Inherent and CPTu-measured scale of fluctuation of undrained geomaterials: a numerical perspective

Lluís Monforte[1#] and *Stefano* Collico[2]

[1]*Centre Internacional de Mètodes Numèrics en Enginyeria, Barcelona, Spain*
[2]*DMT GmbH, Essen, Germany*
[#]*Corresponding author: lluis.monforte@upc.edu*

ABSTRACT

The vertical scale of fluctuation of soil parameters is often indirectly estimated through Cone Penetration Testing (CPTu) as the test provides nearly continuous and repeatable data. However, the variability quantified from CPTu might not be representative of a soil parameter intra-site variability, but rather the effect of the variability of soil measured through its response to the cone probe. The objective of this work -of an openly prospective nature- is to examine how the vertical inherent spatial variability of clay-like insensitive geomaterials propagates to the cone tip and friction sleeve resistance by means of numerical modelling. A total stress analysis is presented, in which the undrained shear strength is described according to random field theory. Numerical results show that the scale of fluctuation of the tip resistance and friction sleeve resistance is greater than the one assumed for undrained shear strength.

Keywords: Variability; undrained shear strength; Random Finite Element method; numerical analysis.

1. Introduction

The vertical inherent variability of a soil parameter (e.g., undrained shear strength, friction angle) is usually quantified through some random field correlation model and its key parameter is the scale of fluctuation, θ (Vanmarke 1984; Griffiths et al. 2011).

It is common practice to estimate θ indirectly by means of Cone Penetration Testing (CPTu) (Alonso and Krizek 1975; Uzielli et al. 2005; Wang et al. 2010) since the test provides almost continuous and repeatable data. For clayey materials, such previous studies report that the vertical scale of fluctuation of normalized cone metrics (i.e., normalized tip resistance and friction ratio) varies from 0.13 m to almost one meter with a coefficient of variation (COV) that ranges from 0.02 up to 0.33 Information on horizontal variability is scarce. However, due to the geological processes involved in the formation of soils, the horizontal scale of fluctuation is, at least, one order of magnitude higher than the vertical one (Phoon and Kulhawi, 1999).

However, the scale of fluctuation computed from CPTu is rather representative of the intra-site variability of soil response to the cone probe rather than the inherent variability of a soil parameter itself (Uzielli et al. 2005).

For clayey materials, several evidence suggest that the scale of fluctuation of a soil parameter might deviate from the one of indirect geotechnical parameter computed from cone tip resistance and/or friction sleeve resistance. It is well-known that the cone tip resistance averages the behavior of a large volume of soil around (beneath and below) the tip: total stress, numerical analysis of cone penetration test in Tresca soil (Lu et al. 2004) shows that cone penetration induces plasticity at a soil mass that extends between 5-12 radii of the cone, depending on the rigidity index of the soil. Other insights on how cone tip resistance is an averaged measure can be obtained from current knowledge of soil repose to CPTu in layered soils. As example, the numerical analysis on layered clay-like soils reported by Walker and Yu (2006) showed that during the transition from a soil unit characterized by a higher undrained shear strength to one having a lower strength resistance, the cone tip resistance start decreasing 5.2 cone radii before the soil layer interface. On the other hand, the new steady state cone tip resistance is not encountered after the cone has penetrated 4.4 radii from the interface; as such, the presence of a soil layer interface influences the soil response to cone probe of about 9.9 cone radii.

The aforementioned evidence suggests that, at least in low permeability soils, the scale of fluctuation of a soil parameter could diverge from the one computed from the cone tip resistance due to geometric averaging nature of cone metrics.

This work presents the first numerical attempt to quantify the relation between the scale of fluctuation of direct soil parameter measurements and the one computed from cone tip and friction sleeve resistances. To do this, we report a set of simulations of CPTu testing in unsensitive, undrained clayey materials. We use a total stress approach and model the constitutive response using a quasi-incompressible elastic perfectly plastic model, using a Tresca yield surface. This way, the only two relevant constitutive parameters are the shear modulus, which is assumed to be constant, and the undrained shear strength, which is described as a random field, whose scale of fluctuation in the horizontal directions is infinite.

2. Numerical approach

Numerical simulations have been performed by means of G-PFEM (Geotechnical Particle Finite Element method) (Carbonell et al. 2022), a numerical method specially designed for the analysis of problems involving the penetration of rigid structures into soil masses (Monforte et al, 2017).

In the present work, we restrict our attention to unsensitive, clayey material. This allows us to employ a total stress approach -as clayey materials typically depict a practically undrained response during CPTu testing- and model the soil constitutive response within an elastic perfectly plastic model. In particular, the elastic model is linear and quasi-incompressible, whereas a Tresca surface describes the yield criterion.

The tip and shaft of the cone are rough, and the maximum allowable tangential stress at the cone-soil interface is a fraction, α, of the undrained shear strength of the soil.

As pointed out in the introduction, several field works suggest that the vertical scale of fluctuation of soil parameters is several orders of magnitude smaller than the horizontal scale of fluctuation. Based on that, in the present work it is assumed that the horizontal scale of fluctuation is infinite. Consequently, an axisymmetric model is used, since only the vertical inherent variability is exploited. Considerations about possible inclusion of inter-site variability (i.e., horizontal scale of fluctuation) will only be discussed and not considered here.

2.1. Random field model

In this work, the undrained shear strength is modeled as a second-order stationary random field. As this variable is strictly positive, the undrained shear strength, S_u, follows a lognormal distribution with parameters $\mu_{\ln(S_u)}$ and $\sigma_{ln(S_u)}$. A spherical correlation function is then considered for the logarithm of the undrained shear strength, with a decay rate defined by the scale of fluctuation $\theta_{\ln(S_u)}$ such that:

$$\rho_{\ln(S_u)}(d) = \begin{cases} 1 - \frac{3}{2}\frac{d}{\theta_{\ln(S_u)}} + \frac{1}{2}\left(\frac{d}{\theta_{\ln(S_u)}}\right)^3 & \text{if } d < \theta_{\ln(S_u)} \\ 0 & \text{else} \end{cases} \tag{1}$$

where $\theta_{\ln(S_u)}$ is the scale of fluctuation of the logarithm of the undrained shear strength and d is the (absolute value) vertical distance between two spatial points.

As done by Li et al (2021), the lognormally distributed random field is generated by transforming a standard normal random field as follow:

$$S_u(z) = \exp\left(\mu_{ln(S_u)} + \sigma_{ln(S_u)} \cdot X_{\theta_{\ln(S_u)}}(z)\right) \tag{2}$$

where z stands for the vertical spatial coordinate, $\mu_{\ln(S_u)}$ and $\sigma_{ln(S_u)}$ are the parameters of the lognormal density distribution of undrained shear strength and $X_{\theta_{\ln(S_u)}}(z)$ is a normally distributed random field with null mean, unit variance and the scale of fluctuation $\theta_{\ln(S_u)}$. This random field is constructed using a Cholesky decomposition (see, for instance, Wang et al. 2010).

The relationship among $\mu_{\ln(S_u)}$ and $\sigma_{ln(S_u)}$ and the mean, μ_{S_u}, and standard deviation, σ_{S_u}, of the undrained shear strength reads:

$$\sigma^2_{\ln(S_u)} = \ln\left(1 + \frac{\sigma^2_{S_u}}{\mu^2_{S_u}}\right) \tag{3}$$

$$\mu_{\ln(S_u)} = \ln(\mu_{S_u}) - \frac{1}{2}\,\sigma^2_{\ln(S_u)} \tag{4}$$

2.2. Numerical domain and constitutive parameters

The numerical domain expands 0.8 m in width and has a height of 1.5 m. Null radial displacements are prescribed in the vertical boundaries. On the bottom boundary, null displacements are prescribed. A vertical load of 100 kPa is placed on the top boundary.

In this work, the effect of soil self-weight is neglected. This hypothesis, even if questionable -as the height of the model is 1.5 meters-, comes handy: the trend function of all random fields (e.g. cone resistance, friction ratio,…) is constant.

The in situ horizontal and vertical total stresses are set to 100 kPa. With the only purpose of computing the normalized metrics, it is assumed that the in-situ vertical effective stress is equal to $\sigma'_{v0} = 40$ kPa. This value has been assumed by considering that, for normally consolidated soils, the relation between the undrained shear strength of the soil and the vertical effective stress is typically $\mu_{S_u} \approx 0.25\,\sigma'_{v0}$ (Nova 2002).

In the current work, all simulations assume that the mean value of the undrained shear strength is equal to $\mu_{S_u} = 10$ kPa and the standard deviation is $\sigma_{S_u} = 3$ kPa. In all simulations, the scale of fluctuation, $\theta_{\ln(S_u)}$, is assumed 0.1m. These values are inspired by previously published works. The coefficient of variation of the undrained shear strength is well in the range of those reported by Phoon and Khulawy (1999). The vertical scale of fluctuation falls in the lower range of the vertical scale of fluctuations of the normalized tip resistance and friction ratio reported by Uzielli et al. (2005). Note that the aim of this work is to quantify the relation between the scale of fluctuation of S_u and normalized cone metrics.

The Poisson's ratio is set to 0.495 and the shear modulus is constant and is computed such that the rigidity index of the soil, computed with the mean value of the undrained shear strength, is equal to 100, thus representative of a normally consolidated, clay. The parameter controlling the soil-steel interface is set at $\alpha = 0.75$. This value, that will determine the friction sleeve resistance, can be thought of as adequate for clayey materials (the interested reader is referred to Smith et al. (2023) for a broader discussion).

3. Results and discussion

For results comparison and interpretation, the following notation is introduced. The generated profile of undrained shear strength according to the correlations model of Equation (1) is denoted as synthetic undrained

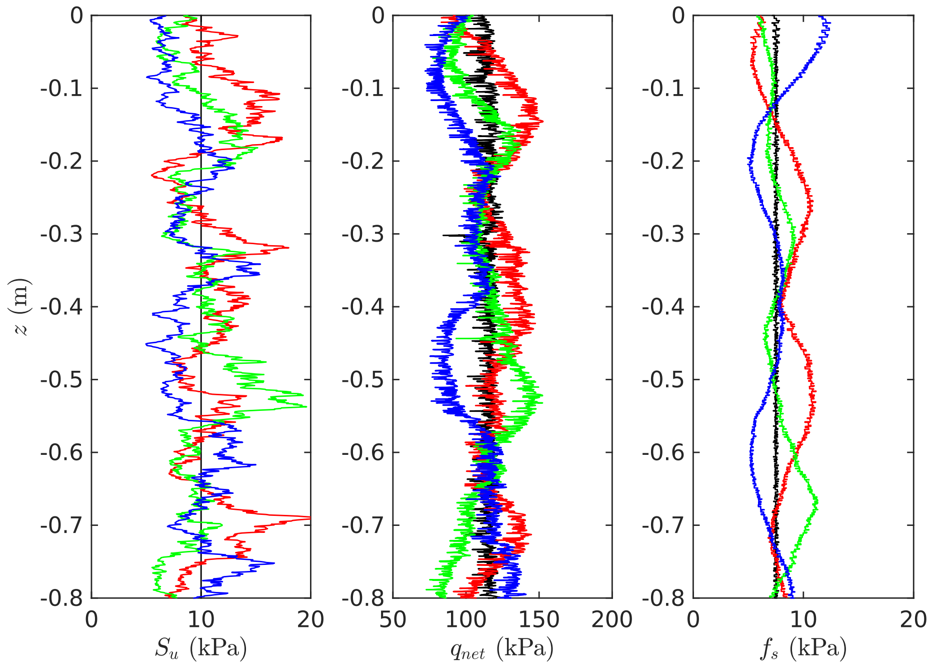


Figure 1. Vertical profile of the undrained shear strength (left), net cone resistance (center) and friction sleeve resistance (right) for the reference solution (black) and three realizations of the stochastic analysis (red, green and blue).

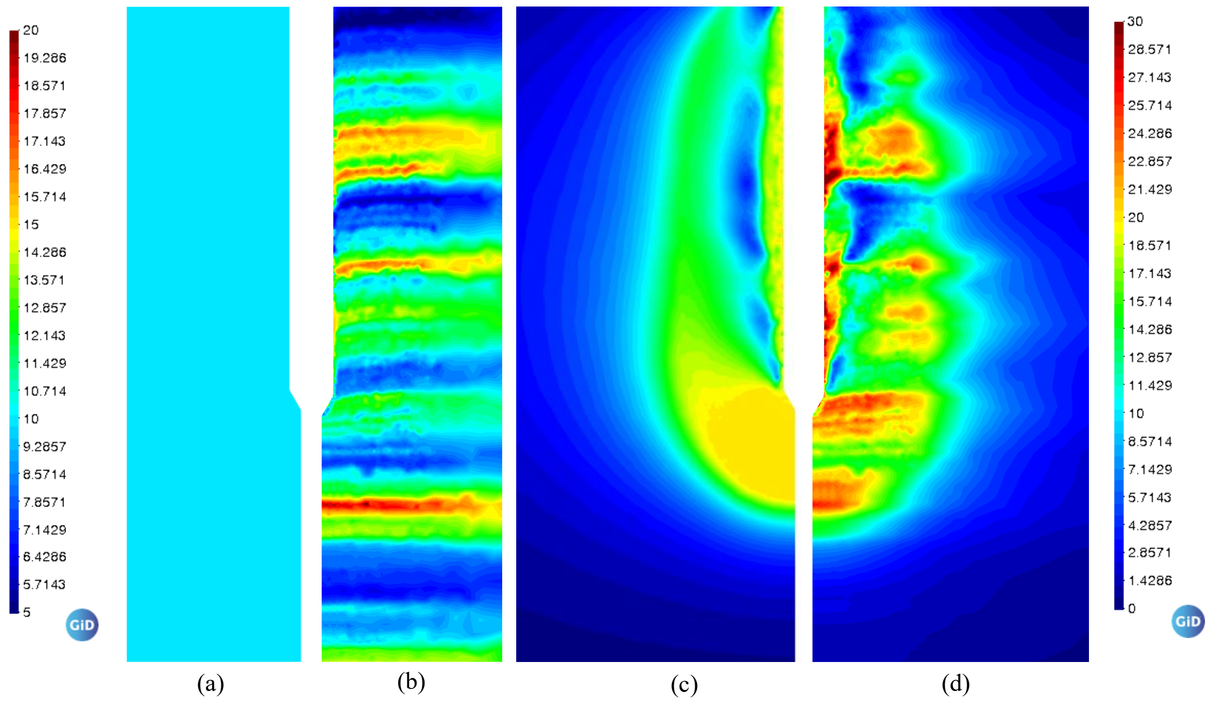


Figure 2. Undrained shear strength (kPa), (a) and (b), and second invariant of the stress tensor (kPa), (c) and (d). Reference solution, (a) and (c), and one stochastic realization, (b) and (d). (Detail around the tip of the cone).

shear strength. Such profile should be well-representative of insensitive, normally consolidated soils. On the other hand, the undrained shear strength derived from numerical outputs is referred to as numerical undrained shear strength.

The outputs of the numerical simulations are analyzed in terms of cone factors and normalized cone metrics. The net cone resistance, $q_{net} = q_t - \sigma_{v0}$, and the friction sleeve resistance, f_s, are related to the undrained shear strength by means of two cone factors:

$$N_{kt} = \frac{q_t - \sigma_v}{S_u} \tag{5}$$

$$N_f = \frac{f_s}{S_u} \tag{6}$$

The normalized cone metrics are represented by the normalized cone tip resistance, Q_t, and the friction ratio, F_r:

$$Q_t = \frac{q_t - \sigma_v}{\sigma_{v0}} \tag{7}$$

$$F_r = \frac{f_s}{q_t - \sigma_{v0}} \, 100\ (\%) \tag{8}$$

Table 1. Sample mean, standard deviation, and coefficient of variation of different cone metrics for the reference simulation and the stochastic analysis.

		Mean (kPa)	**Standard deviation** (kPa)	**Coefficient of variation**
Reference	S_u	10	0	0
	q_n	115.834	4.2	0.0362
	f_s	7.51	0.107	0.0142
	$\frac{q_n}{N_{kt}}$	10	0.363	0.0362
	$\frac{f_s}{N_f}$	10.017	0.14	0.0142
Stochastic	S_u	9.982	3.024	0.303
	q_n	111.204	20.635	0.186
	f_s	7.7122	1.527	0.198
	$\frac{q_n}{N_{kt}}$	9.603	1.782	0.186
	$\frac{f_s}{N_f}$	10.283	2.0365	0.198

3.1. Reference simulation

First, a reference solution is presented, in which the synthetic undrained shear strength of the soil is constant and equal to the mean value assumed in the stochastic simulations (μ_{S_u} =10 kPa). The variance of undrained shear strength, $\sigma^2_{S_u}$, is assumed to be null.

Figure 1 reports the numerical evolution of the net cone resistance and friction sleeve resistance in terms of the penetration distance. As the medium is assumed homogeneous and characterized by a constant undrained shear strength, these cone metrics are constant in depth, with slight noise, which stems from the employed numerical approach. The mean value of the net cone tip resistance is 115.834 kPa (see Table 1). As such, the cone factor is equal to N_{kt} = 11.5. This value is well in the range of previous numerical simulations (Lu et al. 2004) and in good agreement with current knowledge on CPTu interpretation for insensitive materials (Robertson 2009).

The value of the friction sleeve resistance is equal to 7.5 kPa. This does not come as a surprise, as in the numerical simulations reported in this work it is assumed that the cone is rough, and the maximum tangential stress acting at the soil-steel interface is 0.75 times the undrained shear strength of the soil. Consequently, the cone factor of the sleeve friction is N_f = 0.75.

In terms of the stress state of the soil, the deviatoric stress is equal to twice the undrained shear strength in a large region around the cone (see Figure 2). Consequently, it can be inferred that the soil is in plastic state in a region that expands 8.9 cone radii in the radial direction and 7.8 cone radii in the vertical direction from the center of the cone.

Finally, Figure 3 reports the numerical results in Robertson (2009) chart. The numerical result of this analysis falls exactly within the soil behavior type of soft, normally consolidated, unsensitive clays in agreement with current empirical knowledge.

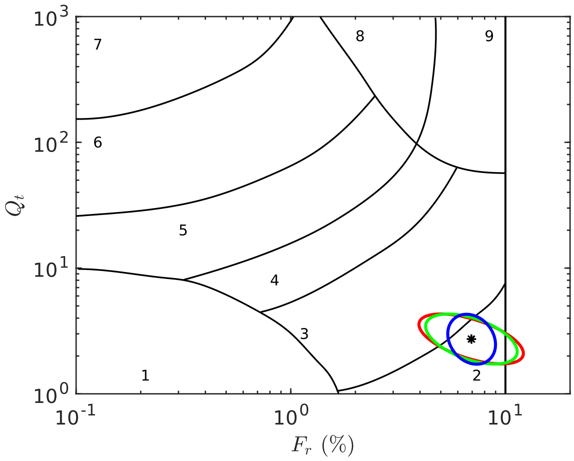


Figure 3. Numerical results on Roberston (2009) chart: reference solution (black dot) and ellipse enclosing 0.95 of the probability of the fitted multivariate joint distribution. Three correction methods are employed to compute the friction ratio: $h = 0$ m (red), $h = 0.03$ m (green) and $h = 0.12$ m (blue).

3.2. Stochastic modelling

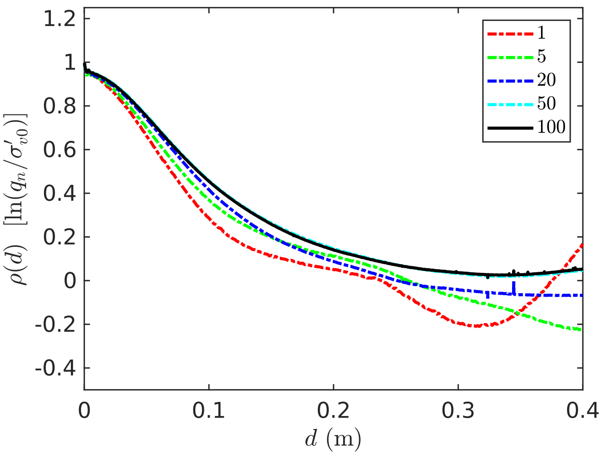
A series of realizations have been computed, each accounting for different synthetic undrained shear strength profiles generated from the same statistics: i.e., $\mu_{S_u} = 10$ kPa; $\sigma_{S_u} = 3$ kPa; $\theta_{\ln(S_u)} = 0.1$ m.

Figure 1 presents the representative results (the vertical profile of the undrained shear strength, net cone resistance and friction sleeve resistance) for three simulations (realizations). Visually, the numerical net cone resistance follows the same variability trend of the synthetic undrained shear strength profile: at depths in which the undrained shear strength is higher than the mean value, the net cone resistance is also higher than the cone resistance of the reference solution. However, the averaging effect of CPTu can also be appreciated: spikes of the undrained shear resistance are not always transferred into the cone resistance.

Figure 1 also reports vertical profiles of the friction sleeve resistance, without correcting for different depth of the position of the tip and shaft resistance measurements. Overall, the friction sleeve resistance follows the same variability trend of undrained shear strength, shifted of about 0.15 m downward.

Even if the synthetic undrained shear strength is described by a random field, the area in which CPTu induces plastic state is comparable to that of the reference solution (see Figure 2). Therefore, even for random field simulation, the cone resistance is an averaged measure of the constitutive response of the soil that is roughly 9 cone radii from the tip of the penetrometer.

Due to the high computational cost and memory requirements of these simulations, the computational domain has been set to 1.5 meters height. The first 0.2 m and the last 0.15 m of simulation are discarded due to the influence of top and bottom boundary conditions, for a final total length of 1 meter approximately. The number of necessary realizations has been established based on the spatial autocorrelation function of the cone and shaft resistance.



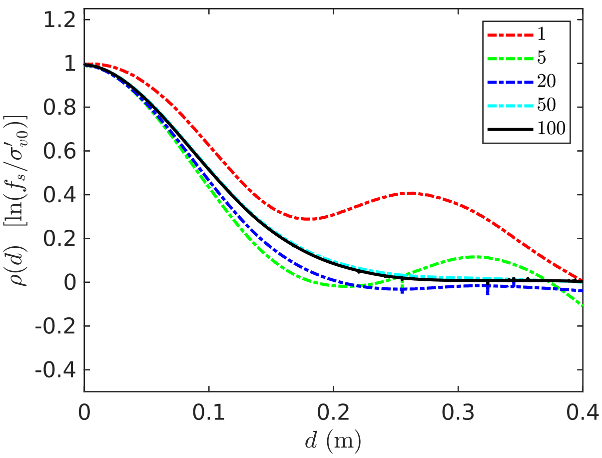


Figure 4. Spatial autocorrelation function of the logarithm of the net cone resistance normalized by the in situ vertical effective stress (i.e. normalized tip resistance) (top) and logarithm of the friction resistance normalized by the in situ vertical effective stress (bottom) in terms of the number of realizations.

Figure 4 reports the spatial autocorrelation function of the logarithm of the normalized net cone resistance and the logarithm of the shaft resistance normalized by the in situ effective vertical stress as function of the number of simulations performed. We have opted to compute spatial autocorrelation functions in terms of the logarithm of cone measurements and metrics as the undrained shear strength is described by a lognormal distribution and because usual CPT interpretation charts are in logarithmic scale.

The experimental spatial autocorrelation function of variable x is computed as usual (Uzielli et al. 2005). The fluctuating component of x is given by $w(z_i) = x(z_i) - \bar{x}$, as in this work the trend of all variables is constant. Therefore, the jth component of the autocorrelation function is:

$$\hat{\rho}_x(d_j) = \frac{\sum_{i=1}^{n_d-j} w_i\, w_{i+j}}{\sum_{i=1}^{n_d-j} (w_i)^2} \qquad (9)$$

where n_d is the number of measurements. Since a Monte-Carlo approach is employed, all experimental autocorrelation functions reported in this work correspond to the average correlation function of all realizations. For the net cone resistance, this curve seems independent from the number of realizations performed, as the shape of the curve is almost the same whether one realization (1 m of continuous simulation) or 100 realizations are run (100 simulations of 1m of CPT). This is not the case of the friction sleeve resistance: results are slightly different if computed with 1 realization or 100.

Both the logarithm of the net cone tip resistance and of the friction resistance seems to have a scale of fluctuation in the order of 0.2 to 0.25 m, twice of the one set for the synthetic undrained shear strength ($\theta_{\ln(S_u)} =$ 0.1m).

The tip and the shaft resistance are measured at the same time, but there is a depth discrepancy between both measurements. Additionally, both sensors have different heights, and the readings are the consequence of shearing a different volume of soil. When calculating the ratio between both measurements, several options are possible and -from a theoretical standpoint- plausible to correct the depth difference. Three distinct correction methods are considered, which are labelled in terms of relative shift applied to the friction sleeve measurement with respect to the cone tip resistance, and denoted as h. In the first case, no correction is applied, meaning that the friction ratio is calculated with the cone and tip resistance measured at the same instant ($h = 0$ m). The second case considers that the friction resistance is computed with the cone and shaft resistance as the tip of the cone or the u_2 position passes a given depth ($h = 0.03$ m). The third method assumes that at a given depth, the representative value of the cone and shaft resistance are those measured when the midpoint of the cone and the shaft are at that given depth ($h = 0.12$ m).

To analyze the effect of the correction methods, Figure 5 reports the vertical profile of the corrected net cone resistance, the friction ratio and the friction ratio of a representative realization. Of the three correction methods considered, only for one of them the cone and friction sleeve resistances exhibit the same tendency (i.e., they both increase or decrease at the same depth). This is the case in which the friction sleeve resistance is corrected so that the pair cone resistance – friction sleeve resistance corresponds to those measured when the midpoint of the cone and the shaft are at the considered depth. In the other two cases, cone resistance and friction sleeve resistance are specular.

The friction ratio computed using the third correction method is the one that has a lower variance (Figure 5). This result can be justified based on the assumed interface behavior and the size of the plastic region - which has a height comparable to the friction sleeve and extends equally upwards and downwards from the cone. However, proposing a depth correction method to compute the friction ratio is far beyond the scope of this work and therefore not exploited beyond.

Each of these correction methods has a noticeable effect on the spatial autocorrelation function of the friction ratio (Figure 6). The first two options produce similar results, and the scale of fluctuation is in the order of 0.1 m, in agreement with the value assumed for the synthetic undrained shear strength. On the other hand, if

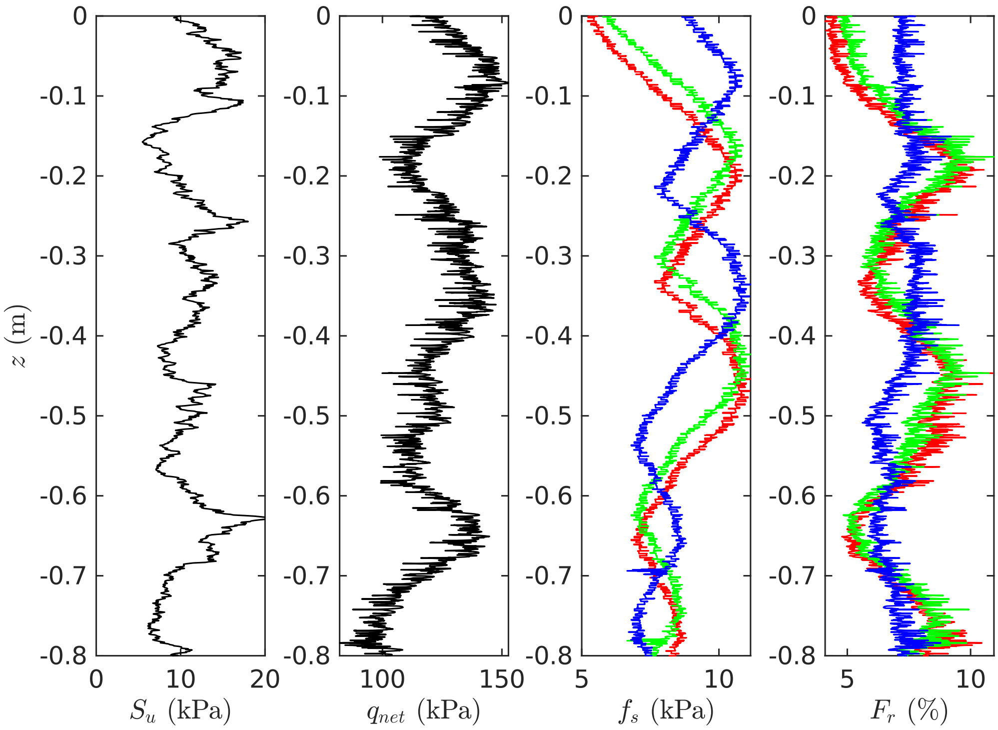


Figure 5. Vertical profile of the undrained shear strength (left), net cone resistance, friction sleeve resistance and friction ratio. The colors correspond to the applied correction tackle the depth mismatch between the tip and shaft resistance: $h = 0$ m (red), $h = 0.03$ m (green) and $h = 0.12$ m (blue).

the friction ratio is computed considering that the representative value of the cone and shaft resistance at a given depth is the one at the middle of each sensor, the scale of fluctuation is quantified of about 0.2 m (approximately). This value coincides with the scale of fluctuation of the cone resistance and the friction sleeve resistance.

The spatial autocorrelation matrix of the logarithm of the normalized cone resistance is not reported as, due to its definition, coincides with the analysis of the net cone resistance, reported in Figure 4.

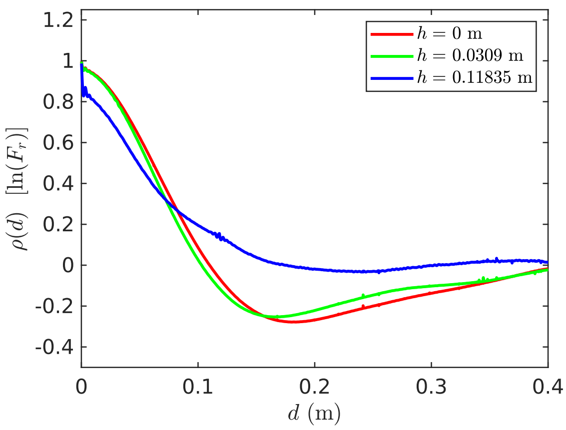


Figure 6. Spatial autocorrelation function of the logarithm of the friction ratio in terms of the applied correction to tackle the depth mismatch between the tip and shaft resistance.

Figure 3 reports the numerical results of this analysis in Robertson (2009) chart. The pair $\ln(Q_t) - \ln(F_r)$ is assumed to follow a bivariate joint normal distribution (Collico et al. 2023), that is fitted to the numerical results. The mean value of the logarithm of the normalized tip resistance and of the friction ratio are almost coincident with those of the reference simulation and mostly independent of the method employed to correct the depth difference. Differences appear in the covariance matrices of $\ln(Q_t) - \ln(F_r)$ measurements (see isolines of joint bivariate density distribution in Figure 3). If the friction sleeve resistance is computed with the first two discussed methods, the variance of $\ln(F_r)$ is higher and the covariance between $\ln(Q_t) - \ln(F_r)$ is negative. Calculating the friction sleeve resistance with the third approach reduces both the variance of $\ln(F_r)$ and the covariance of the pair $\ln(Q_t) - \ln(F_r)$.

The obtained scale of fluctuation of the normalized cone metrics is in the lower range of those reported by Uzielli et al. (2005), who analyzed a database of 40 CPTu in clayey and sand materials. The mean value of the normalized metrics is slightly lower than those reported by Uzielli et al. (2005), but the coefficient of variation of these metrics are in agreement with that field data.

The histogram of the synthetic undrained shear strength and the interpreted shear strength from the numerical results either using N_{kt} or N_f obtained for the reference case, are reported in Figure 7. All distributions have a mean value around 10 kPa (see Table 1); however, the undrained strength interpreted from the cone tip resistance is slightly lower than 10 kPa whereas that from the friction sleeve resistance is slightly higher than 10 kPa. More research should be conducted to better explain such differences, since it might not be related to an insufficient number of simulations in the Monte-Carlo, but rather to some analogous effect of thin clayey layers interbedded in sandy materials on CPTu response. The variance of the synthetic undrained shear strength is higher than the variance obtained from numerical results using cone factors. This can be explained by the averaging nature of CPTu and the assumed scale of fluctuation, which could be considered as small, since it falls within the lower bound of those reported by Uzielli et al. (2005).

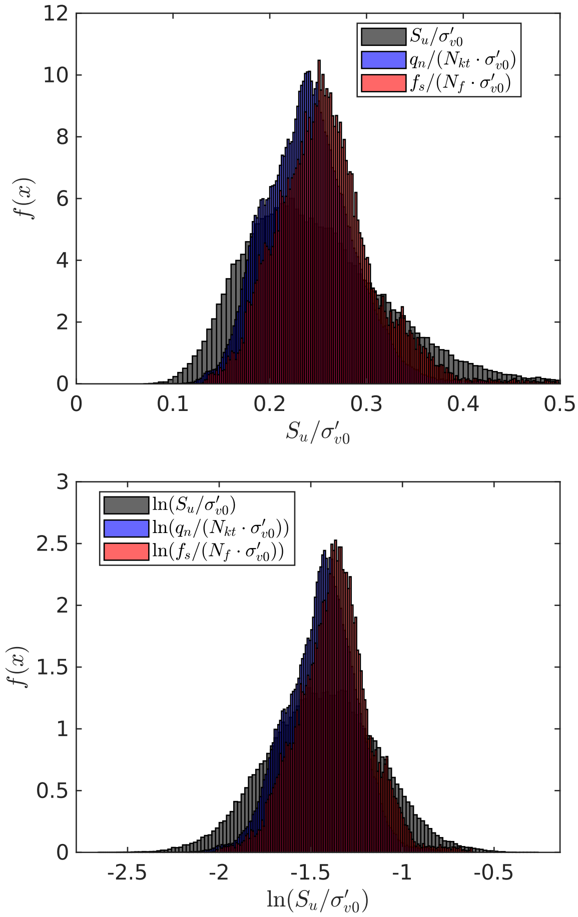


Figure 7. Histogram of the synthetic undrained shear strength and the interpreted strength using the tip and shaft resistance from numerical analyses. Natural scale (top) and logarithmic scale (bottom).

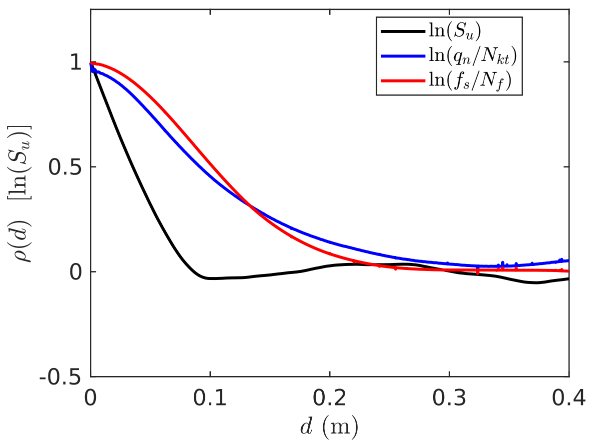


Figure 8. Spatial autocorrelation function of the synthetic (in situ) undrained shear strength and the interpreted undrained shear strength from the net cone tip resistance and friction sleeve resistance.

This averaging effect is also responsible for the differences of the spatial autocorrelation function of the synthetic and numerical undrained shear strength, see Figure 8. The scale of fluctuation of numerical S_u almost doubles the synthetic undrained shear strength. Not only the scale of fluctuation is increased, but the shape of the autocorrelation function of the input (spherical model) is different from the shape of the autocorrelation function of the interpreted S_u.

4. Conclusions

This work has been set out to investigate the relation between the inherent variability of the soils and the variability of the soil as measured by cone penetration testing.

In the analyses presented, the scale of fluctuation of the net cone resistance and friction sleeve resistance almost doubles the scale of fluctuation of the undrained shear strength of the soil. This can be explained by the average nature of cone metrics. The scale of fluctuation of the friction ratio heavily depends on the method employed to tackle the depth difference between the tip and the shaft of the cone.

Due to the openly prospective nature of this work, all conclusions should be understood as those of a work in progress. The aim of this work is to raise awareness on the differences between the scale of fluctuation of geotechnical parameters and that measured using CPTu. Further research is required to fully quantify the differences between them.

Even the naïveté of the reported analysis (total stress, perfect plasticity, infinite scale of fluctuation in the horizontal directions, relatively small scale of fluctuation in the vertical direction...), the information presented in this paper may provide a useful reference framework for further studies.

Acknowledgements

This paper is a tribute to our mentor, Professor Marcos Arroyo, on the occasion of his birthday. We are grateful for his guidance and support, and we hope he finds this paper enjoyable to read. ☺.

The first author gratefully acknowledges financial support of Ministerio de Ciencia e Innovación of Spain (MCIN/AEI/10.13039/501100011033) through the Severo Ochoa Centre of Excellence project (CEX2018-000797-S) and research project PID2020-119598RB-I00.

References

Alonso, E.E. Krizek, R.J. “Stochastic formulation of soil properties”. Proc 2nd Int. Conf on Applications of Statistics & Probability in Soil and Structural Engineering, Aachen. Vol 2, pp. 9-32, 1975.

Carbonell, J.M., Monforte, L., Ciantia, M.O., Arroyo, M., Gens, A. “Geotechnical particle finite element method for modelling of soil-structure interaction under large deformation conditions”, Journal of Rock Mechanics and Geotechnical Engineering, 14(3), pp. 967-983, 2022. https://doi.org/10.1016/j.jrmge.2021.12.006

Collico, S., Arroyo, M., Devicenzi, M., “A simple approach to probabilistic CPTu-based geotechnical stratigraphic profiling" Computers and Geotechnics, 165, 105905, 2024. https://doi.org/10.1016/j.compgeo.2023.105905

Griffiths, D.V., Huang, J., Fenton, G.A., "Probabilistic infinite slope analysis", Computers and Geotechnics, 38(4), pp. 577-584, 2011, https://doi.org/10.1016/j.compgeo.2011.03.006

Li, Y., Fenton, G.A., Hicks, M.A., Xu, N. "Probabilistic bearing capacity prediction of square footings on 3D spatially varying cohesive soils". Journal of Geotechnical and Geoenvironmental Engineering, 147(6), 04021035, 2021. https://doi.org/10.1061/(ASCE)GT.1943-5606.0002538

Lu, Q., Randolph, M.F., Hu, Y., Bugarski, I.C. "A numerical study of cone penetration in clay", Géotechnique 54(4), pp. 257-267, 2004. https://doi.org/10.1680/geot.2004.54.4.257

Nova, R. "Fondamenti di meccanica delle terre", 1st ed., McGraw-Hill Education, 2002.

Monforte, L., Arroyo, M., Carbonell, J.M., Gens, A., "Numerical simulation of undrained insertion problems in geotechnical engineering with the Particle Finite Element Method (PFEM)" Computers and Geotechnics, 82, pp. 144-156, 2017. https://doi.org/10.1016/j.compgeo.2016.08.013

Phoon, K.K. & Kulhawy, F.H. "Characterization of geotechnical variability" Canadian Geotechnical Journal, 36, pp: 612-624, 1999. https://doi.org/10.1139/t99-038

Roberston, P.K. "Interpretation of cone penetration tests– a unified approach", Canadian Geotechnical Journal, 46, pp, 1337-1355, 2009. https://doi.org/10.1139/T09-065

Smith, K., Fanni, R., Reid, D., "On the accuracy of inferring residual undrained shear strengths from CPTu friction sleeve resistance" Mine Waste and Tailings Conference, pp. 110-127, 2023.

Uzielli, M., Vannucchi, G., Phoon, K.K. "Random field characterization of stress-normalised cone penetration testing parameters" Géotechnique 55(1), pp. 3-20, 2005. https://doi.org/10.1680/geot.2005.55.1.3

Vanmarcke, E.H. "Random Fields: Analysis and Synthesis" MIT Press, 1984.

Wang, Y., Au, S.K., Cao, Z. "Bayesian approach for probabilistic characterization of sand friction angles", Engineering Geology, 114(3-4), pp. 354-363, 2010. https://doi.org/10.1016/j.enggeo.2010.05.013