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Verification of a pedestrian simulation tool using the NIST recommended test cases

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Abstract

In an attempt to develop a verification and validation standard for building fire evacuation models, Ronchi et al. (2013) at the United States' National Institute of Standards and Technology (NIST) recommended a set of seventeen verification tests. We found that the application of these verification tests allowed us to make rather significant improvements to our simulation code (PEDFLOW) for approximately half of the recommended tests (Table 1). In some cases, we added capabilities that did not exist before. In other cases, we found anomalous behaviors and adjusted the existing code to remove these unexplained behaviors. This paper summarizes the work on the verification tests, highlighting the lessons learned and modifications made. We also discuss some modifications we recommend to the NIST verification tests, as well as demonstrate how to make these tests suitable for all pedestrian flow models (not just building fire evacuation).

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1. Introduction

With no international standards for verification and validation of pedestrian flow and crowd dynamic simulation tools, researchers often apply inconsistent procedures, use unreliable data, or only partially test the simulation tools. In an attempt to develop a verification and validation standard for building fire evacuation models, Ronchi et al. (2013) at the United States' National Institute of Standards and Technology (NIST) recommended a set of seventeen verification tests spanning five core components: 1) pre-evacuation time, 2) movement and navigation, 3) exit usage, 4) route availability, and 5) flow constraints. The application of these seventeen verification tests to a pedestrian flow simulation tool (PEDFLOW) led to some rather significant improvements to the code for approximately half of the recommended tests (Table 1). In some cases, we added capabilities to PEDFLOW that did not exist before. In other cases, we found anomalous behaviors and adjusted the existing code to remove these unexplained behaviors. This paper summarizes the work on the verification tests, highlighting the lessons learned and modifications made to the

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code as a result. In addition, we discuss some modifications we recommend to the NIST verification tests and briefly demonstrate how to make these tests suitable for all pedestrian flow models, rather than just building fire evacuation.

Table 1: Verification test summary.

Core Component	Sub-Element	Existed	Modified	Added	Remarks
1	Pre-evacuation times			✓	
	Speed in a corridor	✓			
	Speed on stairs		✓		Adjusted pedestrian speed
	Movement around a corner	✓			
	Assigned demographics		✓		Modified distribution
2	Reduced visibility walking			✓	
	Occupant incapacitation			✓	
	Elevator usage	✓-			Not used for evacuation
	Horizontal counter-flows		✓		Path definitions
	Group behaviors	✓			
	People with disabilities		✓		Use ramp/wheelchair speed
	Exit route allocation	✓			Nearest exit
3	Social influence			✓	
	Affiliation			✓	
4	Dynamic availability of exit	✓			
5	Congestion	✓			
	Maximum flow rates		✓		Anomalous behavior discovered

2. Description of NIST Verification Tests

In November 2013, researchers from the United States' NIST Fire Research Division in conjunction with researchers from the Department of Fire Safety Engineering and Systems Safety at Lund University published a set of seventeen hypothetical verification test cases for use in quantitatively and qualitatively verifying results produced by the fire evacuation models. The NIST researchers developed this set of test cases, some of which were based on similar verification tests developed by the International Maritime Organization (2007), as a means to open a debate and contribute to an on-going effort by the International Standards Organization (2008) to develop an overall assessment standard for evacuation models.

Although PEDFLOW was not specifically designed for fire evacuation, we felt confident that applying these recommended test cases to PEDFLOW would serve four main purposes. First, the test cases provide a basic set of simple geometries and pedestrian populations which allows those unfamiliar with PEDFLOW to get used to setting up scenarios and running simulations. Second, the comprehensive nature of the tests will identify capability shortfalls within PEDFLOW and prompt the addition of capabilities that did not exist before. Similarly, the quantitative and qualitative expectation associated with each test case easily highlights anomalous behaviors and identifies the need for code modifications to remove these unexplained behaviors. Lastly, once run, the results obtained from the test cases provide a benchmark for future post-development versions.

3. Brief Description of the Pedestrian Simulation Tool

The pedestrian flow simulation tool (PEDFLOW) used in this study is a discrete model where each pedestrian is treated individually and motion is influenced by Newtonian dynamics. Within PEDFLOW, global movement is controlled by the individual's desired destination, modeled as an internal will force. Local movement is controlled by additional internal forces such as intermediate collision avoidance, near-range (contact) collision avoidance, and wall/obstacle avoidance forces, as well as external pedestrian-pedestrian and pedestrian-object contact forces. For a complete description of the forces, their interactions, data structures, and example simulation capabilities of PEDFLOW see Löhner (2010).

Although PEDFLOW has been in development for more than fifteen years, we found that the application of the NIST verification tests led to some significant improvements. PEDFLOW contains a complete suite of pre- and post-processing tools. The computer aided design tool included in PEDFLOW allows the user to input all information required to set up the test case including the geometric definitions; boundary conditions; pedestrian types, characteristics and desired paths; as well as any scenario-specific information (such as evacuation). In addition, the user may use the computer aided design tool to specify required diagnostics as a means of collecting all necessary quantitative and qualitative information during the simulation run for analysis during post-processing. Once pre-processing is complete, the PEDFLOW tool runs the simulation and outputs all requested diagnostic information to data files for post-processing.

4. Existing Capabilities

Of the seventeen verification tests listed in Table 1, PEDFLOW had the capability to complete seven of the tests with little-to-no modifications. PEDFLOW successfully accomplished four of the ten verification tests associated with movement and navigation, the second core component of evacuation models: 1) speed in a corridor; 2) movement around a corner; 3) elevator usage (although not available for evacuation scenarios); and 4) group behaviors. PEDFLOW also had the capability to complete three other verification tests outside the movement and navigation core component: 1) exit route allocation from the exit choice/usage core component, 2) dynamic availability of exit from the route available core component, and 3) congestion from the flow constraint core component.

4.1. NIST Verification Test 2.1: Speed in a Corridor

Speed in a corridor is a quantitative verification test in which PEDFLOW simply confirms that a pedestrian walks the length of a corridor at his/her assigned speed. Given a corridor 2 meters wide by 40 meters long and one pedestrian with a horizontal walking speed of 1 m/s, PEDFLOW confirmed that the pedestrian traverses the entire length of the corridor in 40 seconds. We also recommend testing this scenario with an input flux of 1 ped/sec, assigning each pedestrian a walking speed of 1 m/s. The expected result would be a line of pedestrians spaced approximately 1 meter apart walking along the entire length of the corridor with an average velocity of 1 m/s.

4.2. NIST Verification Test 2.3: Movement Around a Corner

Movement around a corner is a qualitative verification test where PEDFLOW demonstrates that twenty uniformly distributed pedestrians can successfully navigate a corner. For the purposes of this verification test, we took "uniformly distributed" to mean evenly distributed and used an input file to initialize the pedestrians at specific locations rather than randomly distributed uniform locations. Post-processing the data using Paraview as the visualization tool, we were able to visually confirm that all twenty pedestrians navigate the corner without penetrating any barriers (Figure 1). We found this verification test to be an excellent tool to use in order to illustrate the differing methods of defining paths in PEDFLOW.

(a) Geometric definition and pedestrian initialization. (b) Qualitative verification of successful corner navigation.

Fig. 1: Verification Test 2.3

4.3. NIST Verification Test 2.7: Elevator Usage

Although PEDFLOW previously included an elevator sub-model, the elevator is currently not a viable egress component within PEDFLOW. As currently coded, in evacuation situations everyone heads towards the nearest exits (defined as an in/out boundary condition) following a 'time-to-exit' gradient direction that is applied to the geometric mesh. The elevator is excluded from this mesh (the assumption was that people are not supposed to use elevators in an evacuation/ fire situation) and therefore is not available in evacuation simulations. However, in a non-evacuation simulation, we were successful in quantitatively and qualitatively verifying elevator usage as outlined in the NIST paper (Figure 2). Including the elevator as a viable means of evacuation within PEDFLOW is an area requiring further development.

(a) Geometric definition and pedestrian initialization. (b) Awaiting elevator arrival.

Fig. 2: Verification Test 2.7

4.4. NIST Verification Test 2.9: Group Behaviors

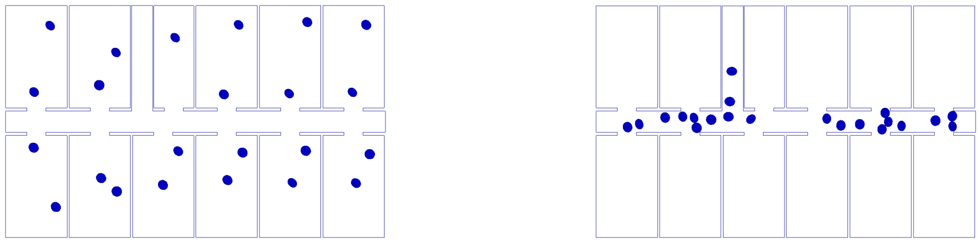
Group behaviors is a qualitative test of PEDFLOW's ability to replicate group dynamics, namely the ability of a group of individuals to stay together while exiting a room. PEDFLOW provides the user an opportunity to define many group types with various behaviors. Some of the choices for groups behaviors include: 1) try to go to leader; 2) try to go parallel to leader; 3) try to go behind the leader (sophistic group, see Plato's "Protagoras"); 4) try to form a row (loose connection) behind the leader; 5) try to form a chain (strong connection) behind the leader; and 6) amoeba, force based group association. In addition, the user is able to set the maximum separation distance allowed before leaders begin to slow, as well as the maximum separation distance allowed before a group is split and/or separated for both low and high densities. For the purposes of the NIST Verification test, we used a maximum separation distance of 3 meters before the leader slows and a group split distance threshold of 15 meters for low densities and 14 meters for high densities. These values ensured the group stayed together and all members of the group exited the room within 8 seconds of each other (Figure 3).

(a) Geometric definition and pedestrian initialization. (b) Qualitative verification of group cohesion through exit.

Fig. 3: Verification Test 2.9

4.5. NIST Verification Test 3.1: Exit Route Allocation

In the exit route allocation verification test, PEDFLOW successfully demonstrated that, in evacuation mode, all pedestrians exit the building via the nearest exit (exit route is dynamically selected based upon shortest time to exit). The pedestrians were distributed among the twelve rooms as shown in Figure 4 in accordance with Figure 8 from Ronchi et al. (2013). The pedestrians were randomly assigned horizontal walking speeds of 1.25 m/s +/- 10%, with a relaxation time of 0.5 m/s. The minimum exit time was 3.05 seconds and the maximum exit time was, on average, around 17 seconds.



(a) Geometric definition and pedestrian initialization. (b) Qualitative verification of exit selection.

Fig. 4: Verification Test 3.1

4.6. NIST Verification Test 4.1: Dynamic Availability of Exit

Dynamic availability of exit is a qualitative verification test which demonstrates PEDFLOW's ability to close an exit and have the pedestrian(s) dynamically find an alternate exit. The user has the ability to define a scenario-dependent file in PEDFLOW which limits outflow fluxes. Setting an outflow flux to zero effectively closes off the exit and the evacuee will dynamically find an alternate exit. In addition, PEDFLOW has the ability to define paths that are modified in time, making it possible to not only close an exit, but cut-off an entire exit route within a building. When a path is interrupted PEDFLOW dynamically finds an alternate path to an exit.

4.7. NIST Verification Test 5.1: Congestion

The congestion verification test is a qualitative verification test intended to verify how well the simulation tool simulates congestion. In this case, this verification test intends to verify flow constraints in a staircase. The capability to simulate congestion previously existed in PEDFLOW; however, the specifications of this test failed to form congestion at the base of the stairs as intended (Figure 6). As can be seen from Figure 6, congestion does form at the exit of the room, but the flow limitation through the opening from the room simply prevents congestion on the stairs. Although the test specified in Ronchi et al. (2013) was intended to test movement in the downward direction, we felt it beneficial to perform the test in both directions.

5. Modified Capabilities

In addition to the seven pre-existing PEDFLOW capabilities, there were five capabilities which existed but needed improvement. Four of these capabilities were from the movement and navigation core component, namely speed on stairs, assigned occupant demographics, horizontal counterflows, and people with movement disabilities. The fifth capability came from the flow constraint core component where we discovered anomalous behaviors.

5.1. NIST Verification Test 2.2: Speed on Stairs

The speed on stairs verification test quantitatively confirms a pedestrian's ability to travel up or down a flight of stairs at his/her assigned speed. Since each pedestrian is assigned only a horizontal (desired) velocity in PEDFLOW,

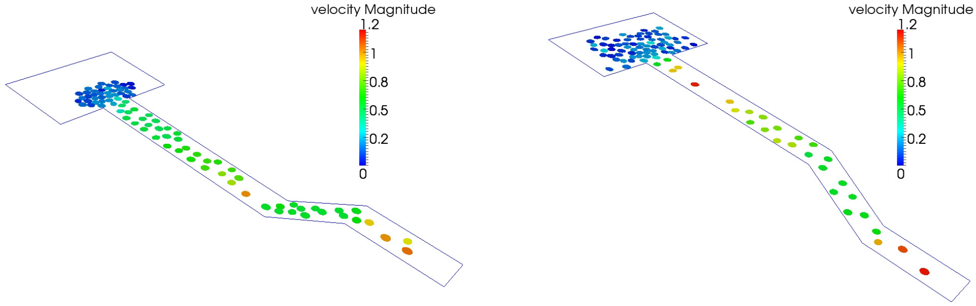


Fig. 5: Comparison of velocity in both the upward (left) and downward (right) directions for Verification Test 5.1 (notice the lack of congestion at the top of the stairs).

the code makes appropriate velocity corrections for travel up/down both ramps and stairs. While not explicitly specified by the NIST verification requirements for this test case, we found value in first conducting two additional tests which compute pedestrian adjusted velocities on ramps in the upward and downward directions. In fact, by doing this test we found an anomaly for pedestrians traveling down the ramp and modified the subroutine in PEDFLOW to ensure accurate quantitative results.

Once convinced that the pedestrians were traveling with appropriate velocities up and down the ramp, we focused on the actual verification requirement to verify the speed on stairs. In order for PEDFLOW to properly compute the speed on stairs, we must know the step height and tread depth which was not given in the NIST paper. A typical riser height used is 7 inches (0.18 m) and tread depth is 11 inches (0.28 m), which results in a stair gradient of approximately 32.7°. To maintain the 30° gradient already established in the ramp verification test, we used a step height of 0.154 meters and a tread length of 0.267 meters. Choosing these values allowed us to use the same geometric definition already established for the ramp with the inclusion of the stair steps. We immediately recognized two anomalies. First, the velocity of the pedestrian was the same when traveling up or down the stairs and secondly, the speed of the pedestrian was significantly reduced (by more than 60%) when traversing the staircase. A pedestrian with an unimpeded horizontal velocity of 1.0 m/s was restricted to a velocity of 0.289 m/s when traveling the stairs in either direction.

In 2004 at the 10th International Conference on Mobility and Transport for Elderly and Disabled People, Fujiyama and Tyler (2010) presented a rather complete set of empirical stair data. Their study consisted of two subject groups: a group of 6 healthy men and 12 healthy women (Group 1) between the ages of 60 and 81, and a second group consisting of 7 healthy men and 8 healthy women between the ages of 25 and 60 (Group 2). They measured the normal walking speeds and fast walking speeds of each participant on a horizontal surface and when ascending/descending four individual flights of stairs. The stairs had differing step riser heights and tread lengths, resulting in stair gradients ranging from 24.6° to 38.8°. Fujiyama and Tyler noted that the participants in their study showed a high correlation between horizontal walking speed and speed on stairs and hypothesized that this is somehow related to the individual's step frequency.

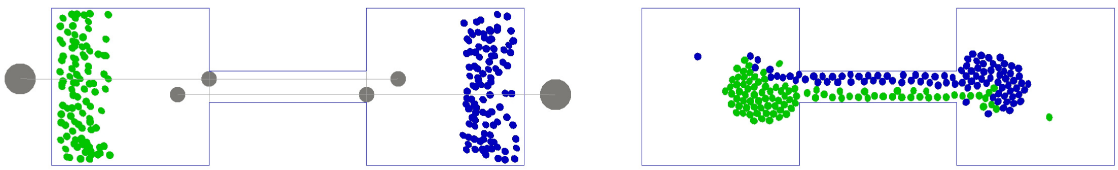
Exploring this theory, we sought to devise a new formula based on parametric values obtained from Fujiyama and Tyler's empirical data. In general, a pedestrian's step frequency is simply the product of a person's desired velocity and the inverse of their step size. On a horizontal surface, the often assumed step size value is 0.8 meters which equates to a step frequency of 1.25 steps per second for a person with a desired horizontal velocity of 1.0 m/s. Using the data provided by Fujiyama and Tyler (2010), we found that the corrected step size for a person traveling up a flight of stairs is approximately 0.5 meters and the corrected step size for a person descending a flight of stairs is approximately 0.66 meters. Using these values and our modified PEDFLOW subroutine equates to observed simulation values of 0.463 m/s when the pedestrian is ascending the stairs and 0.502 m/s when descending.

5.2. NIST Verification Test 2.4: Assigned Occupant Demographics

The next modified verification test, assigned occupant demographics, is a quantitative verification of the simulation tool's ability to properly assign pedestrian characteristics. To provide maximum flexibility, numerous pedestrian demographic options exist within PEDFLOW. Occupant types can be defined as either (1) pedestrians or (2) wheelchairs with user-specified averages and variations (defined as a percentage) available for the following characteristics: 1) velocity, 2) relaxation time, and 3) pedestrian size (radius). In addition, the user may also specify limits (max/min) for the following additional characteristics: 1) ellipticity, 2) pushiness, and 3) desired comfort zone. In order to verify the assignment process within PEDFLOW, we simply output the characteristic data to a file and verified that the assigned values are consistent with the distribution desired. By completing this verification test, we found that PEDFLOW was assigning all pedestrian characteristics using a uniform distribution when we, in fact, desired a Gaussian distribution for the velocity assignments. Without the benefit of these verification tests, this anomaly may have remained undiscovered.

5.3. NIST Verification Test 2.8: Horizontal Counterflows

Horizontal counter-flows, tests PEDFLOW's ability to simulate and reproduce emergent behaviors in uni-directional and bi-directional flows in a corridor. Upon initial testing of the uni-directional verification test, we found the some of the pedestrians displayed anomalous behaviors (such as moving to a random corner of the room prior to exiting the room). The problem was traced to one of the object collision algorithms, which was subsequently improved. While testing the bi-directional flows using opposing paths, we found that the simulation would typically display what Helbing et al. (2002) called ”freezing-by-heating”, or a complete stalemate, whereby none of the pedestrians could move. To solve this problem, we modified our path input instructions, requiring paths to be defined on half the corridor for



(a) Geometric definition and initial pedestrian positions. (b) Horizontal counterflow with 'cultural behavior'.

Fig. 6: Verification Test 2.8.

pedestrians moving in each direction. Defining the paths this way does not prevent the pedestrians from using the entire corridor, but gives each group of pedestrians a tendency to stay to a particular side of the corridor. This may also be seen as a 'cultural behavior' (preferring the right side) that requires demographic information.

5.4. NIST Verification Test 2.10: People with Movement Disabilities

The final verification test in the movement and navigation core component is the people with movement disabilities verification test. This test is intended to verify the simulation tool's ability to simulate a pedestrian with reduced mobility and increased space requirements (such as a wheelchair). The ability to define pedestrians as wheelchair occupants was pre-existing in PEDFLOW. In order to perform the test as outlined in the paper by Ronchi et al. (2013), we first had to modify the geometry since, as published, the ramp was too steep for a wheelchair. According to Fruin (1971), the ramp should not exceed a 8.33% grade. Since the change in height between the two rooms was prescribed

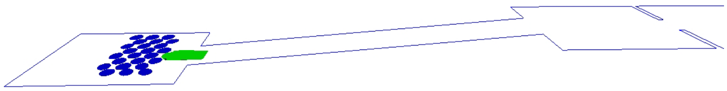


Fig. 7: Geometric definition and initial pedestrian placement for Verification Test 2.10.

as 1 meter, we modified the geometry shown in Figure 7 of Ronchi et al. (2013) and made the ramp 12 meters long (rather than 2 meters). As can be seen from Table 2, the pedestrians took, on average, approximately five seconds longer to exit the room when following the wheelchair up or down the ramp.

Table 2: Room exit time statistics after thirty PEDFLOW simulation runs.

Lead Occupant	Ramp Direction	Minimum (s)	Maximum (s)	Average (s)
Wheelchair	Up	46	52	48.83
Pedestrian	Up	43	46	44.23
Wheelchair	Down	45	50	47.97
Pedestrian	Down	41	45	43.37

5.5. NIST Verification Test 5.2: Maximum Flow Rates

The maximum flow rate verification test confirms the simulation tool's ability to set flow rates. The user must place 100 occupants in the room, assign a specific maximum flow rate for the exit and ensure that the flow rate never exceeds the established threshold. During our initial attempt at this verification test, we discovered an anomalous behavior in our pedestrian initialization subroutine and corrected the code. Once corrected, PEDFLOW confirmed that, with a limiting exit flux of 1 person per second, it takes 100 seconds for 100 pedestrians to exit the room (versus just 55 seconds when no limiting flux is present).

6. Added Capabilities

Completion of the seventeen verification tests led to the addition of five capabilities which did not previously exist in PEDFLOW. The ability to assign pre-evacuation time delays did not exist nor did the ability to slow/incapacitate pedestrians due to reduced visibility or the inhalation of toxic materials. Exit route/choice was also severely limited by PEDFLOW's singular ability to force a pedestrian to select the nearest exit without consideration for social influence or route familiarity (affiliation). In addition, PEDFLOW uses a lot of random number generation. By specifying a different identifier in an initialization file, the initialization of the random numbers is changed, so that statistical data can be obtained from many PEDFLOW runs that use the same deterministic data but use different random data. We believe the addition of these capabilities has significantly improved the robustness of PEDFLOW.

6.1. NIST Verification Test 1.1: Pre-evacuation Delay Times

The first core component of evacuation modeling concerns pre-evacuation time. In an evacuation scenario, pre-evacuation time is often categorized as the time an individual needs for recognition and response, or in other words, the time elapsed from the initial sounding of an alarm to the time when the individual decides to act (evacuate, shelter-in-place, seek additional information, etc.). The verification test for this component confirms the simulation tool's ability to distribute a set of pre-evacuation time delays among the population. Prior to the application of this particular test, the capability to assign evacuation delay times did not exist in PEDFLOW. With the capability now added, users may now choose one of three delay options during an evacuation run: 1) no delay, 2) a delay based on a Gaussian random number, or 3) a delay based on a table of user-defined probabilities.

6.2. NIST Verification Tests 2.5 & 2.6: Reduced Visibility vs. Walking Speed & Occupant Incapacitation

Prior to completing these verification tests, PEDFLOW had limited abilities to account for the physical impacts of smoke and other toxic materials on the pedestrian (smokeinhale previously existed). The user is now able to input a maximum smoke concentration level which leads to zero movement, or total impedance, as well as a value for toxic material inhalation which leads to incapacitation for each pedestrian type. Given these values, PEDFLOW reads in smoke and toxicity data from an input file, interpolates concentrations across the domain, and then updates pedestrian health. The inhalation of toxic material is still monitored, but the pedestrian now becomes incapacitated if the levels exceed the established threshold. Using an established respiration rate of 15 liters per minute, PEDFLOW accumulates the total amount of toxic material inhaled based upon the pedestrian's current position in the domain and the interpolated toxicity levels at that location. After each update, PEDFLOW checks the pedestrian's current toxic inhalation levels and marks the pedestrian incapacitated if the level exceeds the established threshold. In order to limit the pedestrian's walking speed in conditions where visibility is limited, a new subroutine was created which corrects the pedestrian's desired velocity for conditions of dense smoke. For our purposes, we assume that even in the most dense smoke (as long as the pedestrian doesn't succumb to an inhalation injury), the pedestrian is still able to crawl until becoming incapacitated due to smoke.

6.3. NIST Verification Tests 3.2 & 3.3: Social Influence & Affiliation

The completion of these verification tests led to numerous additions to our scenario-specific simulation inputs. Prior to the completion of these tests, the only exit-choice behavior available during evacuation scenarios was exit selection based on shortest time to exit. PEDFLOW now has the ability to include social influence, computed as an average motion of neighbors, and affiliation, modeled as a pedestrian's desired to choose his/her usual path, to evacuation scenarios, both capabilities which simply did not exist before. In the case of social influence, the pedestrian follows the direction of pedestrians that ”know” where they are going; however, if there are no ”knowing” neighbors around, the pedestrians continue to the nearest available exit. For the affiliation case, the pedestrian always follows his/or her assigned (affiliated) path to the exit. The evacuation subroutine also includes the ability to define a mixture of these exit-choice behaviors for each pedestrian type.

7. Conclusions

The application of the seventeen verification tests recommended by NIST to the PEDFLOW simulation tool allowed the identification of several errors and anomalies. This led to rather significant improvements for approximately half of the recommended tests. Several cases led to new capabilities that did not exist before. And in other cases, anomalous behaviors were found, which led to an adjustment and corrections of the existing code. Overall, this was a very valuable exercise. It is recommended that similar codes be also tested against this set of problems.

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