrade layer that is the main point of interest in this analysis. During the simulations, these body elements sets follow their corresponding vehicle axle as the vehicle moves along the cartographic profile of the track. Table 2 presents the values used for the modal parameters of the infrastructure elements sets. The implemented modal stiffness and damping values are within the respective ranges used on consulted bibliography, as are the specific mass values used to calculate the mass for each layered element of the infrastructure ^[16].

Table 2: Main parameters used in the Simpack® infrastructure model

Infrastructure Parameters	Value
Layered elements diameter (θ)	1.070 m
Subgrade layer height (h_{su})	3 m
Ballast layer height (h_{ba})	0.3 m
Subgrade specific mass (ρ_{su})	2000 kg/m ³
Ballast specific mass (ρ_{ba})	1500 kg/m ³
Subgrade layer mass ($m_{i,su}$)	5152.9 kg
Ballast layer mass ($m_{i,ba}$)	386.5 kg
Sleeper mass ($m_{i,sl}$)	330 kg
Sleeper spacing (L_{sl})	0.5 m
Subgrade layer spring element stiffness ($k_{i,su}$)	225 kN/mm
Ballast layer spring element stiffness ($k_{i,ba}$)	450 kN/mm
Sleeper spring element stiffness ($k_{i,sl}$)	1800 kN/mm
Layered elements damping ($C_{i,l}$)	37,600 Ns/m

3.2 Simulation setup

Having created the base model for the UDDT assessment simulations, there was the need to define the baseline case (BC) and the damaged cases (DC). Do to this, a specific set of parameters were selected and tuned to create significantly different simulation conditions to provide the data for these cases. Naturally, the main parameter of interest was the vertical track stiffness of the subgrade element ($k_{i,su}$). This was the parameter used to primarily define the proper track conditions for the BC, and the damaged scenarios for the DC. Then some additional parameters were selected to imposed changes in the EOVS, to synthesise a wider range of realistic condition, thus forging more robust and statically significant BC and DC. The three parameters selected were: vehicle running speed, vehicle sprung mass and the track irregularities profile. All of the values implemented in the aforementioned parameter changes were either directly from experimental data (the three EOVS related parameters) or at least heavily based on such data (the implemented subgrade track stiffness variation profiles).

The data for the three EOVS related parameters came from different sources, all related with

experimental campaigns conducted on Portuguese tracks. The running speed profiles used in these simulations were all related with a base speed profile obtained from an Alfa Pendular train, with just overall amplitude variations to create significantly different EOV conditions. The vehicle sprung mass values used were all based on different average passenger occupation levels of an BNN Alfa Pendular carriage. Like the running speed profiles, the track irregularities profiles used were developed from a base profile collected on a Portuguese track, with just an amplitude change to simulate progressively worse track irregularity conditions. The more severe profiles went slightly over the warning level values indicated by the norm EN13848-5^[17], to create simulations where superstructure conditions did not represent a completely adequate track. These would also be used in the BC so that the UDDT could learn that these types of damage were to be discarded, since they are not the focus of this tool.

As for the subgrade track stiffness variation profiles, this data was collected in an experimental campaign using the *portancemetre* method^[5]. This method can measure local vertical stiffness values at surface level of a track or road being assessed. These values were then slightly adjusted to be within a specific range of admissible track stiffness values based on consulted bibliography. While there is no consensus on this topic, there are rough stiffness ranges mentioned by several authors as requirements for proper support conditions on a track^[18]. The admissible stiffness range considered here was between 100 and 180 kN/mm, per equivalent axle of a railway vehicle. These values were then converted into equivalent subgrade track stiffness values based on an intrinsic knowledge of the developed numerical model, where the overall stiffness of the modelled track was decomposed on the multiple intervening stiffness components, one of which is the stiffness of the subgrade layered element. For subgrade track stiffness, the equivalent admissible stiffness range was from 150 to 375 kN/mm. This base stiffness profile, converted from the experimental data, was used for the BC. Then synthesized damage situations were added to the base profile to create the different DC. The main logic behind using this procedure to create these track stiffness profiles was to at least have a fairly realist behaviour of the stiffness changes along the track, since direct subgrade track stiffness profiles is something that is not currently monitored by currently employed methods.

Combinations of the several profiles designed for the aforementioned parameters resulted in the simulations batches that were run for the BC and the DC. More specifically, as shows in the schematic presented in Figure 3, these combinations resulted in 100 different simulations for the BC and 144 simulations for the DC (18 simulations for 8 DC). The BC simulations all shared the same base subgrade track stiffness profile, and each batch of the 8 DC shared their own version of a modified subgrade track stiffness profile.

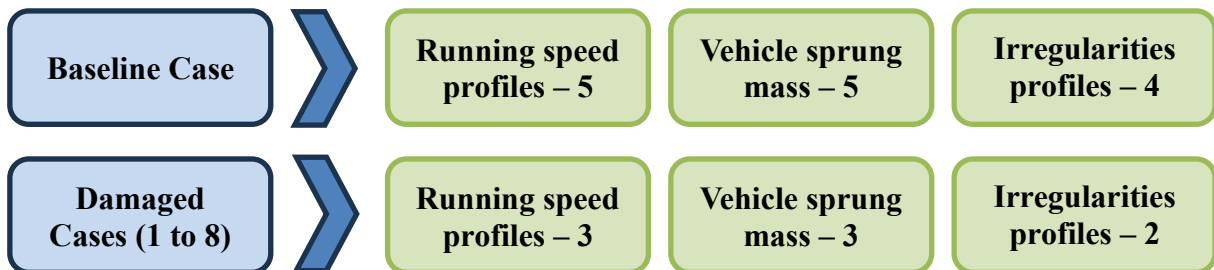


Figure 3: Parameter configurations used to generate the Baseline and the Damage Cases (types and number of profiles used for the EoV parameters in each configuration)

Since the nominal track stiffness profile was designed to be within the aforementioned admissible range, the profiles used for the 8 DC were created so that the imposed subgrade stiffness changes in the damaged sections actually went outside the admissible range. Figure 4 shows the stiffness profiles used in the BC (blue curve) and DC. The design logic for the DC was to have stiffness variations where the damage or anomaly caused an increase in stiffness above the admissible range (cases 1 to 4), and other were the damage caused a decrease in stiffness below the admissible range (cases 5 to 8). There was also the intent of analyzing if the UDDT would provide different results if the stiffness changes from the damage occurred more abruptly (cases 2, 4, 6, 8) or more gradually (cases 1, 3, 5, 7). Finally, in each combination of the two previously mentioned aspects, two intensity level of damage were defined, where the first one represented damage just over the admissible stiffness limit and the second corresponded to a more severe damage situation. All of these DC were created just by manipulating the base track stiffness profile in the same track section, by increasing or decreasing the local stiffness values to maintain the realistic variability of the base profile.

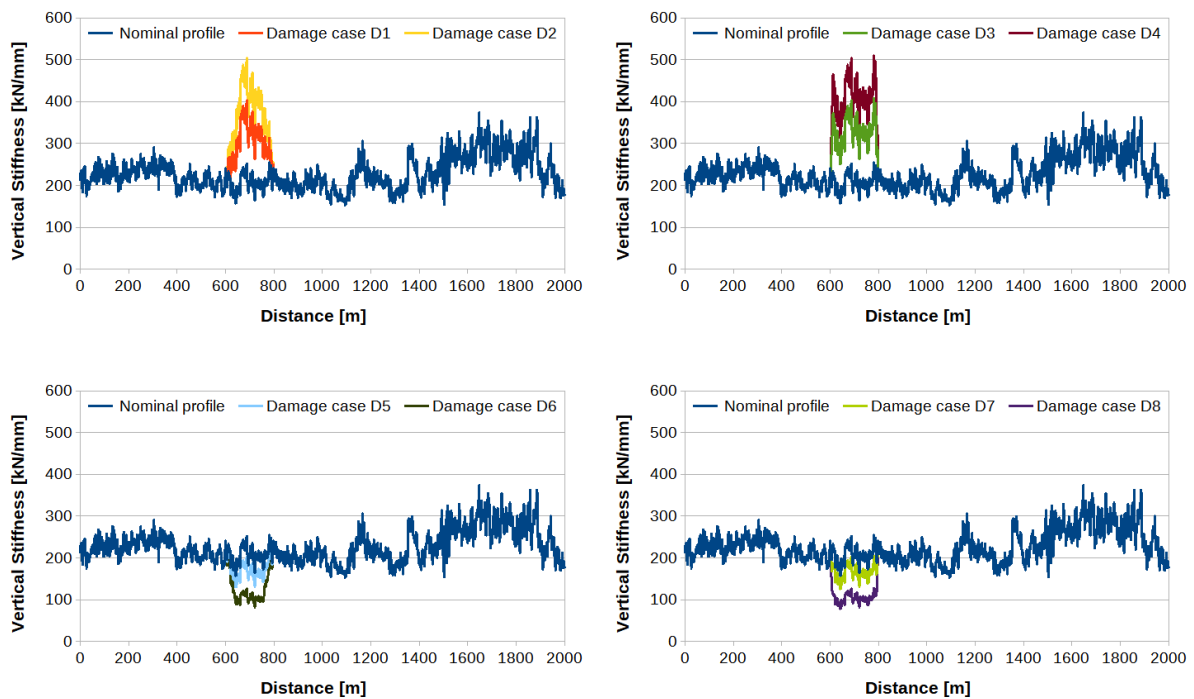


Figure 4: Subgrade stiffness profiles used in the BC and DC simulations

The cartographic profile used in each simulation was a straight track with a length of 2000 m, since this aspect does not interfere with the frequency content of interest ^[14]. Each simulation was conducted using a data sampling frequency of 1000 Hz. Nine virtual accelerometers were placed in several points along the vehicle (three on the chassis, two on the bogie and one in each wheelset) to provide the data that was used to test the UDDT.

4 RESULTS

Having acquired the data from the 244 previously described simulations (i.e., BC and DC),

the UDDT was applied to it in bulk. The routine just as to be informed of which data it as to consider as BC and which is meant to be analysed as possible DC. Thus, for simplicity and better graphical interpretability, in the following graphs the first 100 simulation correspond to the BC and the remaining 144 to the DC (Damage cases 1 through 8, in sequential order).

Figure 5 presents the obtained results from a partial application of the UDDT where the data fusion step as just been applied to the data collected from each individual sensor (chassis sensors, then bogie and then wheelset). This colour map is just a compact way to graphically represent the obtained results to check upon the accuracy of the UDDT in the damage identification task. Each cell in the colour map that is coloured in green represents a negative damage identification and a yellow cell represents a positive damage identification, resulting from the application of the Outlier Analysis step to the results from each sensor with a confidence boundary value of 0.99. These results provide a solid base for the potential merits of the UDDT since the BC were all correctly classified as undamaged cases and only a few DC were incorrectly classified as undamaged (i.e., only six false negative results in total).

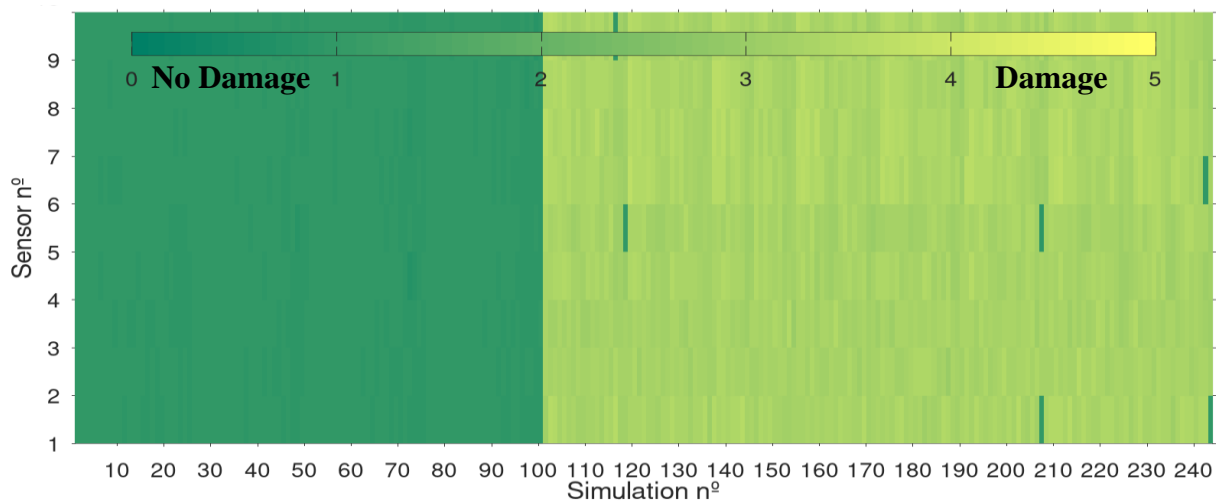


Figure 5: Damage identification results from each sensor used in the simulated railway vehicle

Following the previous results, Figure 6 presents the final output of the UDDT from the data collected in the aforementioned 244 simulations, having merged the features from each sensor into their final form. The horizontal line represents the threshold value given by the Outlier Analysis for these results, thus any point below the line represents a negative damage classification and a point above the line represents a positive damage classification by the UDDT. Here the results were even better than those shown in Figure 5, since there are no false negative results and only 2 false positive results in the BC. This represented an overall accuracy of 99.18 % in the damage identification task. While these results are just from a single case study with simulated data, they still demonstrate the significant potential of the developed UDDT in solving the complex issue of correctly identifying cases where there is a damage on a subgrade layer only based on acceleration data collected with a moving instrumented railway vehicle. Thus, it also represents a very promising solution for a proper implementation of the proposed TMM since it potentially solves the aforementioned difficulties of extracting reliable results from realistic data.

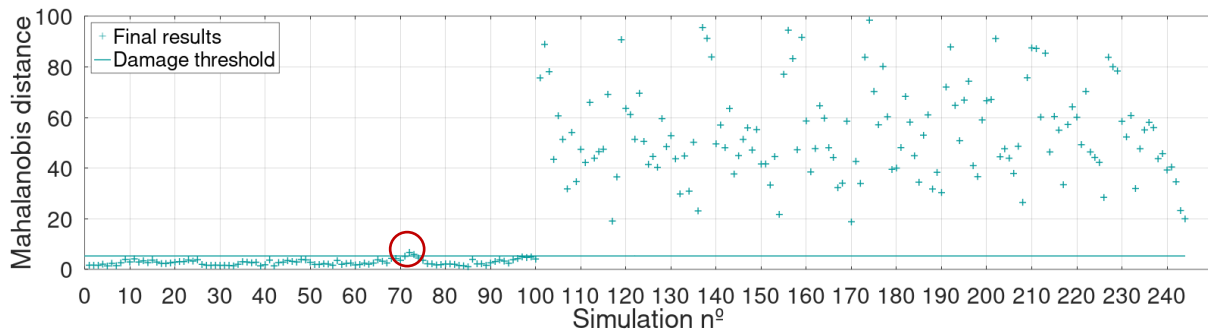


Figure 6: Final damage identification results, after combining the features obtained from every sensor. The red circle indicates the only false identifications obtained from the analyzed data

4 CONCLUSIONS

A novel methodology to assess railway track support conditions is currently under validation. This methodology states that by monitoring the natural frequencies of the subgrade layers, it's possible to obtain usable data on the overall support conditions of a track. But recent results demonstrated that the methodology was not always reliable when working with data from realistic scenarios. Thus, a new UDDT was developed to solve this issue.

This paper presents a brief description of the proposed TTM and on the UDDT, followed by a description of a numerical model used to perform a preliminary assessment test on the developed tool. The scenarios presented here, that used typical values for the modal parameters of both the vehicle and the infrastructure elements, demonstrated that the UDDT can indeed solve the previously mentioned issue and provide accurate results in the damage identification task with simulated data. Something that the current version of the UDDT does not provide is a way to classify the identified DC depending on their severity. This will be the topic for future work on the development of this tool.

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