Mean curvature flow for generating triangular meshes with piecewise constant mean curvatures

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ABSTRACT

With the development of computer-based analysis techniques and the diversification of feasible architectural forms, many attempts have been made to design spatial structures whose shapes are determined by specifying boundary conditions and geometric invariants. In particular, constant mean curvature (CMC) surfaces are uniquely determined by specifying a smooth boundary shape and a target mean curvature. Since the minimal surfaces are at equilibrium by uniform membrane stress, it is utilized in form-finding methods to obtain reasonable shapes of tensile structures.

Although it is more desirable to combine different CMC surfaces to generate piecewise CMC (p-CMC) surfaces satisfying C^0 or G^0 conditions along the connections to provide various design options, most previous studies are limited to the cases where a single CMC surface patch is periodically arranged. Moreover, even though continuous surfaces are often converted to discrete surfaces during structural analysis and construction phases, there are relatively few design methods that directly handle discrete surfaces.

In this study, we propose a method for generating p-CMC meshes using the mean curvature flow (MCF) [1]. p-CMC surfaces can be obtained as the stationary point of an energy functional of multiple patch surfaces and auxiliary surfaces between them. A new formulation is presented for the MCF as the negative gradient flow of the energy functional for multiple patch continuous surfaces, which are further discretized so as to determine the change in the vertex positions of triangular meshes on the surface as well as along the internal boundaries between patches. The internal boundary that is at equilibrium in its perpendicular direction is also determined based on the formulated MCF. Numerical examples show that multiple patch surfaces approximately reach the specified mean curvatures through the proposed method.

REFERENCES

[1] K. Hayashi, Y. Jikumaru, M. Ohsaki, T. Kagaya, and Y. Yokosuka, "Mean curvature flow for generating discrete surfaces with piecewise constant mean curvatures", *Computer-Aided Geometric Design*, to be published