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Modeling and characterizing locally subsiding ground for the analysis and design of mat foundations

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ABSTRACT

Mat foundations are often used as a means of protecting buildings and other structures from excessive distortion due to differential settlements in the underlying ground. Once soil bearing capacity concerns have been addressed, the analysis of these foundations becomes a soil-structure interaction problem where the bearing pressure from the mat induces settlement in the underlying ground while localized settlement distorts the mat and redistributes the bearing pressure. An accurate representation of this soil-structure interaction is necessary to facilitate computations of the shear and flexural stresses in the mat and to develop an appropriate structural design.

However, modeling and characterizing this system has long been a source of confusion and contention among both geotechnical and structural engineers. The soil response is typically characterized using the modulus of subgrade reaction, k_s (also known as the coefficient of subgrade reaction) which describes a certain mechanical soil-structure interaction model known as a Winkler foundation. However, k_s is arguably one of the most misunderstood and misapplied parameters in geotechnical practice, and proper assessment of this parameter is more complex and nuanced than might be expected.

Further complexities are introduced when locally subsiding ground is present. This is because the Winkler model assumes settlement occurs in the soil only in response to an applied bearing pressure, whereas local subsidence introduces additional settlement (with associated shear and flexural stresses in the mat) which is independent of that caused by the applied structural loads.

Methods of modeling and characterizing the subsurface conditions for the purpose of developing design values of k_s to be used in mat foundation analysis and design are proposed, then these methods are extended to accommodate sites with locally subsiding ground. These methods are compatible with standard geotechnical assessment techniques as well as standard structural analysis and design software packages.

Keywords: ISC7, modulus of subgrade reaction, mat foundations, subsiding ground

1. Background

A mat foundation is often an excellent choice for buildings and other structures, including those at sites where settlement is a concern. The structural continuity of a mat is better able to accommodate differential settlements than are individual spread footings, often at a lower construction cost than would be incurred with a deep foundation system.

Yet, the associated geotechnical characterization of the supporting ground and modeling of the soil-structure interaction (SSI) are frequently misunderstood and misapplied. The modulus of subgrade reaction, k_s (also called the coefficient of subgrade reaction) is the most commonly used SSI parameter, and when properly applied, should produce appropriate designs. However, evaluating this parameter and utilising it in the analysis and design process requires some finesse and is more nuanced than might be expected.

This process becomes more complex when subsiding ground is present because the deformations in the SSI model now depend on both the applied structural loads and on the subsidence.

This work reviews the state of the art for defining SSI for mat foundation projects on ordinary sites using k_s as the SSI parameter, a design condition we will call “conventional SSI models”, then extends the discussion to include sites with subsiding ground. Only the static loading case and only regular (i.e. not pile supported) mats are being considered here. Dynamic SSI analyses (FEMA, 2020) are fundamentally different and are beyond our scope. The use of k_s for other purposes, such as pavement design, also is beyond our scope.

Proper characterization of k_s requires an understanding of both the geotechnical site conditions and the proposed structure, along with an understanding of how the SSI model works and how this parameter ultimately impacts the structural design of the mat. Thus, in addition to discussing geotechnical characterization methods, this work also explores the role of this parameter in the structural design process and the necessary collaboration between the geotechnical and structural engineers.

The appropriate rigor in analysis and design depends on the complexity of the subsurface conditions, the type and importance of the structure, and other factors, so the rigor used on specific real projects might be greater than or less than that described here.

2. Geotechnical characterization for conventional static soil-structure interaction models

First we will address methods of geotechnical characterization for conventional static SSI models, including k_s, at ordinary sites. These are sites where subsidence is not a consideration. This discussion will then be extended to include the effects of subsidence.

2.1. Bearing capacity

Spectacular bearing capacity failures of heavily-loaded mat foundations have occurred when these foundations are underlain by saturated clays (Peck and Bryant, 1953; White, 1953; Nordlund and Deere, 1970; Skaftfeld, 1998). Fortunately, this failure mode is easily evaluated using conventional geotechnical characterization of undrained strength and classical bearing capacity theory (Coduto, et al, 2016). When drained conditions and sandy soils exist, bearing capacity is generally more than adequate, even for heavily-loaded mats, and again can easily be evaluated using conventional methods.

Loss of bearing capacity due to seismically-induced liquefaction is an entirely different matter that must be evaluated on a site-specific basis when applicable. This mode of failure has occurred at multiple locations across the world (Niigata, Christchurch, Adapazari, etc.).

2.2. Modulus of subgrade reaction

Once bearing capacity considerations have been satisfied, the geotechnical part of the design becomes a deformation problem, while the structural part is mostly a strength problem with a check on deformation (serviceability). This difference in perspectives can be a source of confusion.

The analysis and design is largely governed by the size and location of the mat, the applied structural loads, and the underlying soil characteristics. In practice, the structural engineer analyzes the superstructure and the mat, while the geotechnical engineer analyzes the supporting ground. These two analyses are linked using an SSI model intended to represent the interface between the mat and the soil, and to characterize the influence each has on the other.

The most commonly-used SSI model is the Winkler foundation (Winkler, 1867; Hetényi, 1946; Terzaghi, 1955), also known as the beam on elastic foundation, which uses an array of independent vertical linear springs located between the mat and the underlying ground with pinned connections between each spring and the mat. The stiffness of each analytical spring is defined by the modulus of subgrade reaction, k_s:

$$k_s = \frac{q}{\delta} \qquad (1)$$

where q is the bearing pressure and δ is the settlement (or compression of the spring).

The geotechnical engineer is primarily responsible for evaluating k_s so it is often mistakenly regarded as a soil property, but clearly it is not. Even the name "modulus" gives the mistaken impression that this is some fundamental material property. In reality, Winkler springs are the analytical representation of a certain SSI model, and the magnitude of k_s depends on the characteristics of the underlying soil as well as the geometry and loading of the mat and the location of the spring on the mat. In many cases k_s even varies with time. Thus, the geotechnical engineer is unable to develop design values until the proposed structure has been defined, and any associated geotechnical recommendations apply only to that proposed structure, not to the project site in general.

2.3. Use of a constant k_s value

Historically, SSI was characterized using a constant k_s value across the entire mat. However, because the springs are uncoupled and act independently this is a weak representation of the actual behavior (Ulrich, 1991). A uniformly-loaded perfectly flexible rectangular mat on a uniform subgrade would have a uniform q, but would distort into a concave-up shape with the greatest δ at the center, so the lowest k_s would be at the center and the highest at the corners. This behavior is sometimes described as "dishing". Conversely, a concentrically-loaded perfectly rigid rectangular mat on the same uniform subgrade would have a uniform δ with q and k_s varying from minimum values at the center to maximum values at the corners. Real mat foundations are somewhere between being perfectly flexible and perfectly rigid, and the structural loading is not uniform, so δ, q and k_s all vary across the mat, even if the subsurface conditions are perfectly uniform.

Nevertheless, this method of characterizing k_s as a constant across the entire mat still persists, even for some large and important structures. Although this practice might be acceptable in some cases, in general it produces structural analyses that overpredict the negative moments and underpredict the positive moments in the mat which results in too much top reinforcing steel and not enough bottom reinforcing steel. This misplacement is somewhat offset by flexural capacity provided by the compression steel, but this is an unbalanced design that, at best, is inefficient and at worst is inadequate. This practice also can impact the concrete punching shear analyses, which can lead to an incorrect mat thickness. In addition, this problem cannot be resolved by simply using a different yet still constant value of k_s.

This method also leads to the mistaken belief that the weight of the mat and any uniformly-distributed floor live loads acting directly on the mat do not induce flexural stresses in the mat and thus can be ignored.

Another common misconception is that lower values of k_s are inherently more conservative. This is not necessarily the case. Lower values generally increase the required top steel reinforcement and decrease the required bottom steel, whereas higher values do the opposite (Tabsh, et al, 2020). However, this effect is not sufficient to overcome the fundamental problems with using a constant k_s.

A third common misconception is that the various SSI uncertainties can be overcome by simply conducting a parametric analysis using different but still constant k_s values, as if this practice would somehow encompass the worst case soil conditions. This belief is incorrect. Variations in the distribution of k_s across the mat, as discussed below, have a much greater impact on design than does changing the magnitude of a constant k_s.

As early as 1965, Meyerhof noted the challenges of evaluating the distribution of bearing pressures acting on mat foundations (Meyerhof, 1965), and called for further research so that "a rational method of their design can be developed".

In spite of these complexities, k_s is sometimes treated as an annoying detail. For example, one widely-used structural analysis software package marketed by a major engineering software publisher uses a fallacious correlation between allowable bearing pressure and k_s, so the structural engineer can simply input an allowable bearing pressure from a geotechnical report or even from the Building Code, then proceed with their design process without any geotechnical characterization of k_s! Clearly there is no correlation between these two unrelated parameters. An older but still frequently quoted foundation engineering textbook includes another fallacious correlation between these two parameters. We can and must do much better than this.

Yet, mat foundations have generally performed well, suggesting that customary analysis and design processes are probably conservative. However, in some cases mat performance has not met expectations (Ergun and Uygurer, 1991; Richards and Kartofilis, 2006; Mayne, 2007; Samarajiva and Gosain, 2010; Russo, et al, 2013), although not necessarily because of incorrect SSI modeling. Regardless, in the interest of safety and economy, the analysis and design process should embrace more modern and more rational methods of evaluating SSI.

2.4. Pseudo-coupled method

The problems associated with using a constant k_s value across the entire mat have led to the use of pseudo-coupled SSI models which are replacing this previous practice (ACI, 2002; Coduto, et al. 2016; Sallam and Casey, 2016; Loukidis and Tamiolakis, 2017). These models attempt to mimic coupling by using stiffer springs (higher k_s values) along the perimeter and especially at the corners of the mat and softer springs (lower k_s values) in the central area. When compared to previous methods that used a constant k_s across the entire mat, these pseudo-coupled methods produce much better assessments of bearing pressure distribution across the mat and correspondingly improved computations of the flexural stresses and distortions within the mat. The result is a mat design that is both more robust and more economical.

Currently, the most useful and well-researched of these zoned pseudo-coupled models is that proposed by Loukidis and Tamiolakis (2017). It expresses the spatial distribution of k_s as described in Equation 2:

$$k_s = k_r(0.55 + C_{H1})\left\{1 + 2C_{H2}\left[\left(\frac{x + 0.1e_x}{L/2}\right)^6 + \left(\frac{y+0.1e_y}{B/2}\right)^6\right] + 4\left[\frac{e_x}{L}\left(\frac{x}{L/2}\right) + \frac{e_y}{B}\left(\frac{y}{B/2}\right)\right]\right\} \quad (2)$$

$$C_{H1} = 0.45\ \exp\left(-2.2\frac{H}{B}\right) \quad (3)$$

$$C_{H2} = \exp\left(-0.4\frac{B}{H}\right) \quad (4)$$

Where k_r is reference coefficient of subgrade reaction for a rigid mat of equal size; B and L are the plan dimensions; x and y are the horizontal coordinates in the L and B directions, respectively, with the origin at the center of the mat; e_x and e_y are the net eccentricities of the applied structural loads; and H is the depth to a firm stratum. When H/B is large and the loading is concentric, Equation 2 produces corner k_s values about 4.8 times that in the center and side k_s values about 3 times that in the center. Kirsch (2011) found a similar pattern. This distribution has the desired effect of increasing the positive flexural stresses, especially near the corners of the mat, and decreasing the negative flexural stresses.

Equation 2 should perform well for rectangular or near-rectangular mats with thickness ≤ perhaps 1000 mm located on soil profiles that are fairly uniform laterally and to at least a depth H then underlain by stiffer strata.

2.5. Discrete area method

Projects that include more unusual or complex mat configurations or those with more complex subsurface profiles require a more intensive method of assessing the spatial distribution of k_s.

For example, the 43-story Mandalay Bay Hotel in Las Vegas has an unusual mat configuration consisting of three radial wings joined at a central core, and the subsurface profile includes stiff caliche strata underlain by softer lacustrine deposits. In spite of successful mat foundations for other nearby structures, this 3 m thick mat experienced differential settlements of about 380 mm while the building was still under construction and had to be underpinned (Thompson, 1998; Sun, 1998; Richards and Kartofilis, 2006).

The discrete area method (Ulrich, 1991) is an extension of the pseudo-coupled method that is more appropriate for these more complex projects. This method evaluates the site-specific SSI effects using an iterative solution where results of structural analyses of the mat and geotechnical analyses of the subsurface are passed back-and-forth with different spatial distributions of k_s until the results of the analyses converge. (Abou-Jaoude and Alzoaby, 2022; Estephan, et al, 2022). This method can accommodate a wider variety of site conditions and has been facilitated by software advances, but requires much greater collaboration between the structural and geotechnical engineers. Sallam and Casey (2016) present a case study using this method for a hospital building that was originally planned to be supported on piles. The construction cost savings were significant and subsequent surveys indicated satisfactory mat performance.

2.6. Horvath-Colasanti/Reissner hybrid model

The Horvath-Colasanti/Reissner (H-C/R) hybrid model (Horvath, 2018) is another method of modeling SSI for mat foundations. Instead of using an array of uncoupled vertical springs, the H-C/R method uses two layers of vertical springs joined by a horizontal deformable tensioned membrane. This model is, at least in theory, a better representation of the true mechanical response of the subgrade to the applied structural loads because it couples the springs.

The efficacy of this model has been demonstrated with case studies. However, sites with complex subsurface conditions may be difficult to model.

3. Design load

Soils and structures respond differently to applied loads, and these differences can introduce complexities in determining the design load to be used in SSI analyses. For example, the soil response to short-term loads (hours or days) is different from that for long-term loads (years). Also, the soil response to static loads that oscillate over time, such as those from storage tanks, is different from that to loads that are more consistent over time, such as those from office buildings. Thus, the loading case being considered impacts the geotechnical characterization and associated settlement analyses.

For the purpose of characterizing k_s, the loading conditions to be considered are largely a geotechnical issue. Ulrich (1991) recommends considering at least two loading conditions: *Total* (all loads, including short-term) and *sustained* (dead load plus long-term live load and any other long-term loads), and notes that the sustained load case often governs the design.

For example, at sites where settlements due to the induced stresses from the mat are expected continue well after construction, such as those due to consolidation of saturated clays, then both short-term (undrained) and long-term (drained) settlement analyses are needed, resulting in two k_s profiles. Shah, et al (2006) did so for a mat foundation on varved silt and clay.

For mats founded in excavations that extend well below natural grade, the weight of the excavated soils and recompression effects must be considered. Also, for mats founded below the groundwater table, the role of hydrostatic uplift pressures must be considered.

Once k_s has been established, the structural engineer characterizes the design load differently. Typically they first conduct a strength analysis using ultimate loads and design the mat accordingly, then check the deformation using the service loads as discussed in Section 4. For example, the load combinations outlined in Sections 2.3 and 2.4, respectively, of ASCE (2022) could be used.

Because k_s is distributed non-uniformly across the mat, the weight of the mat itself and any floor loads acting directly on the mat must be included as part of the design load.

4. Suggested process for characterizing k_s at ordinary sites

In practice, the geotechnical engineer is responsible for developing the design k_s values. The structural engineer then uses these values to design the mat foundation.

Plate load tests are sometimes used to assess k_s in-situ for pavement design. However, the zone of stress influence for mat foundations is orders of magnitude deeper than that for pavements, so this test may provide some insights into the near-surface soils, but is not a means of directly assessing k_s for mat design. Assessments of k_s based on elastic properties (Terzaghi, 1955; Vesić, 1961) also are not suitable, nor is the use of simple tabulated values based on soil classification.

A suggested process for characterizing k_s values at ordinary sites (i.e. those that do not have subsiding ground, as discussed later) is as follows or something similar.

1. Define the location, plan dimensions and bottom elevation of the proposed mat along with an appropriate design load.
2. Divide the design load by the plan area of the mat to determine the average bearing pressure, $\bar{q}$. Adjust for excavation depth and hydrostatic uplift, if appropriate.
3. For mats that satisfy the conditions listed in Section 2.4 use a pseudo-coupled method to determine the design distribution of k_s:
   a. Using the results of a subsurface characterization program and standard geotechnical analysis methods, compute the settlement at the center of a perfectly flexible loaded area having the same size, shape, location and loading of the proposed mat, δ_f (Coduto, et al, 2011; Poulos, 2018).

      As discussed earlier, low k_s values impact the structural analysis one way, whereas high values have a different impact. Thus, contrary to intuition, high δ_f values are not necessarily more conservative. Therefore, it is advisable to perform two settlement evaluations using optimistic and pessimistic soil characterization in order to define a range of likely values which then will envelope the design k_s values.
   b. Based on elastic theory, the settlement of a rigid mat is about 0.76 δ_f (Pantelidis, 2021), so k_r, is:

      $$k_r = \frac{\bar{q}}{0.76\,\delta_f} \qquad (5)$$
   c. Using Equation 2 determine the distribution of k_s across the real mat Repeat with the other k_r value.
   d. If needed, adjust the distributions of k_s to account for non-uniformities in the subsurface conditions across the site.

4. For unusual mat configurations, complex subsurface profiles, or very important projects use the discrete area method:
    a. Determine the location of the resultant of the structural loads, including the weight of the mat, then determine the first trial distribution of the bearing pressure *q* assuming it varies linearly across the mat.
    b. Construct a 3D subsurface continuum numerical model or a 3D Boussinesq model that encompasses the vertical and horizontal changes in stratigraphy across the site. Apply the bearing pressures from Step 4a and assume a perfectly flexible loaded area. Compute the first trial distribution of δ across the mat.
    c. Using the results from Steps 4a and 4b, develop the first trial distribution of k_s across the mat.
    d. Using the trial k_s distribution from Step 4c and a structural numerical model of the mat, compute the distribution of *q* across the mat.
    e. Using the results from Step 4d and the geotechnical model from Step 4b with a perfectly flexible loaded area, develop a revised distributions of δ and k_s across the mat.
    f. Repeat Step 4d using the k_s values from Step 4e.
    g. Continue repeating Steps 4d–4f until the two analyses converge.

As with any suggested procedure, this one can and should be modified on a site-specific basis depending on the circumstances and conditions. It is a broad outline intended for skilled engineers who can adapt it as needed.

Once the k_s distribution across the mat has been defined, the structural engineer uses it to conduct the structural strength and serviceability analyses (Burland, et al, 1977). The strength analysis is used to determine the mat thickness and reinforcement required to carry the internal flexural and shear stresses. It is conducted using factored loads, which implicitly cause greater compression in the Winkler springs and thus create a factored bearing pressure. The required mat thickness is typically governed by punching shear stresses. The required primary reinforcement is governed by the flexural stresses, with consideration of temperature and shrinkage stresses. The serviceability analysis uses unfactored loads and is used to compare the deformation of the mat with the allowable deformation, which includes a factor of safety.

5. Locally subsiding ground

Conventional SSI models are based an important but unstated and often unacknowledged assumption: Settlement of the mat is caused exclusively by the applied structural loads. The analytical springs in these models compress only when they are subjected to bearing pressure from the mat, as described in Equation 1, so if there are no structural loads then there is no settlement.

However, in certain situations the ground also experiences settlements and differential settlements apart from those caused by the induced stresses from the structure. This is subsiding ground. In other words, these settlements occur whether or not the structure is present, and they can significantly redistribute the bearing pressures, thus creating additional flexural stresses in the mat that are not predicted by conventional SSI models, and in some cases also produce global tilting of the structure.

There are two kinds of subsidence: *Regional subsidence* occurs over large areas, perhaps multiple square kilometers, and is due to various natural or anthropogenic processes such as regional groundwater drawdown. Examples include the cities of Venice and Jakarta. These settlements are typically consistent across a building site, or at least vary linearly across the site, and thus usually have little impact on individual mat foundations. In contrast, *local subsidence* occurs on a scale roughly comparable to the size of the project site, can vary significantly across the site, and thus can have a impact the mat design. This process is especially important when the settlement profile is non-planar. Examples include sites with any of the following:

- Recently-placed fill underlain by soft compressible soils. The weight of the fill causes primary consolidation and secondary compression settlement in these underlying strata.
- Groundwater elevations that decline significantly and differentially across the site. For example, Shen, et al (2006) report settlements of up to 330 mm due to construction dewatering at the site of a proposed subway station in Shanghai, along with significant differential settlements. Xia, et al (2006) predicted up to 75 mm of differential settlement in existing buildings due to dewatering of a construction site in Toronto.
- Decomposing organic materials (biocompression), including both natural deposits (Baker, 1995) and municipal solid waste landfills.
- Soils prone to long-term secondary compression. These include sites with certain natural soils as well as those with deep fills. Sites where the fill depth varies significantly over short horizontal distances have often been problematic (Coduto, 2024).
- Poorly-constructed fills
- Soils prone to hydrocompression when wetted, such as from rising groundwater or from surface infiltration of storm water or irrigation water.
- Soils subject to seismically-induced settlements.
- New tunnels below the foundation, such as for subways, mines, or other purposes.

- Collapse of mine voids or other underground cavities such as sinkholes in limestone. Structural damage from this process has been extensively documented in many countries. For example, López Gayarre, et al (2010) describe extensive building damage due to collapse of underground mine voids in Spain that occurred decades after cessation of mining and years after construction of the buildings.
- Settlements due to induced stresses from subsequently-constructed adjacent structures. For example, Hannink (1994) describes such cases in Rotterdam.

These scenarios often play out long after the structure is constructed, producing additional and possibly significant post-construction deformations and flexural stresses in both the mat and the superstructure. This problem is most acute when the stratigraphy changes significantly across the site or when significant local non-uniformities are present within individual strata.

Although it is tempting to use deep foundations in such cases, they can introduce another set of problems both in terms of performance and cost. For example, a structure supported by deep foundations seated in deep firm strata will remain at a nearly constant elevation with minimal distortion, but the shallower soils continue to settle, ultimately resulting in a vertical offset between the finish floor and the outside grade. In addition, deep foundations passing through compressing strata are subject to substantial downdrag loads which can significantly increase the cost of construction. Fortunately, at suitable sites and for appropriate structures, a well-executed mat foundation avoids both of these problems. Pile-enhanced mats where the piles act as settlement reducers or mats underlain by rigid inclusions also can be viable options in some cases, but these solutions are beyond our scope.

6. Characterizing local subsidence

Characterization of local subsidence requires a thorough assessment of the stratigraphy, characterization of the various strata, and a viable analytical model of the physical processes. Some of the processes described above are well understood and, with proper subsurface characterization, can be evaluated using classical soil mechanics or published methodologies. Others are less well understood and thus more difficult to assess.

At some sites multiple subsidence processes might be acting simultaneously, and their effects may or may not be cumulative.

Unlike k_s, larger values of the subsidence settlement are always conservative. So, consistent with standard geotechnical practice, these evaluations should be conservative, but do not include an explicit factor of safety.

In some cases the spatial distribution and magnitude of the expected subsidence can be assessed, and thus may be presented on a plan view of the site as post-construction settlement contours. However, in other cases the subsidence risk is recognizable, but the spatial distribution and/or magnitude are much more difficult to characterize. In such cases, multiple design contour maps might be generated based on a range of potential scenarios.

7. Global tilt

Depending on the subsidence pattern, the mat may experience a global tilt that can be important in itself, especially for tall structures. Global tilt also can produce further redistribution of q and k_s.

The standard Winker model uses a pinned connection between the spring and the mat, so there is no rotational restraint. Thus, any global tilt and the associated redistribution of q and k_s should be implicit in the model. This characteristic could be checked by verifying that the resultant of the bearing pressure is coincident with that of the applied structural load.

8. Modified soil-structure interaction model for locally subsiding sites

Post-construction local subsidence beneath a mat foundation results in a redistribution of both bearing pressure and deformation, and this in turn alters the shear and moment diagrams in the mat, and possibly tilting of both the mat and the superstructure. Therefore, the SSI model for ordinary sites must be modified to account for these effects.

Two design conditions must be considered: First, the condition immediately after construction when no subsidence has yet occurred, and second, the long-term condition that includes the effect of subsidence. The final structural design must envelope both conditions. In any case, this design problem is uncharted territory for most geotechnical and structural engineers, so additional care will probably be necessary.

We will consider two methods of modifying the conventional SSI model to accommodate subsiding ground.

8.1. Gap method

A conceptually straightforward way to modify the pseudo-coupled or discrete area SSI model is to add a gap element in series with the Winkler spring, as shown in Figure 1a. The settlement at a point on the mat due to compression of the spring is δ_1 and the settlement due to subsidence δ_2 at that point for a total settlement δ of:

$$\delta = \delta_1 + \delta_2 \qquad (6)$$

$$= \frac{q}{k_s} + \delta_2 \qquad (7)$$

The same mechanism could alternatively be modelled using a bilinear strain-hardening spring with a stiffness of 0 at displacements $\leq \delta_2$, then a stiffness of k_s beyond that point as shown in Figure 1b. The analysis and design process then proceeds as described in Section 4.

Unfortunately, this model is not compatible with currently-available standard structural analysis software packages, particularly those intended specifically for mat design, and thus is difficult to implement in practice.

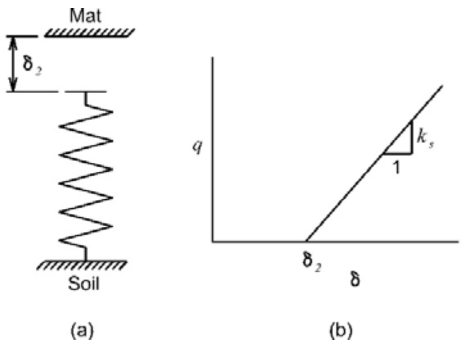

Figure 1. Modified SSI models for locally subsiding ground: a) gap element, b) bilinear spring

8.2. Deaggregation method

An alternative to the gap method is to deaggregate the problem into two components: 1) The response of the mat due to the applied structural loads, including the weight of the mat, and 2) The response of the mat due to the subsidence. The computed flexural stresses in the mat from each component are then combined using superposition. This methodology is not as rigorous and is more tedious to implement, but it is compatible with currently-available standard software.

The deaggregation method can be implemented generally along these lines:

1. Assuming no subsidence, define the distribution of k_s across the mat using either the pseudo-coupled method or the discrete area method as described in Section 4.
2. Using the k_s values from Step 1 and the ultimate structural loads, conduct a structural strength analysis of the mat to determine the required thickness and both the positive and negative flexural stresses. This represents the condition immediately after construction.
3. Evaluate the subsidence potential across the foundation and present the results in the form of a contour map of δ_2 values.
4. Replace the structural loads and the weight of the mat with an array of closely-spaced fixed vertical displacements such that the shape of the deformed mat matches the subsidence contours. Ignoring any soil reaction, compute the associated positive and negative flexural stresses. Multiply these computed stresses by an appropriate load factor.
5. Combine the computed flexural stresses from Steps 2 and 4 by superposition to obtain the total long-term flexural stresses.
6. Envelope the flexural stresses from Steps 2 and 5 to obtain the design stresses for the strength analysis, then design the reinforcing steel accordingly.
7. Using the structural design from Step 6 and the k_s values from Step 1, apply the service loads and evaluate the mat deflection using the structural analysis model. At any location where the computed deflection is less than the subsidence at that point, set k_s=0. At all other locations, use the k_s value from Step 1 multiplied by the ratio of (mat deflection – subsidence) / mat deflection at each point.
8. Revise the mat deflection computation using the new k_s distribution then repeat the adjustment described in Step 7. This process may require multiple iterations to reach convergence.
9. Compare the mat deflection from Step 8 with the serviceability criteria (typically a maximum allowable angular distortion and a maximum allowable global tilt). If the angular distortion is unacceptable, then increase the mat thickness and return to Step 4. If the global tilt is unacceptable, then some other foundation type is probably needed.

Advances in commercial design software to accommodate the gap method would eliminate the need for this deaggregation method.

9. Case study

A 40,100 m^2 single-story reinforced concrete tilt-up warehouse building is to be constructed on a site that was previously an open-pit sand-and-gravel mine. The site has been backfilled to natural grade with engineered fill but is prone to long-term secondary compression settlement as described in Coduto (2024).

Under ordinary circumstances this building would have been supported on spread footings with a 150 mm thick slab-on-grade floor. However, due to the expected subsidence a monolithic mat foundation was chosen.

The mat was analyzed and designed using a deaggregation method similar to that described in Section 8.2 with the Loukidis and Tamiolakis distribution of k_s. One challenge was to make the mat stiff enough to satisfy the design criterion for angular distortion while avoiding a design that is overly stiff. The result was a 460 mm thick mat with top-and-bottom reinforcement both ways.

This case study demonstrates that, at least in this case, a well-designed mat foundation is very effective in supporting a conventional building on subsiding soils and does so at a reasonable cost of construction.

10. Conclusions

Standard practice uses the coefficient of subgrade reaction, k_s, to model soil-structure interaction for mat foundation design. Characterization of k_s requires site-specific geotechnical assessments of the subsurface conditions as well as a knowledge of the size, location and loading of the proposed mat. At ordinary sites, a careful characterization of this parameter and its spatial distribution appears to provide a sound basis for design.

Sites expected to experience local subsidence will experience a redistribution of the bearing pressure between the mat and the underlying ground, which adds further complications to the soil-structure interaction problem. In some cases, the impact of subsidence on the flexural stresses is greater than that due to the applied structural loads. This behavior can be characterized in the SSI model by adding a gap element in series with the Winkler spring at each node or by deaggregating the problem by separately computing flexural stresses due to the structural loads and those due to subsidence, then combining them using superposition.

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Dr. Weian Liu, SE and Dr. Rafik Gerges, SE developed the deaggregation methodology and implemented it in the project described in the case study. The author subsequently modified and extended this methodology to that presented in Section 8.2.

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