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SEQUENTIAL SENSITIVITY ANALYSES ON MASONRY MECHANICAL PARAMETERS IN THE SUBSTRUCTURE ANALYSIS OF A MONUMENTAL CONSTRUCTION

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Summary. Assessing the safety of ancient structures is particularly challenging primarily due to the difficulty of thoroughly understanding construction phases and material properties in contexts where destructive testing must be avoided. The inherent uncertainties in material parameters can generally be reduced through updating procedures, which have recently gained ground in structural engineering. In this paper, a Bayesian probabilistic approach is adopted, using existing crack patterns to deduce masonry mechanical properties and evaluate the plausibility of different construction sequences. The case study is the Baptistery of San Giovanni in Pisa, whose construction phases are mostly unknown, as masonry mechanical properties. Initially, sensitivity analyses on the uncertain masonry mechanical characteristics were performed through nonlinear models simulating possible past stages of the building. In this context, response surfaces generated through the Polynomial Chaos Expansion technique were used. Subsequently, the calibration of stiffness and strength parameters leveraging Bayesian techniques based on the observed crack patterns on the dome allowed a first validation of hypotheses regarding the construction phases. This article demonstrates how Bayesian updating techniques, coupled with sensitivity analyses, can contribute to understanding ancient structures and developing awareness of their static behavior.

1 INTRODUCTION

The structural analysis and diagnosis of historic constructions present a significant challenge primarily arising from the need to model complex geometries, uncertainties in materials, limited understanding of history-related phenomena and construction methods, and inherent complexity of capturing the nonlinear behavior of masonry. In particular, identifying the construction sequence with intermediate stages where the structure might have been in precarious equilibrium conditions is crucial to explain the origin of current deformation and crack patterns.

Finite element (FE) modeling has become a common practice for assessing the structural health of historic constructions. However, due to uncertainties in geometrical dimensions, material properties, and boundary conditions, numerical models may fail to predict the static behavior of these structures accurately. Sensitivity Analyses (SAs) are crucial to understand the impact of limited material knowledge [1] and in deciding whether to conduct in-situ tests or select parameters more easily refined through updating procedures based on measurements of the structural response. Such procedures can be either deterministic or probabilistic [2]. The latter has recently gained attention in structural engineering [3], proving particularly promising compared to deterministic methods. Traditional methods involve assigning probability density functions (*pdfs*) to uncertain input parameters and applying sampling techniques, such as Monte Carlo Markov Chain [4], to solve the inverse problem. Using metamodels, such as general Polynomial Chaos Expansion (gPCE)-based response surfaces, which analytically approximate the response of the original numerical model, highly expedites the computations and the problem-solving [1]. Proxy models are thus beneficial especially for monumental structures, characterized by complex FE models with nonlinear masonry material properties, facilitating the updating of prior distributions for inputs based on real data [5].

In the literature, only a few papers model historical constructions with the effects of history-related phenomena and material uncertainties [6,7]. Anyway, to the best of the authors' knowledge, there are no cases in which material uncertainties are included in modeling historical structures while also accounting for the building process in a probabilistic framework.

This paper adopts a probabilistic approach incorporating material uncertainty to evaluate the plausibility of potential intermediate historical configurations of a monumental construction. The focus is on the Baptistery of San Giovanni in Pisa (Fig. 1), where the construction phases and masonry mechanical properties are mostly unknown. The Baptistery is a cylindrical masonry structure covered by two nested domes. Understanding the construction sequence of the monument is crucial for assessing its safety and the reason for the current deformation and crack patterns. Attention is given to the mechanical properties of the arched drum supporting the inner dome due to the large uncertainty regarding construction material and the key role it plays in transmitting the dome's thrust to the underlying pillars and external walls.

The paper presents an original approach using SAs to study the model response in terms of crack length at the edges of the dome, thereby facilitating the understanding of the relationship between the most likely values of the drum's properties for crack development and potential past phases. An initial evaluation allowed the reduction of the problem's size by identifying the parameters that do not influence the monitored quantity. A subsequent SA based on the calculation of Sobol' indices (SIs) was performed with *pdfs* suitably modified to ensure the consistency between model damage and the actual crack patterns. Then, input parameters were calibrated through Bayesian inference. Uncertainty quantification (UQ) was carried out by reconstructing the gPCE-based analytical surface. Subsequently, it was assessed whether the updated values of the drum's mechanical properties were practically realistic in this context. In this sense, UQ was crucial to evaluate the existence of the past phases represented by the analyzed substructures. This paves the way for further staged construction analyses of the entire structure, thus enhancing the understanding of the global current conditions of the monument.

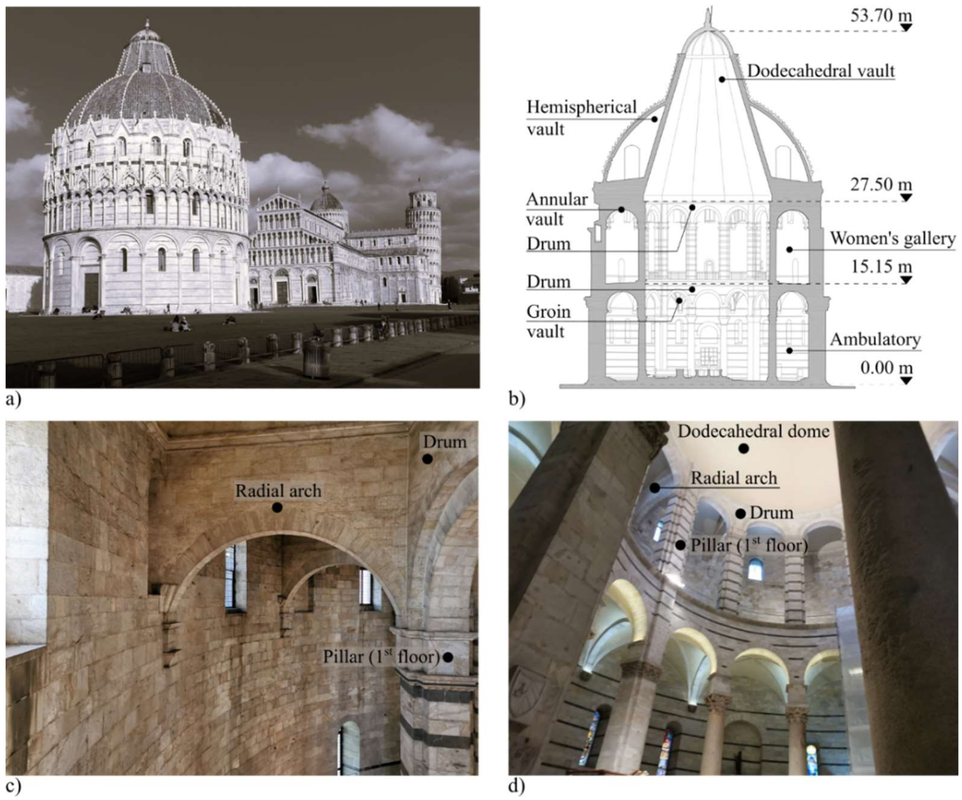


Figure 1: a) San Giovanni Baptistery within the Cathedral Square in Pisa; b) Vertical section of the Baptistery; c), d) Internal views.

2 CASE STUDY

The Baptistery of San Giovanni, located in the Cathedral Square UNESCO site in Pisa, Italy, stands as a remarkable example of medieval architecture. It is an integral part of the cathedral complex, which also includes the renowned Leaning Tower and the Cathedral. The Baptistery is a masonry structure notable for its unique blend of Romanesque and Gothic architectural styles, reflecting the evolution of design during its construction period, and for its dimensions and peculiar covering system. Indeed, its circular plan with an external diameter of 35.4 m and a height of approximately 54 m make it the largest baptistery in Italy, and one of the biggest in the world. The external circular walls, made of San Giuliano marble, have a thickness of around 2.60 m. Internally, the monument features a circular colonnade made of 8 granite columns and 4 stone masonry pillars, thus defining an ambulatory covered by groin vaults in the resulting space between the external walls and the 12 vertical elements arranged circularly. At the upper level, the internal colonnade, here made of 12 pillars, is replicated, and the area between it and the circular walls defines the women’s gallery with a toroidal vault covering it. Two non-intersecting helical stairways cut out of the width of the stone wall connect the two levels. The structure is topped with a unique covering system made of two peculiar nesting domes: the internal one has a pyramidal trunk geometry with a dodecahedral base of approximately 20 m wide and is supported by the colonnade; the external one has a hemispherical shape and rests on the external walls, other than, partially, on the internal dome. At the level of the women's gallery, 12 radial masonry arches link the drum of the prismatic dome to the external walls, thereby contributing to counteracting its horizontal thrust. The construction of the Baptistery began in 1152 in a Romanesque style under the direction of the architect Diotisalvi, whose name can be found inscribed in the building. By the early XIII century, after some interruptions of the works, the lower part of the structure up to the height of the arcade, was completed. However, the project experienced a significant transformation in 1260, when Nicola Pisano, an influential sculptor and architect, took over. The external dome was completed in the XIV century after other intermediate phases in which the construction site was halted and the Baptistery might have been called to withstand precarious/ unexpected equilibrium conditions.

Currently, the crack pattern affects the edges of the inner dome and the intrados surface of the groin vaults covering the ambulatory. The deformation pattern, in turn, affects the pillars of the women's gallery, which exhibit an outward tilt, deviating from plumb. The causes of the current crack and deformation patterns, which could lie both in the inherent mechanical properties of the employed building materials, and construction sequence, should be better clarified to give reason to the current situation. Notably, the process leading to the completion of the monument with the second-floor drum, the radial arches connecting it with the external walls, and the covering system is thought to be a likely part of the cause(s) yielding to the occurrence of cracks at the edges of the internal dodecahedral structure.

On this ground, the work aims to delve into the equilibrium conditions of possible past intermediate stages involving the realization of the internal dome and radial arches.

3 MATERIAL AND METHODS

3.1 Substructure's FE model

The first investigated FE model is representative of a potential past configuration of the Baptistery, particularly, it attempts to reproduce the monument right before its completion by hypothesizing the internal dome to be constructed before the radial arches. According to this assumption, the monument could have been realized up to the second floor (external walls and internal colonnade), including the internal dome, without the radial arches.

To justify the adoption of the substructure's FE model, thus reducing the computational burden of the numerical model of the construction, some preliminary evaluations were performed between the displacements of the top of the second-floor pillars within the model of the whole Baptistery (Fig. 2a) and those in the substructure (Fig.2b), under the self-weight load, to check their consistency. Particularly, it was verified that the displacements above in the substructure do not differ more than a tenth of a millimeter from those of the model of the entire structure when the boundary conditions of the substructure included a fixed base for pillars. Successively the model was imported into Comsol Multiphysics and a computational mesh with a total of roughly 190000 10-node serendipity solid tetrahedral elements was created (Fig. 2b), having minimum and maximum edge lengths of about 0.4 m and 0.8 m respectively.

The material assignment took advantage of the work of Lezzerini et al. [7] that identified the stone of the buildings in Cathedral Square based on their macroscopic properties. Accordingly, pillars were considered as made of marble, and the internal dome as constituted of brick masonry. Mechanical properties of such materials were defined considering the experimental results of the in-situ tests performed on the Tower's masonry [8], in that analogous materials of different buildings of Cathedral Square were found to be extracted from the same quarries.

As for the arched drum (Fig. 2b), it was taken as made of an uncertain material because the one constituting it is not clearly identifiable due to an annular vault partially covering it.

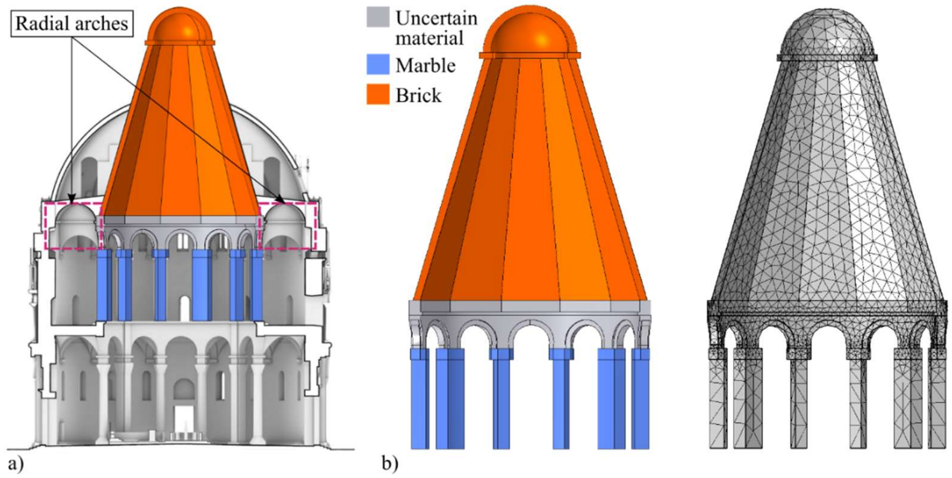


Figure 2: FE model of the Baptistery: a) entire construction; b) Substructure (mesh and material assignment).

The uncertain drum's mechanical properties to be calibrated in the study are reported in the following subsection (see Table 2), while the deterministic characteristics of marble and brick are summarized in Table 1.

Table 1. Deterministic parameters.

Material	f_m [MPa]	f_t [MPa]	E_m [MPa]	ν	ρ [kg/m^3]
Marble	6	0.5	50000	0.36	2600
Brick	2.6	0.05	1200	0.20	1800

As regards the mechanical law, the non-linear, isotropic, strain-based Mazars' material was employed [9]. Here damage is linked to the history of total positive strains through a state variable *d*. The latter tracks the increasing damage by modifying the original stiffness tensor Λ_0 with a reduced one $\Lambda(d)$ according to the following equation where *d=0* for the undamaged material and *d=1* for the completely damaged one.

$$\Lambda(d) = (1-d)\Lambda_0 \tag{1}$$

The rationale behind Mazars' material is such as that positive strains in every point of the model, which are due to tension and, indirectly, compression, can cause cracks when exceeding a certain threshold given by

$$k_0 = \frac{f_t}{E_0} \tag{2}$$

where f_t is the tensile resistance and E_0 the initial (undamaged) modulus.

Damage is irreversible, which means that once it is initialized, i.e. positive strains in a point exceed the threshold, damage increases according to the equation

$$d = \alpha_t d_t + \alpha_c d_c \tag{3}$$

where α_t, $\alpha_c \in [0, 1]$ are the combination coefficients

$$\alpha_t = \sum_{i=1}^{3} \frac{\left\langle \varepsilon_i^{(t)} \right\rangle \langle \varepsilon_i \rangle}{\tilde{\varepsilon}^2}, \qquad \alpha_c = \sum_{i=1}^{3} \frac{\left\langle \varepsilon_i^{(c)} \right\rangle \langle \varepsilon_i \rangle}{\tilde{\varepsilon}^2} \tag{4}$$

with ε_i the principal strain in each point, along direction i; $\varepsilon_i^{(t)}$ and $\varepsilon_i^{(c)}$ the eigenvalues of strain tensors accounting for either their tensile or compressive components respectively. Macaulay brackets express the ramp function. $\tilde{\varepsilon}$ is the equivalent strain relative to the local principal strain ε_i, which corresponds to the current value of the state variable k ($k = \max(\tilde{\varepsilon}, k_0)$) in a point. In particular, $\tilde{\varepsilon}$ reads as follows

$$\tilde{\varepsilon} = \sqrt{\textstyle\sum_i \langle \varepsilon_i \rangle^2} \tag{5}$$

Then, damage components d_t and d_c in (3) are calculated with the following expressions

$$d_t(k) = 1 - \frac{k_0(1-A_t)}{k} - A_t e^{-B_t(k-k_0)} \tag{6}$$

$$d_c(k) = 1 - \frac{k_0(1-A_c)}{k} - A_c e^{-B_c(k-k_0)} \tag{7}$$

Scalar parameters A_t, B_t, A_c, B_c can be experimentally gathered through uniaxial tests on the material (or equivalent simulations). The ideal behavior of a specimen subjected to uniaxial tension-compression tests is illustrated in Fig. 3.

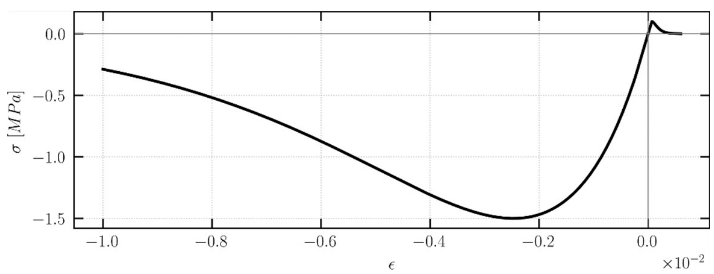


Figure 3: Uniaxial stress-strain curve

3.2 Probabilistic framework

A probabilistic framework was adopted to evaluate the plausibility of the hypothesized past configuration. More pointedly, the setting was instrumental in assessing whether a past stage might have existed, also in light of the required mechanical properties of the involved material with respect to the currently attainable ones before reaching the maximum strength of a specific collapse mode. Thus, the approach can facilitate the mutual understanding between the most likely values of some uncertain parameters and the plausibility of a past stage, by comparison with reality. In this case, it is assumed that cracks at the internal edges of the dodecahedral dome arose in the past intermediate stage configuration represented by the analyzed substructure. The study goal is to find the drum's mechanical parameters causing a damage pattern consistent with the observed one.

The procedure followed to pursue the aim of the work builds upon (i) the assignment of *pdfs* to the drum's uncertain mechanical properties; (ii) the generation of an analytical model replacing the response of the FE one in terms of crack length at the dome's edges; (iii) performing global SA; (iv) the FE model calibration through Bayesian inference based on real data (length of cracks at the dome's edges). In the FE model, cracks are represented by elements with Young's modulus reduced by 80% or more compared to the original value.

Prior distributions of drum's material properties. Based on expert judgment and available data, compression strength f_m and Young's modulus E_m were appointed uniform *pdfs* (see Table 2). The upper and lower values of the distributions were taken from those provided in [10] with reference to the category "*squared stone blocks masonry*". More pointedly, the upper limit was increased by 1.2 to account for the good quality of mortar. The tensile strength f_t was taken as a percentage of f_m, namely around one tenth of f_m for monolithic elements (i.e. marble columns), and one fiftieth of f_m for stone masonry, as already assumed in other papers [11].

Table 2. Statistical data of the drum's uncertain parameters.

Parameter	Distribution	Interval
E [MPa]	uniform	[2500 5000]
f_m [MPa]	uniform	[6.2 10]

Surrogate model and sensitivity analysis. An analytical response surface surrogating the response of the FE model of the substructure in terms of the occurrence of cracks at the edges of the internal dome was created through a fourth-grade gPCE-based approximation. The number of uncertain parameters and other surrogation choices, i.e. the adoption of the pseudo-spectral projection with a full tensor grid as the quadrature rule to compute the expansion coefficients [1], necessitated 25 deterministic evaluations of the FE model in the quadrature points. Successively, the expansion coefficients were used to compute the first-order SIs, which helped in understanding the effect of the variation of the single inputs on the output by decomposing the output variance as a sum of contributions of each input [1].

Model calibration through Bayesian updating. Bayes theorem was leveraged to calibrate model input parameters thanks to the availability of measurements, e.g. the length of cracks at the dodecahedral surveyed at the dome's edges. We considered here its mean value, which is equal to 3.90 m. Lacking a closed-form solution of the input posteriors, the Markov Chain Monte Carlo method was adopted for the updating [12].

4 RESULTS AND DISCUSSION

The analysis performed with the substructure described in Sec. 3.1. (for the combinations of uncertain parameters sampled from their *pdfs*) reports similar damage patterns at the edges of the internal dome, with some difference only in the extension of damaged areas. Fig. 4a illustrates the pattern, which is, overall, consistent with the surveyed cracks at the dome's edges. In addition, Fig. 4b shows the response surface in terms of crack length underlining that f_m determines the greater variation in the crack length, while E_m yields to a slighter change. These results are in line with the SA in that the first-order SIs reported in Figure 4c emphasize the relevance of the compressive strength f_m with a SI of 0.97, in contrast to the low influence of E_m having a SI equal to 0.03.

As a further step of the probabilistic setting, model calibration through Bayesian updating was carried out, which gave the results in Fig. 5 in terms of posterior *pdfs*. The updating led to a significant reduction of the uncertainty in the inputs: the posterior of E_m has a peak around a value of 2600 MPa, while the peak of f_m is shifted towards the upper limit of the investigated interval, close to 10 MPa. The latter corresponds to a tensile strength of 0.5 MPa, falling in the upper limit of the assumed values of such a parameter.

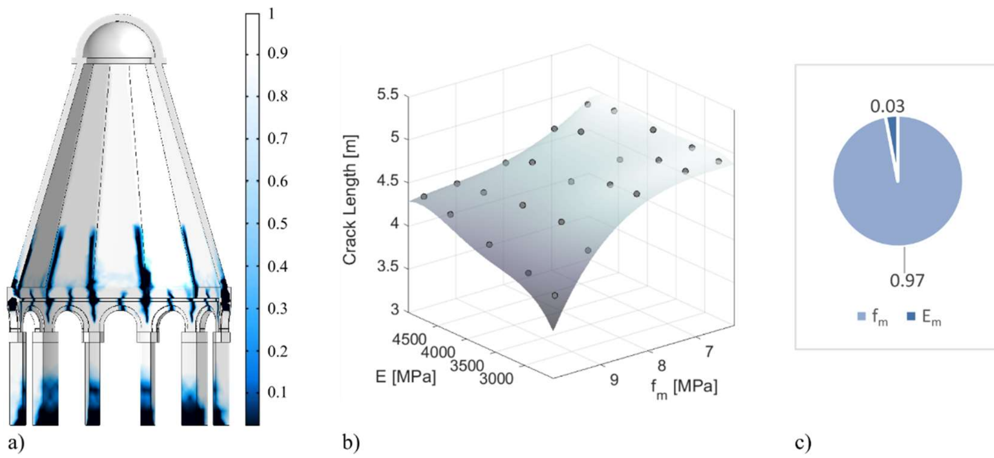


Figure 4: a) Damage pattern; b) gPCE-based surrogate model of the crack length; c) First-order SIs

That value is to be compared to the tensile strength that the structure can actually provide before a certain failure mode occurs. Thus, the evaluation of the substructure's plausibility begins with a critical discussion of the obtained results. In particular, the tensile strength of the drum in this configuration is governed by bed-joint sliding. The mechanism of interlocking and friction between the stones allows the masonry to transmit significant tension stresses, so developing crosswise tensile strength [13]. Such a strength (f_v) can be evaluated through the following equation deriving from the well-known Mohr-Coulomb failure criterion

$$f_v = f_{v0} + \mu\, \sigma_n \tag{8}$$

where:

- f_{v0} is the shear strength of masonry without normal stresses. It is 0.34 MPa for the "*squared stone block masonry*" category in [10], considering good quality mortar;
- μ is the tangent of the internal friction angle, variable for different types of masonry, but widely accepted to be equal to 0.4 for good quality existing masonry;
- σ_n is the compression stress given by the acting loads, here given by the self-weight of the dodecahedral dome and it is equal to 0.18 MPa.

The previous considerations yield a maximum sliding resistance of 0.41 MPa, which is however lower than the value that emerged from the updating as the one required to have a damage pattern consistent with the surveyed one. Therefore, the hypothesized configuration simulated through the analyzed substructure and representative of the case in which the internal dome was constructed before radial arches connecting it to the external walls seems unlikely to have existed due to the implausible high material properties that it would require. On the contrary, a different assumption entailing the simultaneous construction of the dome along with radial arches appears to be more likely and should be investigated.

Thus, starting from this gained knowledge, a FE model representative of the latter potential past intermediate stage was realized by including, other than the structural elements constituting the first model, also the radial arches and the external circular walls on the second floor to simulate the simultaneous realization of the dome and arches (Fig. 6). The displacements along the three directions of both the base of pillars and the wall were restrained, and a computational mesh made of 10-node serendipity solid tetrahedral elements was created in Comsol Multiphysics (Fig. 6a), having minimum and maximum edge lengths of about 0.4 m and 0.8 m.

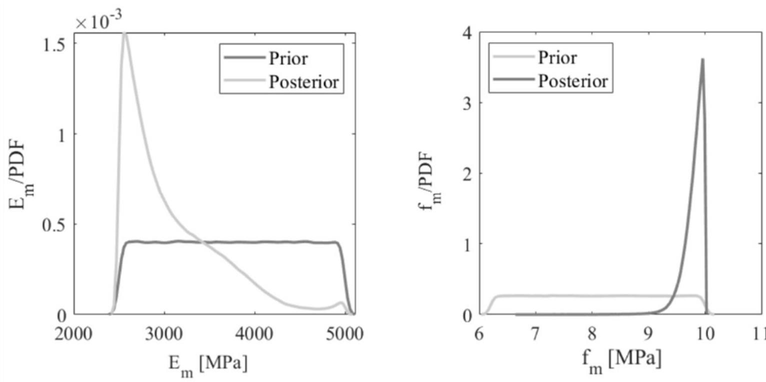


Figure 5: Updating of the uncertain input parameters.

Successively, after checking the consistency between the displacements of the top ends of the pillars of the new substructure and those obtained with the FE model of the entire Baptistery, the plausibility of the configuration was assessed by following the steps of the probabilistic procedure already adopted for the analysis of the first hypothetical past stage. More pointedly, a global SA was performed, and both direct and inverse problems were solved.
As regards material assignment, the drum is still considered as made of an uncertain material, while radial arches are in brick masonry, as the internal dome. Apart from marble pillars and brick masonry dodecahedral dome, whose mechanical characteristics were already set (Table 1), the model required the definition of the material constituting the external walls (Fig. 6b). These are made of three layers, where the two outer ones simulate marble masonry, and the internal one reproduces mixed masonry according to some past experimental tests on the Tower's masonry [8]. Table 3 states the input parameters of walls.

Table 3. Deterministic parameters.

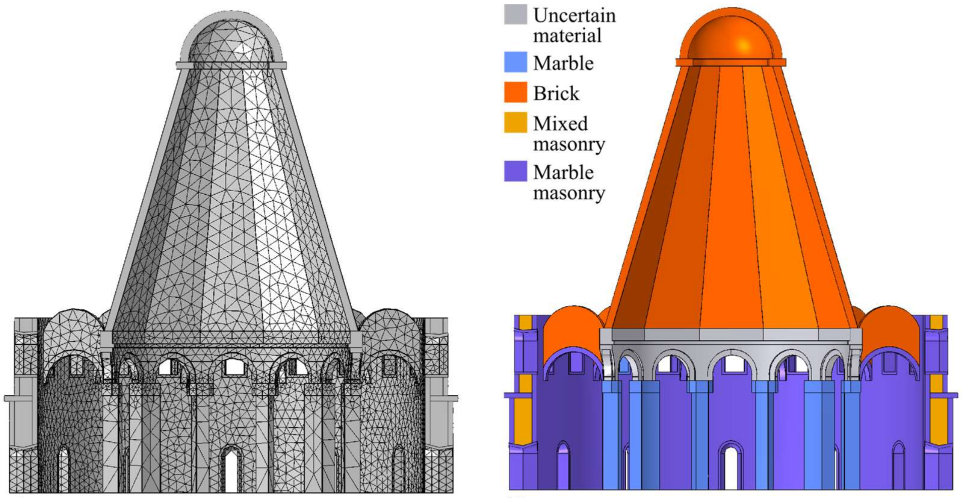
Material	f_m [MPa]	f_t [MPa]	E_m [MPa]	ν	ρ [kg/m^3]
Mixed masonry	4	0.08	7000	0.20	1800
Marble masonry	6	0.12	50000	0.20	2400

After some preliminary evaluations, the pdfs of f_m and E_m of the drum were assumed as uniform with reduced lower limits with respect to the intervals set in Table 2 for the first analyzed substructure to better reproduce the cracks at the dome's edges (Table 4).

Table 4. Statistical data of the drum's uncertain parameters.

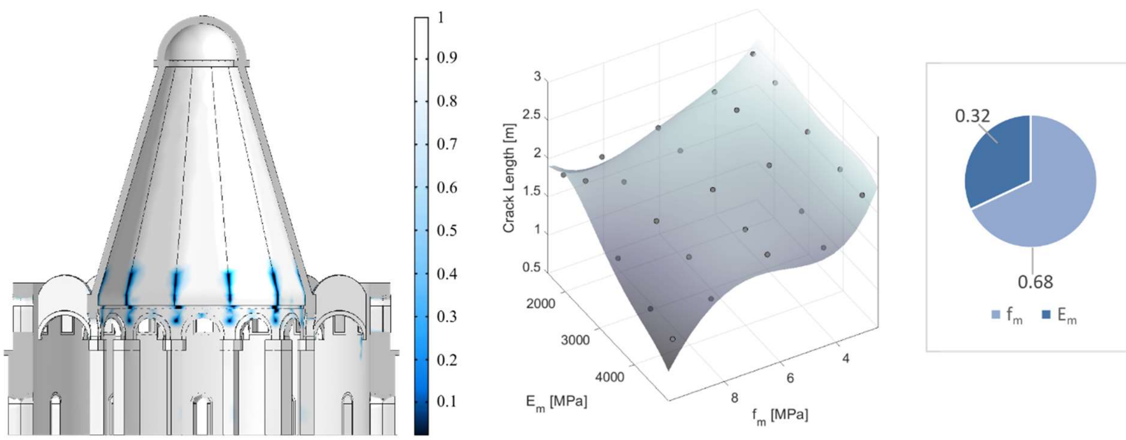
Parameter	Distribution	Interval
E [MPa]	uniform	[1500 5000]
f_m [MPa]	uniform	[2.5 10]

Then, after performing 25 deterministic calls of the FE model (quadrature points), a 4th-grade gPCE-based response surface was generated to surrogate the response of the new substructure in terms of the occurrence of cracks at the edges of the dodecahedral dome (Fig. 7b).



a) b)

Figure 6: FE model of the substructure of the Baptistery under the hypothesis of simultaneous construction of the dome and radial arches: a) mesh; b) assigned materials.



a) b) c)

Figure 7: a) Damage pattern; b) gPCE-based surrogate model of the crack length; c) First-order SIs.

The results show a crack pattern even more consistent with the observed reality, if compared to that provided by the first analyzed substructure. Indeed, damage is now mainly located at the edges of the dome, while disappearing at the keys of the drum's arches, according to surveys. The response surface and 1st –order SIs highlight the importance of both E_m and f_m (thus indirectly f_t), with the second being predominant having an SI equal to 0.68. However, the analyzed substructure does not reproduce the experimental value of the cracks' length precisely. Indeed, as highlighted by the analytical posterior of crack length (dark solid line in Fig. 9) based on the obtained posterior of inputs (Fig. 8) and confirmed by the FE model run with the updated values of the inputs, cracks take a length of roughly 2.6 m, while the experimental ones have a mean length of 3.80 m. This may mean that cracks at the dome's edges are partially due to the equilibrium conditions reached in the analyzed configuration, other than to the addition of further load corresponding to the construction of the external hemispherical dome along with the viscous effects of masonry during the almost millennial existence of the monument.

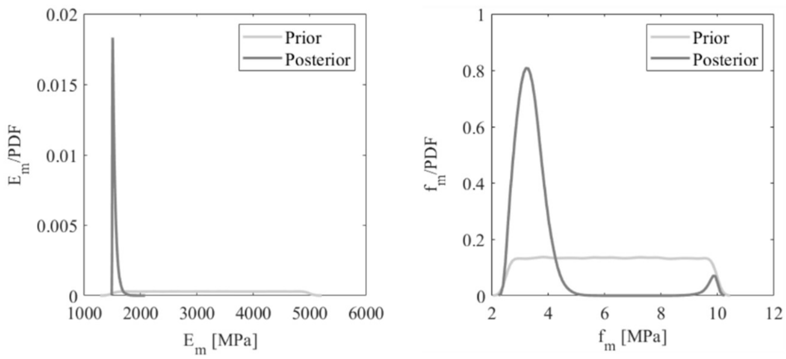


Figure 8: Updating of the uncertain input parameters.

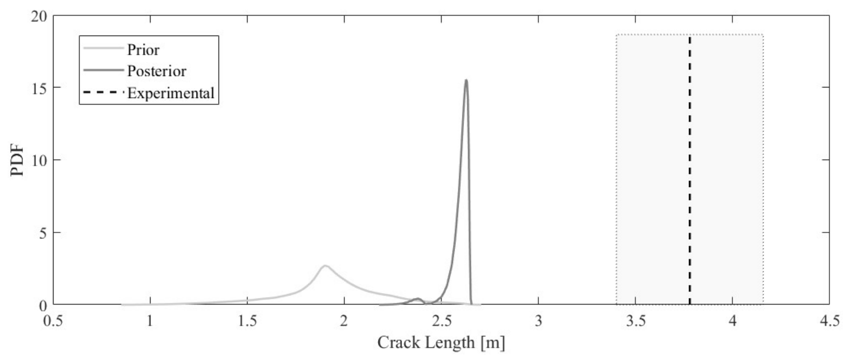


Figure 9: Prior and posterior distributions, and experimental value of the crack length.

5 CONCLUSIONS

This study encompasses SAs on the mechanical parameters of masonry within the nonlinear FE modeling of a substructure of the Baptistery of Pisa, representing a possible intermediate past configuration. The work explores a novel approach that incorporates epistemic uncertainties in a probabilistic framework to assess the plausibility of the analyzed substructure by identifying the most likely mechanical properties of masonry necessary for its equilibrium. Specifically, the application of modern probabilistic theories revealed that certain structural configurations, such as the first one examined, are highly unlikely to have existed. Although the analysis cannot account for all factors—such as masonry creep, initial settlement with increased deformability, or long-term settlement—it effectively establishes a range of values of the mechanical parameters required to ensure the equilibrium in the hypothesized configuration. Furthermore, with more reliable mechanical parameters obtained from experimental testing, these analyses could provide even greater insights into the construction process, and its impact on current stability, equilibrium, crack, and deformation patterns. In the broader context, the proposed probabilistic analysis helps determine whether past interventions occurred and offers valuable insights, especially to archaeologists, where traditional methods may be insufficient.

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