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A method to estimate the state parameter from CPTu soundings using Pocket G-PFEM

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ABSTRACT

The state parameter is the main variable employed for predicting the undrained behaviour of artificial soils such as tailings and hydraulic fills. Current practice involves using screening methods and CPTu data (e.g. Robertson (2010)) or cavity expansion based methods (e.g. Shuttle and Jefferies (2016)). However, these methods have drawbacks: they are based on empirical correlations for clean sands and do not consider the effect of partial drainage. This paper presents a site-specific procedure to determine the state parameter of tailings, inspired in the work by Monforte (2022). The procedure consists of: i) calibrating the CASM constitutive model using triaxial tests for different state parameters; ii) determining the plausible range of hydraulic conductivity based on dissipation tests; and iii) conducting numerical simulations of CPTu tests using the Pocket G-PFEM tool for different combinations of state parameters and hydraulic conductivities. By comparing the results of the simulations with the real CPTu data, a site-specific relationship between the state parameter and the CPTu measurements can be established. To validate the procedure, the method is applied to a real tailings deposit and contrasted against routine screening methods. In the particular case studied in this paper, the method predicts more contractive behaviour than the screening methods. While the method is still in an early stage of development, it looks very promising because it allows for using the raw CPTu data to calibrate a constitutive model, without resourcing to any kind of correlations or empirical transformation models.

Keywords: CPT; State Parameter; Pocket G-PFEM; CASM.

1. Introduction

Upstream-raised Tailings Storage Facilities (TSFs) depend on the strength of tailings for their stability, which can make them susceptible to flow liquefaction. Industry practice employs limit equilibrium methods for drained, peak undrained and residual undrained conditions (ANCOLD, 2019), complemented with numerical deformation analyses to evaluate the robustness of the dam and the risk of progressive failure leading to loss of containment.

The in-situ state of the tailings is generally determined by the state parameter ψ (Been and Jefferies, 1985), defined as the void ratio difference between the current state of the soil and the critical state locus (CSL) at the same mean effective stress p'. A positive ψ denotes a contractive tailings, while a negative value indicates a dilative response. Given the difficulty of obtaining undisturbed samples, several approaches have been developed to estimate ψ from CPTu soundings (e.g. Been et al. (1987); Plewes et al. (1992); Robertson (2010); Shuttle and Cunning (2007); Jefferies and Been (2016); Shuttle and Jefferies (2016)).

Being a poorly graded rock flour, the behaviour of tailings in undrained shear can change dramatically with very small changes in state parameter, well within the error of the cited screening methods, see for instance a discussion by Torres-Cruz (2021) on the uncertainty of Plewes' method. Even the CPT-Widget (Shuttle and Jefferies, 2016) employs a transformation model from cavity expansion to CPT tip, which introduces uncertainty, and forces the user to pick between drained and undrained penetration, with no allowance for partial drainage.

The robustness and reliability of deformation analyses of TSFs would be greatly improved if the raw data from CPTu tests could be directly employed to calibrate constitutive models, without going through correlations. This approach would require the simulation of the CPTu penetration, employing the same software and constitutive model that will be later employed in the deformation analyses, as the parameters yielding the closest simulation of the CPTu penetration are the most likely representative parameters for simulating the behaviour of the TSF as a boundary-value problem.

A first step towards adopting this modelling strategy is presented in this paper. The Pocket G-PFEM tool (Monforte et al., 2017) is employed to simulate CPTu penetration. Following a rationale similar to that of the CPT-Widget, Pocket G-PFEM is then employed to estimate the state parameter. A discussion of the uncertainties of this method, compared with industry-practice screening methods is provided and tentative next steps are discussed as closure.

2. Estimation of the in-situ state parameter from CPTu data

The first attempt to estimate ψ from CPTu data was proposed by Been et al. (1987) for drained penetration. Plewes et al. (1992) proposed a simplified procedure to estimate ψ while accounting for excess pore water pressure. The method was updated by Jefferies and Been (2016) and correlates the normalised sleeve friction ratio F and the slope λ_{10} of the Critical State Locus (CSL) in the $e - \log p'$ plane. Recent additions to the $\lambda_{10} - F$ database suggest that the uncertainty of the correlation is high (Reid (2015); Torres-Cruz (2021)), rendering the Plewes method unreliable for producing estimates of ψ accurate enough for numerical modelling purposes.

A more rigorous method that has gained acceptance is the one proposed by Shuttle and Cunning (2007), further developed by Shuttle and Jefferies (2016), and also described in Jefferies and Been (2016). This procedure relies on CPT results, knowledge (or estimates) of horizontal stress field, laboratory testing of disturbed samples, and numerical simulations of triaxial tests and spherical cavity expansion. This approach is commonly known as the "CPT-widget method". Unfortunately, recent investigations have shown that the CPT-widget method, when utilised in drained and undrained cavity expansion tests, leads to a large range of values for the estimated state parameter (Reid and Smith, 2021). This wide-ranging variability diminishes the practical utility of the method, as is not capable of reproducing the intermediates scenarios between drained and undrained penetration that are typical for tailings.

Ayala et al. (2023) proposed a characteristic surface for estimating ψ from partially drained CPT penetration. The surface, in the space of the initial state parameter ψ_0 - normalised penetration velocity $V = v\ d/c_v$ - normalised tip resistance $Q(1 - B_q) + 1$, considers two bounding straight lines, corresponding to drained and undrained penetration, that can be determined using the widget. For velocities in-between, each line smoothly changes its slope, ultimately coinciding with the other, creating a surface that corresponds to partially drained penetration. This surface allows for the estimation of ψ_0 from the pair V ; $Q(1 - B_q) + 1$ obtained from CPTu data.

Several research groups are working on the numerical simulation of CPTu penetration (Ayala et al. (2023); Bird et al. (2023); Boschi et al. (2023); Moshfeghi et al. (2023); Wu et al. (2023)). The tool examined in this paper, Pocket G-PFEM, has been employed for the evaluation of the effect of the roughness factor of the interface (Monforte et al., 2017), effects of brittleness, permeability and partial drainage (Monforte et al., 2021), numerical examination of the performance of empirical methods for determining both peak and residual undrained shear strengths (Monforte et al., 2022), the effect of OCR (Monforte et al., 2023a), or the determination of the state parameter from CPT under undrained conditions (Monforte et al., 2023b).

3. Proposed procedure

3.1. Brief description of Pocket G-PFEM

The Particle Finite Element Method (PFEM), introduced by Oñate et al. (2004), is a Lagrangian continuum technique ideally tailored for addressing multi-physics scenarios characterised by substantial displacements, extensive deformations, and occasional body splits and opening of contacts. PFEM achieves this by augmenting the Finite Element Method with effective re-meshing strategies.

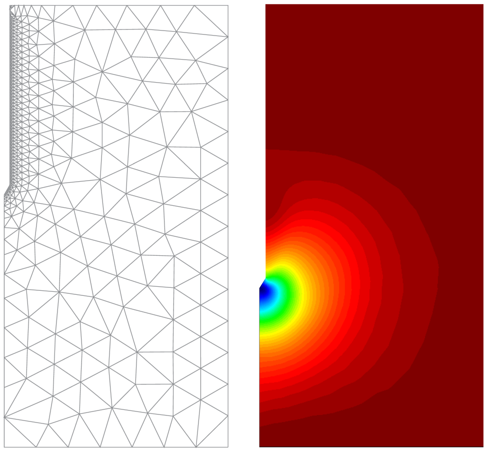

Figure 1. Pocket G-PFEM mesh at the starting position of the cone (left), and typical excess pore pressure field at the end of undrained penetration (right).

Pocket G-PFEM stands for Geotechnical Particle Finite Element Method. It is a computational framework developed by the International Centre for Numerical Methods in Engineering (CIMNE), the Universitat Politècnica de Catalunya (UPC) and TU Graz, utilised for solving large deformation problems in geotechnics (Monforte et al., 2017). Pocket G-PFEM has been integrated into Kratos Multi-physics (Dadvand et al., 2010), an object-oriented platform for multi-disciplinary numerical analysis. Currently, Pocket G-PFEM has only one critical state constitutive model, namely the critical state constitutive model Clay and Sand Model (CASM) (Yu, 1998).

A fully coupled hydro-mechanical model with a single axisymmetric mesh is employed to simulate the deformation of soil around the cone during the penetration (Fig. 1). The cone tip is initially positioned at an approximate depth of 16 cone diameters, to avoid the numerical problems that may arise at the first steps, and pushed at a standard velocity of 2 cm/s. Boundary conditions are: i) zero displacement, free flow at the bottom; ii) no radial displacement, free flow at the right boundary; and iii) constant vertical stress, free flow at the top of the mesh.

3.2. Brief description of CASM

The Clay and Sand Model (CASM) (Yu, 1998), is an isotropic, elasto-plastic critical state constitutive model designed to encompass the behaviours of both clays and sands. Departing from conventional original and modified Cam-clay models, CASM redefines these frameworks in terms of a state parameter, enabling it to capture the responses of clay and sand under various loading conditions, including both drained and undrained shear.

The yield surface is controlled by the model parameters r (spacing ratio) and n (shape parameter). CASM defines the CSL and the reference consolidation line as straight and parallel lines. While other models, such as NorSand, explicitly consider an initial state parameter as input, in CASM this can be achieved modifying parameters n and r. The CASM features a non-associated flow rule where the plastic potential was originally derived from Rowe's stress dilatancy relation (Rowe, 1962). Manica et al. (2022) proposed a plastic potential introducing the parameter m. CASM has been adapted to finite deformations (Monforte Vila, 2018), adopting the hyper-elastic model by Houlsby (1985).

3.3. Steps for computing the state parameter

This work proposes site-specific approach for determining the in-situ state parameter by inversion of CPTu data. The procedure, strongly inspired by the contributions by Monforte et al. (2023a) and Jefferies and Been (2016), involves the following stages:

i) Calibrate CASM to represent different initial state parameters ψ_0, following the typical values of field conditions, from a set of drained and undrained triaxial tests; the model should match both peak and residual shear strength ratios from the tests.

ii) Run CPTu simulations with Pocket G-PFEM for different values of ψ_0 within the range of interest, and for different dimensionless penetration velocities V, typically in the range between 0.01 and 30 (Finnie and Randolph, 1994).

iii) Use the various sets of tip resistance, sleeve friction and excess pore pressure resulting from the simulations to produce a cloud of points in the ψ_0 - V - $Q(1-B_q)+1$ space and parameterise the characteristic surface.

iv) Calculate V along the CPTu soundings using the dissipation test data.

v) With V and $Q(1-B_q)+1$, calculated from the raw data, and using the parameterization of the characteristic surface, compute the state parameter at each data point within the CPTu sounding.

4. Validation of the proposed procedure

4.1. Calibration of CASM with lab tests

Triaxial tests of a real TSF have been interpreted and calibrated using CASM. At its current status, CASM in Pocket G-PFEM is not able to reproduce dilative behaviour (i.e. $\psi_0 < 0$), and has numerical stability issues for high values of ψ_0, when the mean pressure drops too close to zero during undrained penetration. The model was calibrated for three different initial state parameters, namely 0.02, 0.05 and 0.08. The parameters are shown in Table 1.

The triaxial test simulations at element level are shown in Fig. 2 for the three state parameters. Please refer to (Arroyo and Gens, 2021) for a definition of the various parameters of CASM.

4.2. Simulation of CPTu penetration

CPTu penetration was simulated for the three values of ψ_0 and for seven values of V: 0.01, 0.10, 1.0, 3.0, 10, 30 and 100. As the rate of penetration is fixed in Pocket G-PFEM to $v = 2\ cm/s$, different values of V were achieved by modifying the hydraulic conductivity k using the equation:

$$V = v\ d/c_h \tag{1}$$

where d is the diameter of the cone and

$$c_h = (E_{oed} \cdot k)/\gamma_w \tag{2}$$

In this paper, c_h is assumed equal to the vertical coefficient of consolidation c_v for simplicity.

The cone modelled has a diameter $d = 35.68\ mm$ and an apex angle of 60°. The model was run for two different initial mean effective stresses, 300 kPa and 500 kPa, and a $K_0 = 0.60$ was assumed.

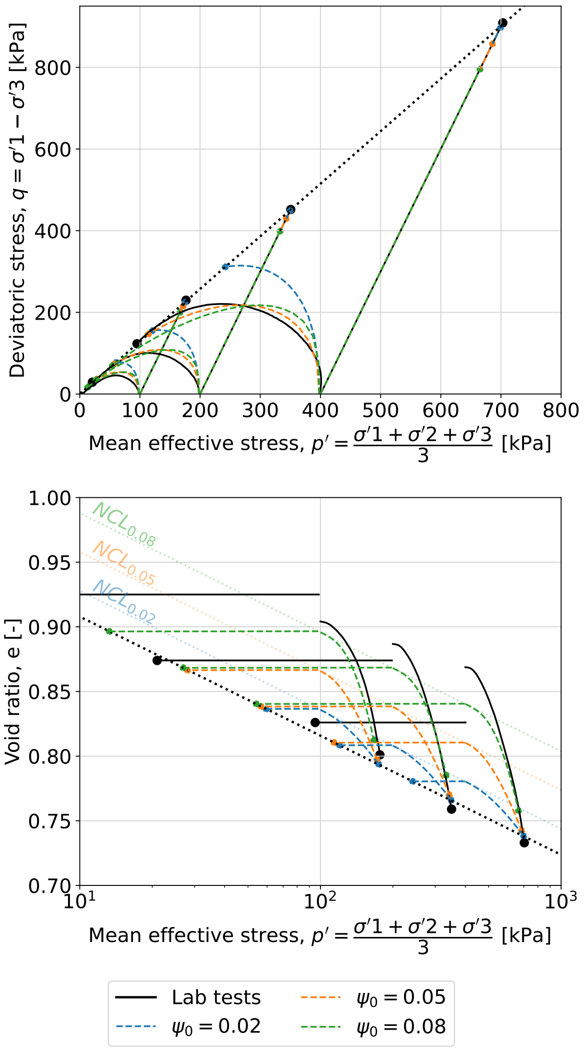

Figure 2. Simulation of drained and undrained triaxial tests with the parameters shown in Table 1.

Table 1. CASM parameters used in Pocket G-PFEM simulations.

Parameter	$\boldsymbol{\psi = 0.02}$	$\boldsymbol{\psi = 0.05}$	$\boldsymbol{\psi = 0.08}$
$\boldsymbol{\lambda_e}$	0.04	0.04	0.04
$\boldsymbol{\kappa}$	0.01	0.01	0.01
$\boldsymbol{\nu}$	0.15	0.15	0.15
$\boldsymbol{M}$	1.287	1.287	1.287
$\boldsymbol{\phi_{tc}}$	32°	32°	32°
$\boldsymbol{m}$	1.75	1.75	1.75
$\boldsymbol{OCR}$	1.05	1.05	1.05
$\boldsymbol{r}$	1.9477	5.294	14.3919
$\boldsymbol{n}$	2.5	2.5	3.4

4.3. Parameterization of the characteristic surface

Simulations of CPTu penetration were performed for each combination of ψ_0 and V. Numerical values of tip resistance, sleeve friction and pore pressure were obtained and employed to compute the normalised tip resistance Q, the third coordinate required to reproduce the characteristic surface. It has been proved numerically that the resulting dimensionless value of $Q(1 - B_q) + 1$ is not dependent on the confinement pressure employed in the simulation.

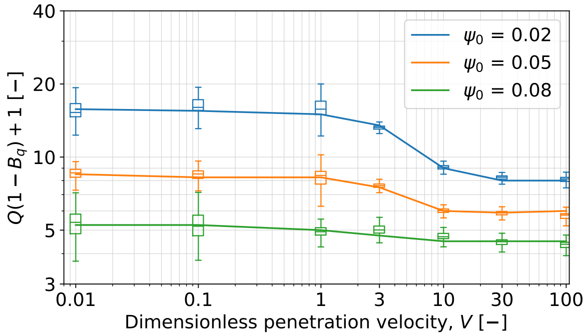

Figure 3. Projection of the characteristic surface determined from numerical simulations: normalized tip resistance vs penetration velocity.

The semi-logarithmic projection of the characteristic surface in the $Q(1 - B_q) + 1$ vs V plane is shown in Fig. 3, where initial state parameters can be mapped to curved lines. The scatter of each value into the stable region is also shown, in contrast with the single value adopted for each combination of ψ_0 and V. At both ends, the curves are essentially parallel, approaching to each other with increasing V. This curved transition between $V_{\min} = 1$ and $V_{\max} = 30$ defines the partially drained region, and shows similarities with the method proposed by Ayala et al. (2023).

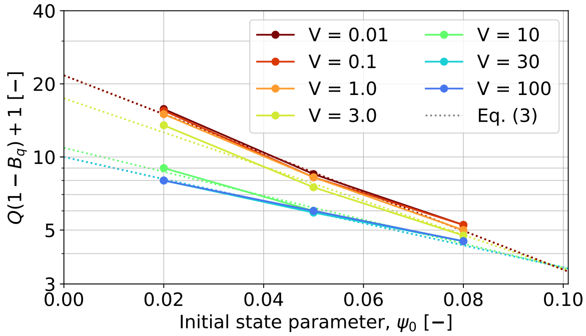

Figure 4. Projection of the characteristic surface determined from numerical simulations: normalised tip resistance vs initial state parameter.

In Fig. 4, the semi-logarithmic projection in the $Q(1 - B_q) + 1$ vs ψ_0 plane is shown. It can be noticed that all the curves, now representing the different dimensionless penetration velocities, are straight lines. The drained lines (i.e. $V < V_{\min}$) are above the undrained lines (i.e. $V > V_{\max}$). This was expected, as drained tip resistance is always higher than undrained tip resistance. Also, it can be noticed that the slope of the drained line is higher than the undrained line, unlike the widget method (Ayala et al., 2023). The transition is also characterised by straight lines that change their slope gradually.

An interpolation function of ψ_0, V and $Q(1 - B_q) + 1$ is proposed to describe the characteristic surface for each value of V between $V_{\min}$ and $V_{\max}$. The shape of this function is:

$$\psi_0 = k_{11} + k_{12}V - (k_{21} + k_{22}V) \cdot \log_{10}[Q(1 - B_q) + 1] \quad (3)$$

where $k_{11} = 0.1622$ $k_{12} = 0.0048$, $k_{21} = 0.1164$ and $k_{22} = 0.0086$ in this particular case. The result of the interpolation is shown as dotted lines in Fig. 4,

4.4. Interpretation of CPTu soundings and method comparison

After calibrating the Eq. (3) for this particular problem, actual CPTu soundings can be interpreted. From CPTu tests, $V = v\,d/c_h$ and $Q(1 - B_q) + 1$ values are calculated. Using the parameterization of the characteristic surface, the state parameter can be determined. Fig. 5 showcases the results of these computations, depicting the c_h, V and $Q(1 - B_q) + 1$ derived from the data, as well as the ψ values computed with this procedure, along with two screening methods used by the industry, namely Plewes et al. (1992) and Robertson (2010). In the rightmost plot, ψ determined for the current dimensionless velocity V is compared with the bounding values of drained and undrained penetration.

Zooming into the sounding interpretation, Fig. 6 provides a closer examination, offering a detailed perspective on specific segments of the different methods. A frequency plot using data from 44 soundings performed in the same TSF is shown in Fig. 7, where the proposed method is also compared with both screening methods.

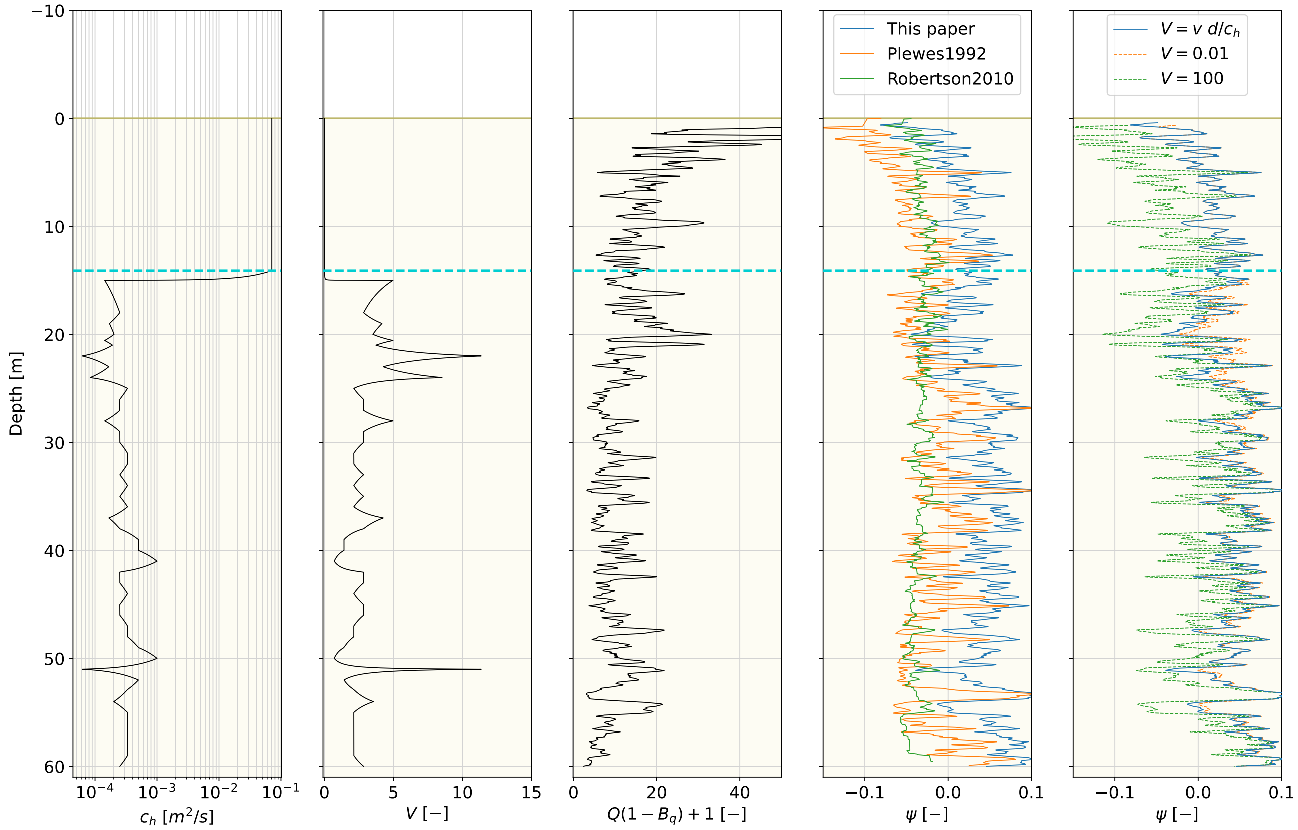

Figure 5. Example interpretation on an actual CPTu sounding in tailings. Comparison between the proposed method and other screening methods.

In the example shown here, the proposed procedure predicts a looser state than Plewes et al. (1992) or Robertson (2010) methods: i) the mean value of ψ is +0.03 vs -0.04 from these two screening methods; ii) 90% of the points have $\psi > -0.06$ vs 50% from the screening methods (Fig. 7). On the other hand, Fig. 5 and 6 show that the distance between the boundaries of fully drained and fully undrained penetration is smaller than, for instance, typical results obtained when employing the CPT-Widget, as discussed by Reid and Smith (2021).

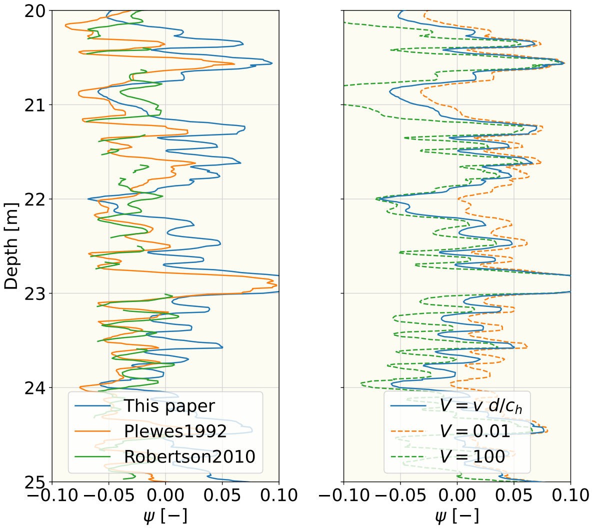

Figure 6. Example interpretation on an actual CPTu sounding tailings, for a zoomed range of depth. Comparison between the proposed method and other screening methods.

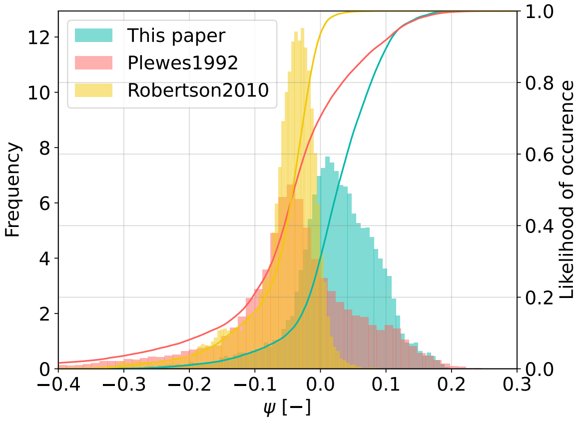

Figure 7. Frequency analysis of the state parameter from 44 soundings performed in the same TSF. Comparison between the proposed method and other screening methods.

4.5. Discussion

The method presented in this paper is currently in its preliminary developmental phase, indicating that further enhancements are necessary for both the tool and the procedural framework. Extensive testing is required before considering its practical adoption.

Nevertheless, its initial outlook is highly promising, given its utilisation of both laboratory and raw CPTu data to calibrate a constitutive model and estimate the in-situ state parameter. This approach presents a significant advantage compared to the methodologies proposed by Plewes et al. (1992) and Robertson (2010). A second look at Fig. 7 proves the statement: while the probability density function (PDF) of this method includes all the uncertainties of the procedure, the PDFs of the Plewes et al. (1992) and Robertson (2010) methods do not include the (high, but unknown) model uncertainties of the empirical correlations employed in them. While other methods were developed for either drained or undrained penetrations (e.g., CPT-Widget), the proposed method is capable of reproducing the fully coupled flow-deformation phenomenon, considering partial drainage driven by the penetration velocity of the CPT and the consolidation coefficient (i.e. stiffness and permeability) of the material.

The tool (i.e. Pocket G-PFEM and CASM) has some limitations that require further development:

i) While operational, the tool is not robust enough to be used by practitioners not familiar with numerical modelling and advanced constitutive models, and is far from being a replacement of screening methods.

ii) Pocket G-PFEM manual recommends sensitivity analyses on numerical control parameters when simulating drained or partially drained penetration. This requirement should come with a procedure to make these sensitivity analyses more user-independent. In this study, default numerical parameters were kept, and a limited strain localisation was observed for low values of V, which might be a source for the high scatter observed in $Q(1 - B_q) + 1$ at low values of V (see wide error bars in Fig. 3 for $V < 1$).

iii) The tool has severe limitations to simulate undrained penetration when ψ_0 is calibrated to reproduce dilative or very contractive states. Due to this limitation, the range of ψ_0 employed in this exercise has been narrowed to 0.02-0.08, probably influencing the bias towards contractiveness shown in Fig. 3.

iv) CASM is the only constitutive model available in Pocket G-PFEM. In this exercise, CASM was calibrated using CIUC triaxial tests only, a strategy that can induce significant error for other stress paths (Tasso et al., 2024).

5. Conclusions

The in-situ behaviour of tailings is determined by their stress state and their density, commonly characterised by the state parameter (ψ). Given the difficulty of obtaining undisturbed samples and the spatial variability of tailings deposits, the assessment of ψ relies on in-situ tests. Several empirical screening methods have been developed, which have a high, but unknown, model uncertainty. As the response of tailings to undrained shear can dramatically change with very small changes in ψ, this model uncertainty is a major source of concern for practitioners.

To enhance the robustness and reliability of stability and deformation analyses in TSFs, it is imperative to explore avenues where raw data from CPTu can be directly employed. By bypassing the need for correlations and directly utilising CPTu data to determine the in-situ state of materials and calibrate constitutive models, it becomes possible to substantially enhance the accuracy and efficacy of geotechnical assessments.

This work proposes a site-specific approach for determining the in-situ ψ of tailings by inversion of CPTu data. The procedure employs the numerical tool Pocket G-PFEM and the constitutive model CASM, and involves five stages: i) calibrate CASM for different initial state parameters (ψ_0); ii) run CPTu simulations for those ψ_0 and dimensionless velocities V between drained and undrained penetration; iii) parameterise a characteristic surface (ψ_0 - V - $Q(1 - B_q) + 1$ space); iv) calculate V along the CPTu soundings using the dissipation test data; and v) compute the state parameter at each data point within the CPTu sounding.

The proposed approach was applied to a real TSF. CASM was calibrated using CIUC triaxial tests, and data from a CPTu sounding was used to estimate ψ. Results were compared with Plewes et al. (1992) and Robertson (2010) screening methods. In this example, it has been found that the proposed method predicts a looser state than these screening methods.

The method proposed in this paper is in a preliminary stage of development. Nevertheless, it already looks very promising, as it uses raw CPTu data to estimate the in-situ state parameter and to calibrate a constitutive model, including all sources of uncertainty in the resulting probability density functions of ψ.

Some recommendations for future development are provided, namely: i) include other constitutive models in Pocket G-PFEM; ii) extend the range of allowable input state parameters to include dense and very loose states; iii) develop a less user-dependent procedure to select numerical control parameters for simulating drained or partially drained penetrations.

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References

ANCOLD (2019). *Guidelines on tailings dams - Planning, design, construction, operation and closure*. Australian National Committee.

Arroyo, M. and A. Gens (2021). *Computational analyses of Dam I failure at the Corrego de Feijao mine in Brumadinho, Final Report*. CIMNE.

Ayala, J., A. Fourie, and D. Reid (2023). A unified approach for the analysis of cpt partial drainage effects within a critical state soil mechanics framework in mine tailings. *Journal of Geotechnical and Geoenvironmental Engineering 149*.

Ayala, J., M. Martinelli, D. Reid, and F. A. (2023). Cone resistance and soil state of tailing sand deposits using de material point method. In *Proceedings 10th NUMGE 2023*, London. SIMSG-ISSMGE.

Been, K. and M. G. Jefferies (1985). A state parameter for sands. *Géotechnique 35*, 99–112.

Been, K., M. G. Jefferies, J. H. A. Crooks, and L. Rothenburg (1987). The cone penetration test in sands: part ii, general inference of state. *Géotechnique 37*, 285–299.

Bird, R., W. M. Coombs, C. E. Augarde, M. J. Brown, Y. Sharif, G. Carter, K. Johnson, and C. Macdonald (2023). Cone penetration tests (cpts) in layered soils: a material point approach. In *Proceedings 10th NUMGE 2023*, London. SIMSG-ISSMGE.

Boschi, K., L. Monforte, M. Arroyo, J. M. Carbonell, and A. Gens (2023). Effects of drainage conditions on state parameter inversion from cptu. In *Proceedings 10th NUMGE 2023*, London. SIMSG-ISSMGE.

Dadvand, P., R. Rossi, and E. Onate (2010). An object-oriented environment for developing finite element codes for multi-disciplinary applications. *Arch. Comput. Methods Eng. 17*(3), 253–297.

Finnie, I. and M. Randolph (1994). Punch-through and liquefaction induced failure of shallow foundations on calcareous sediments. In *Seventh International Conference on the Behaviour of Offshore Structures*, Volume 1, United Kingdom, pp. 217–230. Pergamon. Punch-through and liquefaction induced failure of shallow foundations on calcareous sediments ; Conference date: 01-01-1994.

Houlsby, G. (1985). The use of a variable shear moudulus in elastic-plastic models for clays. *Computers and Geotechnics 1*(1), 3–13.

Jefferies, M. and K. Been (2016). *Soil Liquefaction. A critical state approach.* Taylor and Francis Group, LLC.

Manica, M. A., M. Arroyo, A. Gens, and L. Monforte (2022). Application of a critical state model to the merriespruit tailings dam failure. In *Proceedings of the Institution of Civil Engineers - Geotechnical Engineering*, pp. 151–165.

Monforte, L., M. Arroyo, J. M. Carbonell, and A. Gens (2017). Numerical simulation of undrained insertion problems in geotechnical engineering with the particle finite element method (pfem). *Computers and Geotechnics 82*, 144–156.

Monforte, L., M. Arroyo, and A. Gens (2022). *Undrained strength from CPTu in brittle soils: A numerical perspective*, pp. 591–597. London: CRC Press.

Monforte, L., M. Arroyo, and A. Gens (2023a). Cone penetration in brittle, lightly over-consolidated soils: a numerical perspective. In *Proceedings 10th NUMGE 2023*, London. SIMSG-ISSMGE.

Monforte, L., M. Arroyo, and A. Gens (2023b). A relation between undrained cptu results and the state parameter for liquefiable soils. Volume 60.

Monforte, L., A. Gens, M. Arroyo, M. Mánica, and J. M. Carbonell (2021). Analysis of cone penetration in brittle liquefiable soils. *Computers and Geotechnics 134*.

Monforte Vila, L. (2018). *Insertion problems in geomechanics with the particle fininite element method.* Universitat Politecnica de Catalunya.

Moshfeghi, S., M. Taiebat, and A. Lizcano (2023). The role of soil constitutive model in simulation of cone penetration test. In *Proceedings 10th NUMGE 2023*, London. SIMSG-ISSMGE.

Oñate, E., S. R. Idelsohn, F. Del Pin, and R. Aubry (2004). The particle finite element method—an overview. *International Journal of Computational Methods 1*(02), 267–307.

Plewes, H. D., M. P. Davies, and M. G. Jefferies (1992). Cpt based screening procedure for evaluating liquefaction susceptibility. In *45th Canadian geotechnical conference, Innovation conservation and renovation*, Toronto, pp. 4. CGS.

Reid, D. (2015). Estimating slope of critical state line from cone penetration test - an update. *Canadian Geotechnical Journal 52*, 45–57.

Reid, D. and K. Smith (2021). Interpretation of state parameter in partially drained tailings: a case history examination. *Géotechnique Letters 11*, 1–5.

Robertson, P. K. (2010). Evaluation of flow liquefaction and liquefied strength using the cone penetration test. *Journal of Geotechnical and Geoenvironmental Engineering © ASCE 136*, 842–853.

Rowe, P. W. (1962). The stress-dilatancy relation for static equilibium of an assembly of particles in contact. In *Proceedings of the royal society*, London.

Shuttle, D. and J. Cunning (2007). Liquefaction potential of silts from cptu. *Canadian Geotechnical Journal 44*, 1–19.

Shuttle, D. and M. Jefferies (2016). Determining silt state from cptu. *Geotechnical Research 3*, 90–118.

Tasso, N., F. L. Rivarola, O. De Santiago, N. Rivas, and M. Sottile (2024). Calibrating constitutive models for flow liquefaction: a word of caution. In *1st International Symposium of Tailing Deposits*.

Torres-Cruz, L. A. (2021). The plewes method: a word of caution. *Mining, Metallurgy and Exploration 38*, 1329–1338.

Wu, X., M. Ghafghazi, Z. Liu, Y. Tian, and S. H. Chow (2023). Numerical modelling of drained and undrained cone penetration tests. In *Proceedings 10th NUMGE 2023*, London. SIMSG-ISSMGE.

Yu, H. S. (1998). Casm: a unified state parameter model for clay and sand. *International Journal for Numerical and Analytical Methods in Geomechanics 22*(8), 621–653.