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MODEL ORDER REDUCTION FOR COUPLED MULTI-DOMAIN SIMULATIONS APPLIED TO HELICOPTERS

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Summary. The consideration of different physical phenomena is often required for a realistic simulative representation of complex systems. An example of such a complex system, which is examined in this paper, is a helicopter in flight. A strong interaction between the structure and the ambient air occurs. For modeling of the mechanical part of the coupled system, the Finite Element method is frequently used, leading to very high-dimensional systems of differential equations. In order to still enable efficient and effective simulations, Model Order Reduction (MOR) is used. By coupling the different domains, though, the individual subsystems often have numerous inputs and outputs which poses a special challenge for many MOR methods. In this paper we present a novel hybrid method that combines modal truncation and moment matching with Krylov subspaces. Significant improvements were revealed for this hybrid method compared to state-of-the-art methods. The hybrid method uses harmonic force amplitudes as inputs for the reduction. Large speed-ups were obtained whereby the reduced models are well suited for the use in transient co-simulation.

1 INTRODUCTION

Helicopters are lightweight structures that are exposed to strong and often periodic forces during operation. This makes helicopters prone to vibrations. For the simulative prediction and investigation of these vibrations, different physical domains have to be considered. These domains are the mechanical structure on the one hand and the ambient air on the other.

Coupled systems of this type can be investigated with co-simulations. In co-simulations, the different domains are solved separately and data are exchanged between them to take into account the interactions. For complex systems, the employed Finite Element (FE) models are very high-dimensional and numerically expensive to solve. Model Order Reduction (MOR) provides a remedy with the purpose to approximate the full order system with one of much lower dimension while keeping the most relevant characteristics. In previous publications in the field of helicopter dynamics, mainly modal truncation was used to reduce the structural models as in [1, 2]. For mechanical systems, however, other methods have gained importance in recent years, as e.g., moment matching with Krylov subspaces, see e.g., [3, 4, 5]. Moment matching has already been successfully applied to FE models of helicopters in [6, 7, 8]. For use in co-simulation, though, there arises a particular challenge: The number of inputs and outputs resulting from the coupling of different domains over large areas is very high.

This paper examines how the MOR method based on moment matching can be adapted for use in co-simulations. We show a hybrid method that combines modal truncation and moment matching and furthermore uses harmonic input forces for moment matching. It is compared with the state-of-the-art modal truncation method and standard moment matching without adaptations. This hybrid method as well as its application to the model of an industrial helicopter are the novelties of this contribution.

The paper is structured as follows. In Section 2, the general framework for the coupled multi-domain simulation of helicopters is described and some challenges are outlined. In Section 3 the most relevant theoretical background on projection based MOR is given. Section 4 deals with the application of MOR to multi-domain simulations. In Section 5, some results for the industrial FE model of the helicopter are presented and compared. Section 6 concludes this article.

2 SIMULATION FRAMEWORK

Co-simulation is a wide-spread method for the simulation of heterogeneous models. The entire system is divided into different subsystems, e.g., into its physical domains. Each subsystem can be modeled using the most appropriate tool and co-simulation coordinates the interactions between these subsystems. This approach is particularly beneficial when dealing with complex systems that are composed of various subsystems, each requiring specialized modeling techniques. However, the challenges associated with integration, synchronization, and data exchange between subsystems need to be carefully addressed to fully leverage the benefits of co-simulation.

An example of a complex system where co-simulation can be beneficial is a helicopter in flight. Strong interactions exist between the structure of the helicopter and the surrounding air. In the following, we refer to the Finite Element model of the fuselage of the helicopter with the structural model and to the surrounding air with the fluid model. We restrict ourselves to steady flight conditions, e.g., hovering or a steady forward flight. The first important step in such simulations is an initial trim to find an equilibrium solution of a steady flight state. This trim defines control variables such as blade pitch angles for the transient simulation. In principle, it is also possible to include the trim in the transient simulation workflow and by this allow for unsteady flight conditions, but this is not in the scope of this paper.

With the trim solution, we can start the transient simulation process sketched in Figure 1. The left box represents the fluid system Σ_{F} and the right box depicts the structural system Σ_{S}. Between those systems, data have to be exchanged and in the course of this coordinate transformations and mappings have to be performed. The simulation of the fluid system provides the pressures $\boldsymbol{p}$ that act on the cells of the Computational Fluid Dynamics (CFD) grid. These pressures are used to compute the forces that act on the FE model of the fuselage. Typically, the spatial discretization used for the fluid system is much finer than the one used for the structural system. Especially at locations with high geometric gradients, where complex aerodynamic

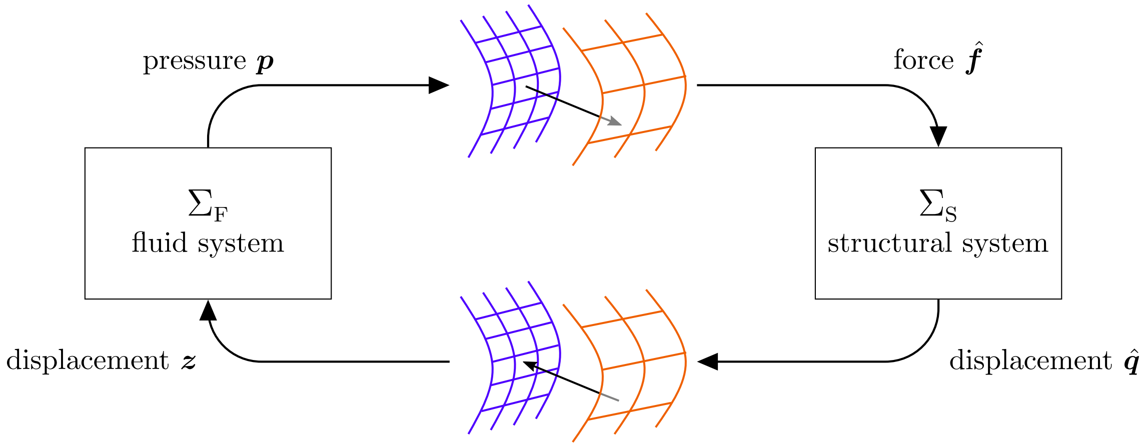

Figure 1: Workflow of the coupled simulation

phenomena are expected, a fine discretization is necessary [1]. The loads have to be transferred from the finer fluid grid to the structural FE model while ensuring conservativity, i.e., the integral loads over the surface must match for both discretizations. For this purpose, the normal force that acts at the center of each CFD surface cell is computed by multiplication of its area and the pressure that was obtained from the CFD simulation step. The structural model is then transformed by translation, rotation, and scaling so that both models overlap. Now, for every cell center, the closest structural node is determined and the force is applied to it. The small deviation between cell centers and FE nodes is considered by applying moments computed with the force and the distance as lever. This leads to the load vector $\hat{\boldsymbol{f}} \in \mathbb{R}^{6n_{\mathrm{FE}_\mathrm{nodes}}}$, which contains the forces and moments that act on the surface nodes. In contrast to concentrated forces often used in FE analyses, this load vector is usually fully populated in the setting of co-simulations, because the exchange surface is large and covers many of the $n_{\mathrm{FE}_\mathrm{nodes}}$ FE nodes.

The load vector is then applied to the structural system and the time step is solved by numerical integration. This leads to an updated displacement $\hat{\boldsymbol{q}} \in \mathbb{R}^{6n_{\mathrm{FE}_\mathrm{nodes}}}$. With this displacement, the position of grid nodes in the CFD grid $\boldsymbol{z} \in \mathbb{R}^{3n_{\mathrm{CFD}_\mathrm{nodes}}}$ is updated by the use of radial basis functions, see [1]. With the updated grid, the next time step is computed.

For the practical application of this tool chain, different software is used. The trim simulation is performed with the comprehensive rotorcraft solver *CAMRAD II* [9, 10]. The FE model is build in *MSC Nastran* and imported to *Matlab* with the MOR toolbox *MatMorembs* [11]. The reduced structural system and the fluid system are coupled with the coupling manager *HeliCats* which is developed at the Institute of Aerodynamics and Gas Dynamics of the University of Stuttgart. The fluid simulation is conducted with the CFD code *FLOWer* [12].

Figure 2 shows the industrial FE model that is used for the structural simulation. Some things are noticeable and should not go unmentioned at this point:

- The main rotor and the fenestron are not contained in the finite element model and just represented by surrogate masses
- Side doors and side windows are replaced by surrogate masses
- The resolution of the tailboom is much finer than the resolution of the cabin

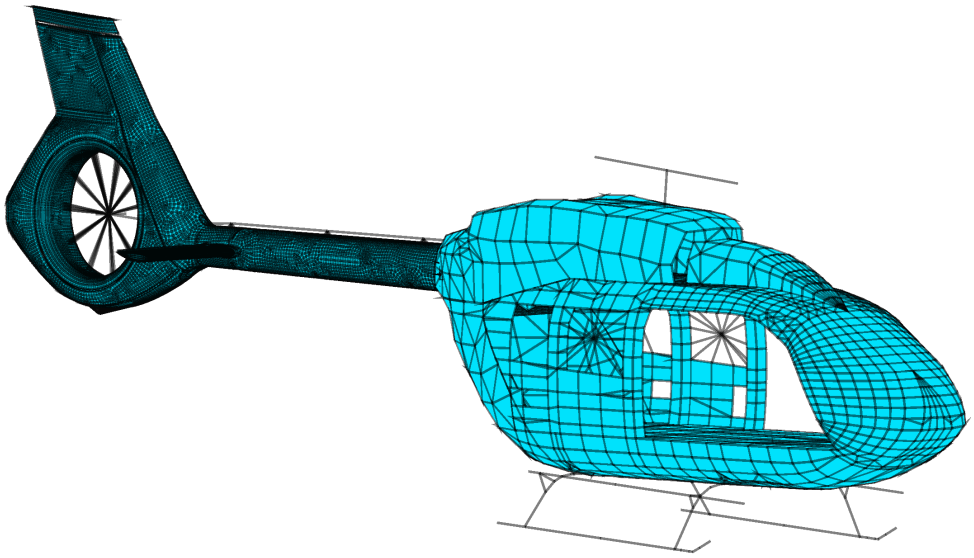

Figure 2: Industrial FE model of the helicopter fuselage

These three observations have an influence on the forces acting on the nodes of the FE model. As some parts are not fully resolved, the force of the surrounding CFD cells is summed up and acts on few substitute nodes that are the closest nodes to many CFD cells. For these reasons we end up with large forces at the hub nodes of the main rotor and of the fenestron, as well as with large forces onto the surrogate nodes in the center of doors and windows as well as at the frame of both. The simplifications in the model are covered by the coupling process, but pose a limitation in the accuracy of the simulation. Nevertheless, the FE model is still very high dimensional exceeding hundreds of thousands degrees of freedom leading to high numerical costs for the transient simulation. This motivates the use of MOR, which aims to represent the full-order system by a low-rank approximation while preserving its key properties.

3 PROJECTION BASED MODEL ORDER REDUCTION

Mechanical systems are often spatially discretized and then described with the linear Finite Element Method (FEM), see [13]. This results in the high-dimensional system of linear second-order differential equations

$$\boldsymbol{M}\ddot{\boldsymbol{q}}(t) + \boldsymbol{D}\dot{\boldsymbol{q}}(t) + \boldsymbol{K}\boldsymbol{q}(t) = \boldsymbol{f}(t). \tag{1}$$

Here, $\boldsymbol{M}, \boldsymbol{D}, \boldsymbol{K} \in \mathbb{R}^{N\times N}$ are the mass, damping, and stiffness matrix, respectively. The state vector $\boldsymbol{q}(t) \in \mathbb{R}^N$ describes the nodal displacements. The notation $\dot{\boldsymbol{q}}(t) := {}^{\mathrm{d}\boldsymbol{q}(t)}/_{\mathrm{d}t}$ and $\ddot{\boldsymbol{q}}(t) := {}^{\mathrm{d}^2\boldsymbol{q}(t)}/_{\mathrm{d}t^2}$ is used for the time derivatives in this paper. The right-hand term $\boldsymbol{f}(t)$ contains all external forces that act on the system. By factorizing the force vector into its spatial distribution and a time dependent input signal with the product $\boldsymbol{f}(t) = \boldsymbol{B}\boldsymbol{u}(t)$ and by extracting system outputs with $\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{q}(t)$, Equation (1) can be interpreted as a linear second order input-output system

$$\begin{aligned} \boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{D}\dot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} &= \boldsymbol{B}\boldsymbol{u}, \\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{q}. \end{aligned} \tag{2}$$

The matrices $\boldsymbol{B} \in \mathbb{R}^{N\times b}$ and $\boldsymbol{C} \in \mathbb{R}^{c\times N}$ are the input and output matrix of the system and the vectors $\boldsymbol{u} \in \mathbb{R}^b$ and $\boldsymbol{y} \in \mathbb{R}^c$ are the inputs and outputs. The time dependency is omitted for better readability.

Linear time-invariant systems of this type can effectively be reduced with projection based MOR, see [14, 15, 16]. Its idea is to approximate the state vector within a subspace $\mathcal{V} = \mathrm{span}\{\boldsymbol{V}\}$ with

$$\boldsymbol{q} \approx \boldsymbol{V}\widetilde{\boldsymbol{q}}, \qquad \dim(\boldsymbol{q}) = N \gg \dim(\widetilde{\boldsymbol{q}}) = n, \qquad \boldsymbol{V} \in \mathbb{R}^{N\times n}. \tag{3}$$

Plugging the approximation from Equation (3) in Equation (2) and left-multiplying by $\boldsymbol{V}^\top$ leads to the reduced model

$$\begin{aligned} \widetilde{\boldsymbol{M}}\ddot{\widetilde{\boldsymbol{q}}} + \widetilde{\boldsymbol{D}}\dot{\widetilde{\boldsymbol{q}}} + \widetilde{\boldsymbol{K}}\widetilde{\boldsymbol{q}} = \widetilde{\boldsymbol{B}}\boldsymbol{u}, \\ \widetilde{\boldsymbol{y}} = \widetilde{\boldsymbol{C}}\widetilde{\boldsymbol{q}}. \end{aligned} \tag{4}$$

The tilde indicates quantities of the reduced model and its quite small system matrices result from

$$\widetilde{\boldsymbol{M}} = \boldsymbol{V}^\top\boldsymbol{M}\boldsymbol{V}, \quad \widetilde{\boldsymbol{D}} = \boldsymbol{V}^\top\boldsymbol{D}\boldsymbol{V}, \quad \widetilde{\boldsymbol{K}} = \boldsymbol{V}^\top\boldsymbol{K}\boldsymbol{V}, \quad \widetilde{\boldsymbol{B}} = \boldsymbol{V}^\top\boldsymbol{B}, \quad \widetilde{\boldsymbol{C}} = \boldsymbol{C}\boldsymbol{V}. \tag{5}$$

The matrix $\boldsymbol{V}$ is called projection matrix and is a core element of projection based MOR. There are different methods to construct it.

A commonly used method that is also very popular in aerodynamics is modal truncation. In modal truncation, the first n eigenmodes $\boldsymbol{\phi}_i$, with $i = 1, \ldots, n$, of the system are calculated and used as columns for the projection matrix

$$\boldsymbol{V} = \left[\begin{array}{ccc} \boldsymbol{\phi}_1 & \ldots & \boldsymbol{\phi}_n \end{array}\right]. \tag{6}$$

However, it is often unclear whether the first modes are the ones that contribute most to the deformation behavior for the load case at hand. Furthermore, local forces can only be described well with many modes.

A method that takes a different approach is moment matching with Krylov subspaces. It aims at approximating the transfer function $\boldsymbol{H}(s) = \boldsymbol{C}(s^2\boldsymbol{M} + s\boldsymbol{D} + \boldsymbol{K})^{-1}\boldsymbol{B}$ of the system, where s is the Laplace variable. Therefore, the transfer function is written as a power series

$$\boldsymbol{H}(s) = \sum_j^\infty (s - s_k)\frac{1}{j!}\frac{\partial^j \boldsymbol{H}(s)}{\partial s^j}\Big|_{s=s_k}, \quad j = 0, 1, \ldots, J_k - 1, \tag{7}$$

around expansion points s_k, also called shifts, as explained in [16, 17]. The first J_k moments are then matched at different shifts s_k. This is implicitly done by the use of Krylov subspaces, as extensively described in [18, 19]. A numerically stable algorithm that produces such subspaces is the second order Arnoldi algorithm. It uses a modified Gram-Schmidt orthogonalization to ensure stability and is presented in [19, 20].

Moment matching was effectively applied to many mechanical systems as in [21, 22] for first-order moments or in [4, 23] for higher-order moments. However, in all these contributions, the number of inputs and outputs is limited to very few. The classical moment matching approach is very inefficient if forces act on many nodes. This is because in its basic form, moment matching produces reduced models with an order that is a multiple of the number of inputs and the number of shifts. For undamped systems with $\boldsymbol{D} = \boldsymbol{0}$, b real vectors are added to the projection matrix for each shift in the frequency domain. For damped systems, the Krylov vectors $\boldsymbol{v}$ are complex and two vectors $\mathrm{Re}(\boldsymbol{v})$ and $\mathrm{Im}(\boldsymbol{v})$ are added for every input at each shift, as explained e.g., in [6]. Although after calculating a new Krylov vector, a Gram-Schmidt orthogonalization is performed and vectors that are almost linearly dependent on the existing basis are truncated, the size of the reduced model grows very fast for many inputs. This property makes moment matching unsuitable for co-simulations, at least if no further adaptions are made.

4 MODEL ORDER REDUCTION IN CO-SIMULATION

High numbers of inputs and outputs are characteristic for co-simulations with exchange of data between different domains. For methods such as modal truncation, which do not depend on the inputs and outputs of the system, this does not pose a challenge and the procedure can be the same as for systems with few inputs and outputs. For methods that depend on the number of inputs, however, adjustments must be made in order to apply them effectively in co-simulation.

In general, the number of inputs have to be somehow reduced. This can be done with tangential interpolation [24, 25] by either using just some of the original inputs or by constructing a new set of inputs of lower dimension e.g., with SVDMOR, see [26] for its general idea and [27] for its application to mechanical systems. Usually, if nothing about the forces acting during the simulation is known previously, all possible inputs are considered independent of each other. This results in a large sparse Boolean matrix $\boldsymbol{B}$. Every column represents one input direction which is one coordinate direction of one surface FE node. If moments and forces act on all nodes of our system, which could well be the case for coupled multi-domain systems, $\boldsymbol{B} = \boldsymbol{I} \in \mathbb{R}^{N\times N}$. The assumption of entirely independent inputs allows completely arbitrary input forces and does not incorporate the distribution of surface loads. In reality, however, it is very unlikely that if one node experiences a very high aerodynamic load, no load will be applied to the neighboring node. If we assume that the forces that act on the mechanical subsystem are known in advance, their distribution can be taken into account. This assumption can be legitimate if, for example, the forces can be calculated in a preliminary calculation with a rigid mechanical system. We then obtain a snapshot matrix

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{b}(t_0) & \boldsymbol{b}(t_1) & \dots & \boldsymbol{b}(t_{\mathrm{end}}) \end{bmatrix} \tag{8}$$

from the simulation with the rigid body from starting time t_0 to final time t_{end} and discrete time steps in between. This matrix is potentially also large, depending on the simulation time and the step size, but we can decompose this matrix with Singular Value Decomposition (SVD) to get an optimal low-rank approximation, see [28]. If the inputs are periodic, another possibility is to compute the harmonic components of the snapshot matrix with a discrete Fourier transformation. In that way, one obtains the amplitudes of the input force at different frequencies. These vectors of amplitudes can be used as input directions for shifts in moment matching at these frequencies.

With these ideas, there are ways to cope with the large number of inputs in co-simulation. Yet, we also have to ensure that the reduced system approximates the displacement of all surface nodes well, i.e., we also have to deal with many outputs. In contrast to the inputs, we do not know the outputs previously. While moment matching mainly captures the behavior of different load paths, modal truncation retains the global properties of the system. It therefore seems natural to combine both approaches. Moment matching to approximate the inputs of the system and modal truncation to ensure approximately correct displacements of all surface nodes. The idea of combining different reduction methods to get appropriate reduced order models for a specific use case is not new but known from component mode synthesis, see [29, 30].

The combination of two projection-based MOR methods to one hybrid method is straightforward. Two reduced subspace from modal truncation and moment matching, spanned by the bases $\boldsymbol{V}_{\text{mod}} \in \mathbb{R}^{N \times n_{\text{mod}}}$ and $\boldsymbol{V}_{\text{mm}} \in \mathbb{R}^{N \times n_{\text{mm}}}$ can be unified by concatenating them to

$$\overline{\boldsymbol{V}}_{\text{hyb}} = \begin{bmatrix} \boldsymbol{V}_{\text{mod}} & \boldsymbol{V}_{\text{mm}} \end{bmatrix} \in \mathbb{R}^{N \times n_{\text{mod}} n_{\text{mm}}}. \tag{9}$$

To ensure full rank of the projection matrix, $\overline{\boldsymbol{V}}_{\text{hyb}}$ is decomposed with a singular value decomposition

$$\overline{\boldsymbol{V}}_{\text{hyb}} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{R} \tag{10}$$

and only those left singular vectors corresponding to singular values larger than a given threshold are kept to span the reduced subspace

$$\mathcal{V} = \operatorname{span}\{\boldsymbol{V}_{\text{hyb}}\} \quad \text{with} \quad \boldsymbol{V}_{\text{hyb}} = \begin{bmatrix} \boldsymbol{u}_1 & \boldsymbol{u}_2 & \dots & \boldsymbol{u}_{n_{\text{hyb}}} \end{bmatrix} \quad \text{and} \quad n_{\text{hyb}} \le n_{\text{mod}} n_{\text{mm}}. \tag{11}$$

The vectors $\boldsymbol{u}_i$ are the first n_{hyb} columns of $\boldsymbol{U}$.

5 RESULTS

We now apply the proposed method of MOR for co-simulation to the FE model of the helicopter fuselage. For perceptible vibrations of the fuselage, frequencies below 40Hz are especially relevant. Thus, we will concentrate on this frequency range in the following. The first 30 elastic eigenfrequencies of the model are in this range. The largest forces are expected to be at the blade passage frequency (bpf) and for that reason a good correspondence of the reduced order model and the full order model at this specific frequency is particularly important.

For the evaluation of the reduced order models we consider the relative error

$$\epsilon = \frac{||\boldsymbol{H}(s) - \widetilde{\boldsymbol{H}}(s)||_{\text{F}}}{||\boldsymbol{H}(s)||_{\text{F}}} \tag{12}$$

in the Frobenius norm of the transfer function. The transfer function is computed for the harmonic input force vectors and some characteristic output points which are translations and rotations at the main rotor hub, the fenestron hub, the pilot seat, the copilot seat, and one point each on the right and left side of the cabin frame. The error in the Frobenius norm also yields bounds for the error in time domain, see [31, 32].

In Figure 3, the relative Frobenius norm errors for three reduced system are compared. All have a comparable reduced order of 40 and 37, respectively. The bpf is marked with the dashed line. The model resulting from modal truncation with the first 40 eigenmodes yields a model where the error is constantly below 18%. Compared to the model created with moment matching, where the error is huge around the bpf, this is good, but an error of 18% is still not satisfying.

For the moment matching model, six available discrete frequencies from the Fourier transformation were used with the corresponding shape of the input force. The harmonic input forces at these frequencies result from a co-simulation run with rigid fuselage to obtain the snapshots

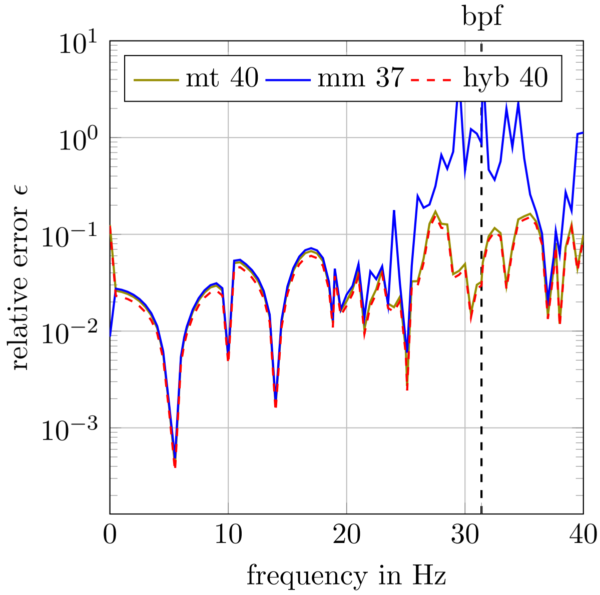

Figure 3: Frobenius norm error for the reduced order models with modal truncation (mt), moment matching (mm) and the proposed hybrid method (hyb) with the first 36 eigenmodes and 4th order moment matching at bpf

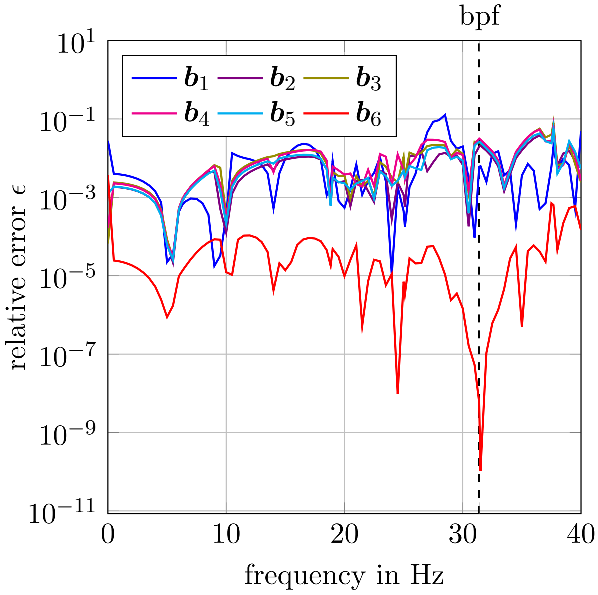

Figure 4: Frobenius norm error for the hybrid reduction separated into different inputs with $\boldsymbol{b}_k$ being the amplitudes of the harmonic forces at the computed discrete Fourier frequencies $f_k = 0, 1/5\,\mathrm{bpf}, 2/5\,\mathrm{bpf}, \dots, \mathrm{bpf}$

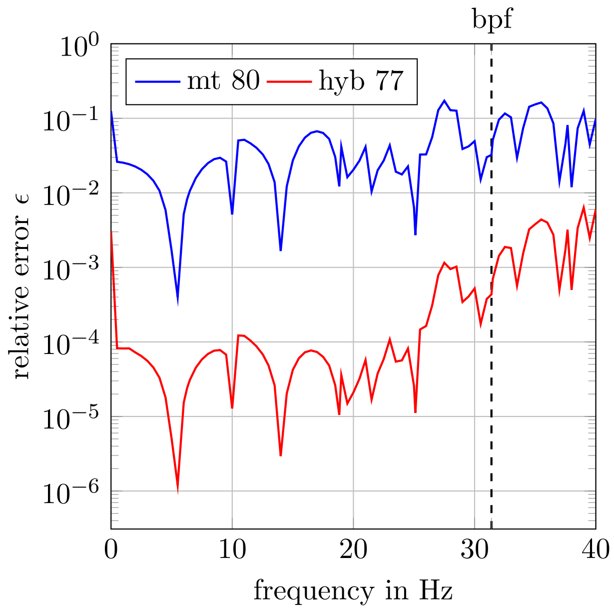

Figure 5: Frobenius norm error for larger reduced models with modal truncation and the hybrid approach

$\hat{\boldsymbol{f}}(t)$. These snapshots are projected onto the constraint system to obtain snapshots $\boldsymbol{f}(t)$. With a Fourier transformation, the amplitudes $\boldsymbol{F}(s)$ are obtained. These periodic input forces are used for the reduction with shifts at $f_k = 0, 1/5\,\mathrm{bpf}, 2/5\,\mathrm{bpf}, \ldots, \mathrm{bpf}$ with $k = 1, ..., 6$. All moments were matched until the sixth order for the respective input force at each of these frequencies to obtain a model with comparable number of degrees of freedom.

If we enrich a modal basis, here the first 36 eigenmodes, with four Krylov modes the red dashed error is obtained. At first glance, it does not reveal a proper advantage compared to modal truncation. However, Figure 4 shows that there is a big difference if we investigate the different input directions separately. The Frobenius norm is shown for every column of the transfer matrix separately. This means, that the error of the transfer function for each input direction $\boldsymbol{b}_k$ to all outputs is displayed over the frequency range. For the input $\boldsymbol{b}_6$, which is the harmonic input at bpf, we see that the error gets very low around this frequency. If the model is excited with the predicted shape of the force at the bpf, the error is very small. Since we expect that this force acts on the model in the coupled simulation, it is much better suited for use in this. Just for other input forces, the error is still big and, therefore, the Frobenius norm over all inputs shown in Figure 3 stays almost unchanged.

If we match every input direction at the respective frequencies of $f_k = 0, 1/5\,\mathrm{bpf}, \ldots, \mathrm{bpf}$, we can improve the Frobenius norm over all inputs. This is demonstrated in Figure 5, where the modal basis with 40 modes is enriched with 37 modes obtained from moment matching at frequencies of $f_k = 0, 1/5\,\mathrm{bpf}, 2/5\,\mathrm{bpf}, \ldots, \mathrm{bpf}$. Here the error is quite low over the whole frequency range. This is not achievable with modal truncation. Although the first 80 eigenmodes are used to span the reduced subspace, the approximation error is not improved significantly compared to modal truncation with 40 modes. This observation shows that we do not obtain satisfying approximation quality with modal truncation unless we keep almost all modes in the reduced basis which, however, is against the purpose of reduction. Standard moment matching is also not appropriate for the model with very many inputs. The combination of modal truncation with Krylov modes created with harmonic force inputs at different input frequencies, provides a good solution.

The reduction of the FE model enables transient simulations which are not possible with the full-order model in a reasonable time. To get an impression of the computational time savings we compare the elapsed time for the computation of the transfer functions. The full model evaluation took 112min while the reduced ones all compute in under 1s. These are speed-up factors in the range of 6500 and larger. Similar speed-ups can be expected in time domain.

6 CONCLUSION

In this paper, it was shown how projection-based MOR can be applied to multi-domain co-simulations. This was done with the example of an industrial FE model of a helicopter fuselage. A hybrid method that combines modal truncation and moment matching with Krylov subspaces was presented and compared to both standard methods. Significant improvements were revealed for this hybrid method which uses harmonic force amplitudes as inputs for the reduction. Large speed-ups were obtained whereby the reduced models are well suited for the use in transient co-simulation.

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