

RITTER-KRIŽAIĆ ITERACION METHOD OF TRUSS CONSTRUCTION

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Abstract. The optimization of the dimensioning of constructive designs is constantly evolving. FEM, evolutionary, and other various methods are being developed, which are implemented with algorithms in computer simulations of building models. The problem with these methods is solving large differential equations, which is inconceivable without computers and large memories. The Ritter-Križaić (RK) iteration method works for both straight and oblique networks with one side, and it can even be used instead of trigonometric and FEM equations. It does this by adding the geometric properties of the networks and outside actions to the directional equations. By creating straightforward monograms of RK-FEM technology with straightforward differential or subspace equations that are simple to calculate by hand or draw with Mathcad tools, the RK-FEM loop enables COD to define various types of trusses and even other supports. RK-FEM COD is therefore used to create simulation games that explain many logical phenomena in the design of external and internal actions of beam supports, which can be compared to a spider thread or an ice plate structure as an RK string and even to the moon.

1 INTRODUCTION

All truss constructions are structural systems consisting of interconnected elements (chords and diagonals) that generate only axial forces due to external loads. Truss structures are created by optimizing [1,2] the use of materials. In optimizing the design of structural systems today, FEM [3] and evolution various other methods [4,5,6] are used to create algorithms that are implemented in computer simulations of structural models. The calculation of plane truss models is easy to solve with the first Ritter method. However, it involves many routine calculations for different dimensions. Therefore, a new calculation must be performed for each change. To solve this problem of routine calculations, mathematical and algorithmic models are created and existing methods are used. The combination and consolidation of these structures leads to a new, seemingly complicated but simple method that is easily supported by computers in software loops. By converting the geometric properties of structural truss systems into line equations, formulas are obtained that even replace trigonometric equations. The contribution of the work is the definition of iterative mathematical models of truss or lattice girders with simple plane equations or substantial equations. The aim of the work is the simple and fast definition of internal actions and, conversely, the creation of external actions based on a constant model that allows the interpretation of monograms for all engineers, including non-experts.

2 STATIC METHODS OF TRUSSES

2.1 Classical methods

The classical mathematical methods for calculating planar structures are based on the analytic-graphic principle. Using the method of joints and sections, the first Ritter method defines equilibrium states of intersecting structural elements by balancing actions in beam members with a certain number of members. By the method of nodal isolation, both analytically and graphically, the Maxwell-Cremona force diagram releases nodes, and the statics in the members are determined by the equilibrium sum of all actions at the node (Fig. 1), where, of course, reactions are first defined as in the classical beam member. The method of cutting through three members using the Ritter method (Eg. 1) eliminates two members with a common moment point, allowing the definition of an equation with a single unknown.

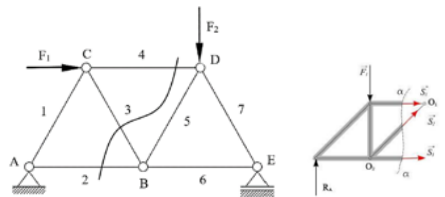


Figure 1: Ritter's method of section and analytical calculation [7].

In a planar system, all actions can be most easily reduced to the axes of the xy coordinate system, which leads to two scalar equations in which the sum of the forces and moments is equal to 0 for a balanced static action.

$$\sum_{i=1}^n F_{xi,yi} , M_{xi,yi} = 0 \quad (1)$$

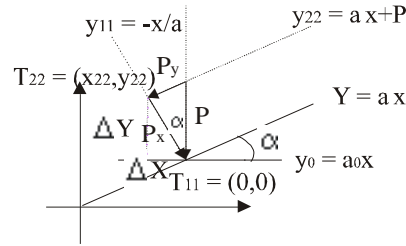


Figure 4: The force components defined by the equation of the line

Solving the system of linear equations, we obtain the coordinates T_{22} , or differentials Δx and Δy , as well as the magnitudes of the forces P_x and P_y

$$x = -\frac{aP}{1+a^2} = \Delta x ; y = \frac{P}{1+a^2} = \Delta y \quad (3)$$

So, the distance between the points of force P and the force P_x is

$$P_x = d(T_{11}, T_{22}) = \sqrt{\Delta x^2 + \Delta y^2} = aP/\sqrt{1+a^2} \quad (4)$$

Which are also replacements for trigonometric equations with quadratic equations expressed using the slope coefficient of the line. From this, it is evident that for a unit force P or vector, the mathematical trigonometric expression is $\sin(\alpha) = a/\sqrt{1+a^2}$ and $\cos(\alpha) = 1/\sqrt{1+a^2}$.

3.2 Iterative element of a single-pitched truss

By applying Ritter's method and substituting trigonometric functions with the slope coefficient of linear functions, you obtain the required dimensions of the truss system for its structural design. The system only allows the substitution of known trigonometric equations. However, the process is still partial. To make the process iterative, i.e. to speed it up, the truss problem is solved by defining all distances of the nodes and members from the iterative model element of the truss system (Fig. 5).

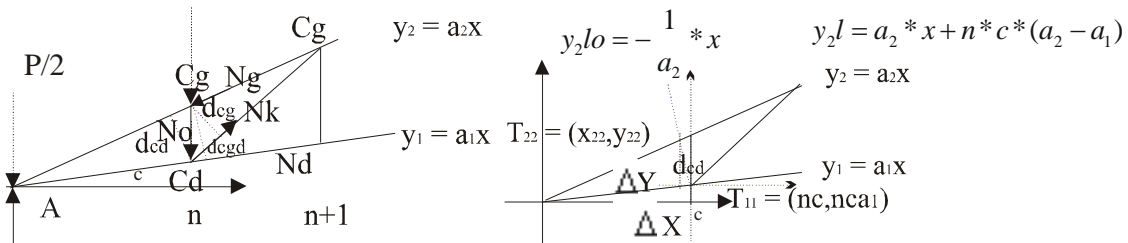


Fig. 5. Representation of forces and nodes in the iterative model of a single-pitched truss

The elements of the iterative model are:

- n - ordinal number of nodes or members
- c - spacing of nodes along the x -axis
- e - total number of iterative elements
- a_1, a_2 - slope coefficient of the bottom and top chord
- q, P - load.

3.3 Defining the forces in the members using force geometry and systems

Using the well-known Ritter method for the static design of truss structures defined by the iterative model, the infinitesimal Ritter-Križaić method is derived as a function of the coefficients of the line equations. All force quantities in the members are obtained from the iterative model using the equilibrium equation of the moment sum at the bottom node. By inserting distances and loads into the known Ritter and Culmann equations using five components of the iterative model, the Ritter-Križaić formula is obtained for the bottom and top chords, verticals and diagonals in the truss of the mentioned model, where each iterative element has a diagonal. The truss is inclined on one side with a flat bottom chord and an inclined top chord. The geometric properties are expressed by inclination coefficients (Eq. 5,6).

$$h_{ni} = (a1 - a0)n c \quad (5)$$

$$l_{okni} = \cos(\text{atan}(a1 - a0)) (n - 1) c (a1 - a0) \quad (6)$$

The final static equation for a truss member with a flat bottom chord, with given variables for the slope coefficients as 0 and $a1$, along with the static reactions for a simple beam, is as follows:

Bottom chord N_{Di} (Eq. 7,8)

$$N_{Di0} = 1/h_{ni0}(A(n - 1)c - \sum_{i=0}^{n-1} Pic) \quad (7)$$

$$N_{Di} = 1/h_{ni}(Anc - \sum_{i=0}^n Pic) \quad (8)$$

Top chord N_{Gi} (Eq. 9)

$$N_{Gi} = -1/l_{okni}(A(n - 1)c - \sum_{i=0}^{n-1} Pic) \quad (9)$$

Vertical member N_{Vi} (Eq. 10,11)

$$N_{Vi0} = -A + iP \quad (10)$$

$$N_{Vi} = N_{KiV} \quad (11)$$

I For the diagonal member N_{Ki} (Eq. 12,13,14)

$$N_{KiH} = N_{Di} - N_{Di0} \quad (12)$$

$$N_{KiV} = N_{KiH} \Delta a \quad (13)$$

$$N_{Ki} = \sqrt{N_{KiH}^2 + N_{KiV}^2} \quad (14)$$

The initial element has a small difference, so it supplements the iterative algorithm at the beginning of the loop.

4 RITTER-KRIŽAIĆ – FEM COD METHOD

The results (Fig. 6) with variable and constant truss elements define new values that behave variably, or functionally. Thus, the external action on the members is defined by a differential equation, and all member forces are functions of a , n , c , and RA (Eq. 15).

$$N_{Di}; N_{Gi}, N_{Ki} = f(a, n, c, RA) \quad (15)$$

By defining constants for n , c , and a and iterating the force for 1 unit increment or the numerator i , the RA is differentiated, and the differential iteration equation (Eq. 16) is defined based on the method of dynamic structural programming (Eq. 17,18,19) from areas operations research.

$$NGi(Pi+1) = f(NGi, + y') \quad (16)$$

$$f_n(S_n) = \max_{0 \leq x_n \leq S} (g_n(x_n) + f_{n-1}(S - x_n)) \quad (17)$$

$$f_{n+1}(S_n) = f_n(S_n) \quad (18)$$

$$f_{n+1}(N_G, N_D, N_K, N_V) = f_n(N_G, N_D, N_K, N_V) \quad (19)$$

The displayed model is for x equal to 10, meaning there are 10 iterative elements in the truss model. For the initial iteration, the equations are N_{D0} , i.e. the 0th element, while the rest form an infinite series with the i-th element.

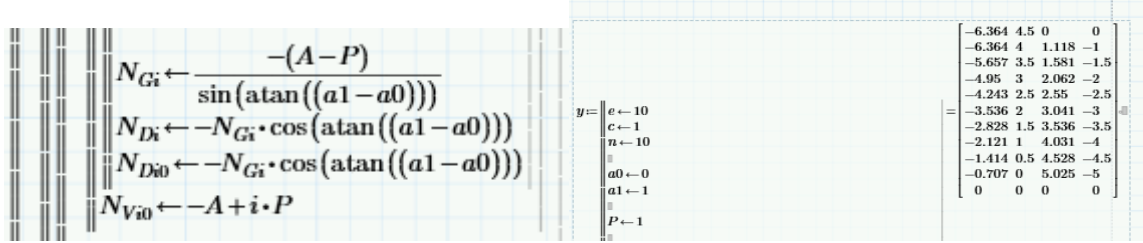


Figure 6. The result and code for RK-FEM technology

5 VECTOR SURFACES RK-FEM COD METHOD

The results with variable and constant truss construction elements define new discrete values that behave variably or functionally. This defines the differential equation for the external action on the members, and all member forces are functions of a, n, c, and RA. By searching for these partial curves from the RK-FEM truss calculation technology.

5.1 N_{Gi}

For a constant l , where $y = 10$, $a_1 = 1$, $a_0 = 0$ and $P = 1$, a partial straight-line curve is defined from the results of the member forces in the chord, in sequential order, with a simulated increase in the force P by 1 per unit length of the truss, up to 10. This results in a series of observable lines defined by the least squares Gaussian method for quadratic (Eq. 20,21,22,23) or linear curves, or a series of differential equations.

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{bmatrix} \quad y = \begin{bmatrix} -6.364 \\ -6.364 \\ -5.657 \\ -4.95 \\ -4.24 \\ -3.82 \\ -2.121 \\ -1.414 \\ -1.707 \\ 0 \end{bmatrix} \quad b = \begin{bmatrix} b1 \\ b2 \\ b3 \end{bmatrix} \quad b1 = \sum_{x=1}^n x, \quad b2 = \sum_{x=1}^n xy, \quad b3 = \sum_{x=1}^n x^2 y \quad (20)$$

$$\begin{bmatrix} n & \sum_{x=1}^n x & \sum_{x=1}^n x^2 \\ \sum_{x=1}^n x & \sum_{x=1}^n x^2 & \sum_{x=1}^n x^3 \\ \sum_{x=1}^n x^2 & \sum_{x=1}^n x^3 & \sum_{x=1}^n x^4 \end{bmatrix} x \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} b1 \\ b2 \\ b3 \end{bmatrix} \quad (21)$$

$$X = bA^{-1}, \quad X = \begin{bmatrix} -7.142 \\ 0.492 \\ 0.016 \end{bmatrix}, \quad a_i = X_i \quad (22)$$

$$N_{G(l10,P1,a1)} = \frac{\Delta N_{G(l10,P1,a1)}}{\Delta x} = a_1 + a_2 x + a_3 x^2 \quad (23)$$

The partial curve with constant P yields the equation (Eq. 24).

$$N_{Gi-con(l10,P1,a1)} = f_x = y' = -7.142 + 0.492x - 0.016x^2 \quad (24)$$

By iterating P for 1 on the plane level, a differentiated curve f_x is obtained (Fig. 7) (Eq. 25,26,27,28).

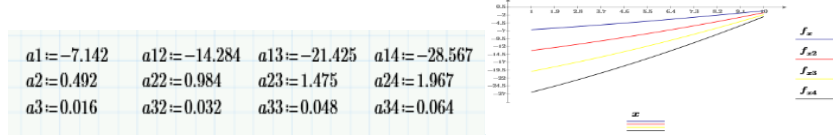


Figure 7. The partials write linearize by reducing the force P to the plane level

The definition of the link between the finite increments on the planes defines the increments of the function as a function of P , leading to equations with two variables $N_{Gi}=f(x,P)$.

$$N_{Gi} = a1 + a2x + a3x^2; \quad (25)$$

$$N_{Gi} = -7.142 + 0.492x + 0.016x^2; \quad (26)$$

$$N_{Gi} = -14.284 + 0.984x + 0.032x^2; \quad (27)$$

$$N_{Gi} = -21.425 + 1.475x + 0.048x^2; \quad (28)$$

$$N_{Gi} = -28.567 + 1.967x + 0.064x^2; \quad (29)$$

By connecting the coefficients $a1i$ into lines and $a2i$ and $a3i$ and deleting $a3i$ due to its small value, a formula is created with two variables $N_{Gi}=f(x,P)$, resulting in a plane (Fig. 8a), (Eq. 30,31,32,33).

$$a1 = -7.142 P \quad (30)$$

$$a2 = 0.492 P \quad (31)$$

$$N_{Gi} = a1 + a2x \quad (32)$$

$$N_{Gi(x,P)} = -7.142 P + 0.492 P x \quad (33)$$

The equalization of values is evident, so the spatial equation replaces the Ritter calculation for the given example of a truss. By checking for larger values of P and x , an invisible state is defined, and the upper equation is suitable for the specified model. Thus, generally, for each truss case, a new equation is defined. The Gaussian method does not provide accurate data for irregular numerous variables xx , so the equation of the line for given forces in the truss members is manually defined. The line on the plane with constant N and P , constant - model for $x=10$ yields (Eq. 34).

$$NP(P, x - con) = 6.5 P \quad (34)$$

The line on the plane with constant N and x - constant $P1$ model for $x=10$ yields (Fig. 8b). From the data points T1,2,3 (10,50,70,100) for x , i.e., length, and for unit increase in force P , the value of force P is respectively -6.63, -34.65, -48.79, and -70. Based on the equation of the line with two points, it is defined as follows (Eq. 35,36,37).

$$y - y1 = (x - x1)(y2 - y1)/(x2 - x1) \quad (35)$$

$$N_{Gi} = -0,7x \quad (36)$$

The value of N increases multiplicatively by the force P , so a or P is inserted into the formula.

$$N_{Gi} = -0,7Px \quad (37)$$

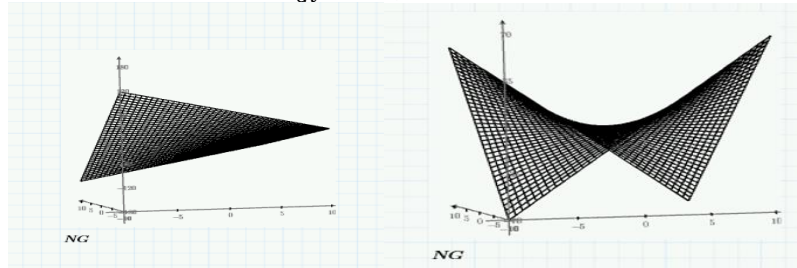


Fig. 8.a) He spatial curve of compressive force N_{Gi} ; 8.b) RK-FEM tehnology ($NG(x,P,a)$; $a = \text{const.}$)

To define all trusses, equations are created as functions of $a1$. By changing the height of the beam defined by $a1$, the model of 10m is affected. From the given data, it can be discerned that increasing the beam height reduces the load on the beam, which is in conflict with the breakdown of the external force, which has an inverse value relative to $a1$. By changing the span, results are obtained that iteratively transmit forces inversely from the largest model to the smallest, depending on the number of elements.

By partially viewing the spatial equation linearly, we obtain the equation (x ,) and $f(P,a)$. So it is for (P,a) for $x = 5, 10, 15$ m (Eq. 38).

$$N_{Gia} = -7 + 0.81a; -14.2 + 1.07a; -20.84 + 2.38a \quad (38)$$

Connecting a and b into formulas creates a plane (Eq. 39,40,41,42,43).

$$a1 = -0.179 - 6.924P \quad (39)$$

$$a2 = -0.15 + 0.785 P \quad (40)$$

$$N_{Gi} = a1 + a2x \quad (41)$$

$$N_{Gi(a,P)} = -6.924 P + 0.785 P a \quad (42)$$

$$N_{Gi(a,P)} = -7 P + 0.8 P a \quad (43)$$

For $N f(x,a)$ for $x=5,10,15$ m (Eq. 44).

$$N_{Gia} = 0 - 2.89a; 0.462 - 6.719a; 10.8 + 1.26a \quad (44)$$

Connecting a and b into formulas creates a plane (Eq. 45,46,47,48).

$$a1 = 7.4 - 5.44x \quad (45)$$

$$a2 = -6.75 + 2x \quad (46)$$

$$N_{Gi} = a1 + a2x \quad (47)$$

$$N_{Gi(a,P)} = 7.4 - 5.44x + 6.75a + 2ax \quad (48)$$

The sum of both $N f(pa$ and xa) results in (Eq. 49).

$$N_{Gi(a,P,x)} = -0.4 - 5.44x + 6.75a + 0.8aP + 2ax \quad (49)$$

By simulating the models 115m and 110m with differentiated increases in the slope coefficient (a) and force P per unit value (1, 2, 3), iterative equations are defined. These are defined through

two points of the curve reduction direction for $l15$, ai , $a=1$ (Eq. 50,51), and $a=2$ (Eq. 52,53), and $a=3$ (Eq. 54,55).

$$y1 = -\frac{10}{15}x; y2 = -\frac{20}{15}x; y3 = -\frac{30}{15}x; \quad (50)$$

Gaussian elimination applied to these three equations yields

$$a1 = -0.65P \quad (51)$$

$$y2 = -\frac{8}{15}x; y2 = -\frac{16}{15}x; y3 = -\frac{23.5}{15}x; \quad (52)$$

Gaussian elimination applied to these three equations yields

$$a2 = -0.507P \quad (53)$$

$$y3 = -\frac{7.4}{15}x; y2 = -\frac{15}{15}x; y3 = -\frac{22}{15}x; \quad (54)$$

Gaussian elimination applied to these three equations (yi) yields

$$a3 = -0.5P \quad (55)$$

Gaussian elimination applied to these three equations (ai) yields (Eq. 56),

$$Y(ai, P) = -0.68 + 0.066 aP \quad (56)$$

This process iterates for $l10$, ai and then Gaussian elimination is applied to these three equations (ai) to obtain (Eq. 57).

$$Y(ai, P) = -0.7 + 0.085 aP \quad (57)$$

Using Gaussian elimination for these three equations (Yi), we obtain $a1$ and $a2$ for the equation $y=a1+a2x$ (Eq. 58,59).

$$a1 = f - (0,68 i - 0,7) = -0.7 + \frac{0.02}{5} aP \quad (58)$$

$$a2 = f(0,066 i 0,085) = 1.23 + \frac{0.19}{5} aP \quad (59)$$

By substituting the coefficients into the NG equation, y yields the function of the decline of $NGa(x,P)$ with respect to the variable a (Eq. 60).

$$NGa(a, x, P) = -0.7 + 0.085 aP \quad (60)$$

The final spatial curve for all positions in space amounts to (Eq. 61,62).

$$NG(a, x, P) = NG(a1, x, P) - NGa(a, x, P) \quad (61)$$

$$NG(a, x, P) = -0.7Px + (-0.7 + \frac{0.02}{5}x + (1.23 + \frac{0.19}{5}x))aP \quad (62)$$

5.2 N_{Di} , N_{Ki} , N_{Vi}

For constant 1, $a1$ and $a0$, a partial straight-line curve is defined from the results of the member forces in the chord, in sequential order, which with a simulated increase in the force value P by 1, yields a series of observable lines defined by the least squares Gaussian method for quadratic or linear curves (Eq. 63).

$$N_{Di} = -0.5x + 5; -1x + 10; -1.5x + 15 \quad (63)$$

Connecting a and b into formulas creates a plane (Eq. 60) and for diagonal and vertical actions,

it is defined (Eq. 64,65,66,67,68,69) (Fig. 9)

$$a1 = 5 P \tag{64}$$

$$a2 = -0.5 P \tag{65}$$

$$N_{Di} = a1 + a2x \tag{66}$$

$$N_{Di} = 5 P - 0.5 P x \tag{67}$$

$$N_{Ki} = -0.406 P + 0.661 P x - 0.008 P x^2 \tag{68}$$

$$N_{Vi} = 0.45 P - 0.652 P x + 0.011 P x^2 \tag{69}$$

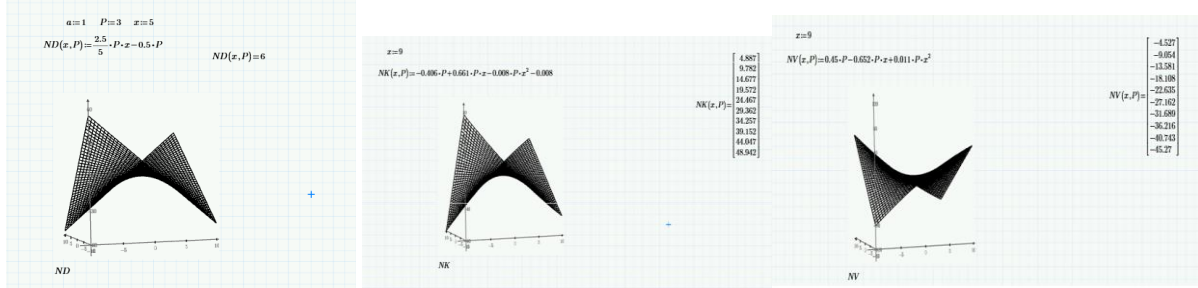


Figure 9. RK-FEM tehnology ($N_{D,K,V}(x,P,a)$; $a = \text{const.}$)

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6 CONCLUSION

This model was created by iterating the idea procedure of the first simpler model truss construction with a flat bottom chord with $a1 = 0$ to a model with $a1 > 0$. The only difference is that it is extended by a constant that defines the directional coefficient of the truss structure. Only by changing some components of the equations, i.e. the direction coefficient of the lines, we obtain n dimensions of a represented model of the truss structure as a beam-static system. To define other types of double-roof, gable-roof and other models, it is necessary to define an iteration model very similar to the above procedure. Well, every designer can also become a programmer, because Mathcad is sufficient for quick calculations. By defining the objective function based on different iteration models and simulations with techno-economic parameters, further optimization of the system is possible. When defining the direction coefficient, i.e. the tangent of the angle by a differential, we also enter the field of infinitesimal methodology, i.e. the field of differential calculus. If we further define the angle of rotation on the iterative model, we obtain the curvature of the lattice constructions. The iterative definition of curvature leads to the iterative formula for the curvature of curves when $x \rightarrow 0$ and $a \rightarrow 0$.

A single roof truss can be solved with the defined system, while other models require a new iteration element or an extension of the given RK-FEM model. The RK-FEM surface vector has the disadvantage that it must be created for each $a1$, since only a few data could be read in the forest of the part $a1$ reduced to the surface plane. It is also recommended to define the path to the megaprojects, i.e. to the spatial domain when $x \rightarrow \infty$ because a larger iteration element in km is defined in a few minutes, while a small element already has problems with processor speed. When solutions simulations $a \rightarrow 0$ is defined spider web which is obtained, i.e. cable replaced with truss construction calculations RK^∞ space technology [12]. Which can be an introduction to AI systems [13] with frequency observation of grid-cable systems [14,15].

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