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INVESTIGATIONS ON THE DESIGN OF MEMBRANE STRUCTURES WITH THE SEMI-PROBABILISTIC SAFETY CONCEPT

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Abstract. The semi-probabilistic safety concept was developed for limit state equations that are linear or mildly non-linear. EN 1990 (Eurocode 0) provides simplified design rules in case of non-linear functional relations between actions and their effects, which are based on a distinction between under- and over-linear behaviour. We discuss circumstances in which this classification can be ambiguous in particular due to pre-stressing of membranes. We then show the effect of the classification on the reliability: the simplified classification of EN 1990 can lead to inconsistent reliability levels for structures with non-linear behaviour. Finally, we discuss the the consequences of our investigations on the semi-probabilistic safety concept for non-linear structures such as membranes.

1 INTRODUCTION

Membrane structures appeal through their elegant double-curved shapes and lightness - which can be achieved by generating pre-stressed equilibrium surfaces through formfinding. These can withstand external loading in pure tension and thus optimally utilize the textile material. The safety assessment of membrane structures is particularly challenging due to the interaction of form and force. The formfound shape and stress state is generally considered as the reference configuration for structural analysis, which typically shows a non-linear relation between external actions and their effects. The effect of the level of pre-stress on the structural behaviour cannot be neglected, especially with respect to the loss of tension in the membrane.

Recently, European efforts towards a harmonized standard in membrane design and verification have yielded in the Technical Specification on the "design of tensioned membrane structures"[1] (TS) with the goal of developing a Eurocode on this basis. The TS[1] provides a framework for the verification of the ultimate limit state (ULS) and serviceability limit state (SLS) depending on the type of non-linearity as defined in EN 1990[2] (Eurocode 0). Furthermore, research on reliability assessment of membrane structures has recently been conducted. Gosling et al.[3] for instance studied the general applicability of reliability analysis on membrane structures and de Smedt et al.[4] performed reliability analysis of membrane structures which are modelled as cable-nets. Other research about membranes in the context of safety concept is concentrated on the determination of partial factors leading to a satisfying safety level, see e.g., Zhang et al.[5] for the investigation of the resistance uncertainty of membrane materials and the recommendation of corresponding resistance partial factors, or de Smedt et al.[6] for the calibration of partial factors for pre-stress, wind and snow especially for membrane structures.

The aforementioned works have in common that they use the partial safety factor concept, which was developed for limit state equations that are linear or mildly non-linear. Thus, the applicability of the partial safety factor concept to membrane structures with distinct non-linear behaviour needs to be studied. To our knowledge there is no research concerning this issue, i.e., there is lack of research on the impact of the design rules of the partial safety factor concept (cf. section 2) on the achievable reliability on the basis of non-linear functional relations between actions and their effects of membranes. The goal of this contribution is to study this aspect based on an exemplary hyperbolic paraboloid. The paper is organized as follows: in section 2 the partial safety factor concept is briefly introduced, section 3 discusses the exemplary case study and section 4 summarizes the contribution and provides an outlook on how the case study can be extended.

2 THE PARTIAL SAFETY FACTOR CONCEPT

The EN 1990[2] (Eurocode 0) distinguishes between four partial safety factors (PSF) γ_f, γ_{Sd}, γ_m and γ_{Rd}. These cover the uncertainties related to the load model, the structural model, the material model and the resistance model, respectively. A PSF design needs to fulfill the following inequality:

$$\gamma_{Sd} \cdot t_S\left(\gamma_f \cdot l_k\right) = e_d \leq r_d = \frac{1}{\gamma_{Rd}} \cdot t_R\left(\frac{m_k}{\gamma_m}\right) \tag{1}$$

where l_k and m_k are the characteristic load and the characteristic material strength, e_d and r_d are the design load effect and the design resistance, t_S and t_R are functions provided by the structural model and the resistance model, which translate the load into the load effect and the material strength into the resistance.

However, the distinction of the four partial safety factors is only theoretical. In practice, the Eurocode 0 combines the PSF as follows

$$\gamma_F = \gamma_f \times \gamma_{Sd} \tag{2}$$
$$\gamma_M = \gamma_m \times \gamma_{Rd} \tag{3}$$

This leads to the question, if γ_F and γ_M should be applied prior (option a) or posterior (option b) to t_S and t_R:

$$(a) \quad e_d = t_S\left(\gamma_F \cdot l_k\right) \qquad \text{or} \qquad (b) \quad e_d = \gamma_F \cdot t_S\left(l_k\right) \tag{4}$$
$$(a) \quad r_d = t_R\left(\frac{m_k}{\gamma_M}\right) \qquad \text{or} \qquad (b) \quad r_d = \frac{1}{\gamma_M} \cdot t_R\left(m_k\right) \tag{5}$$

As long as t_S and t_R are linear functions, option (a) and (b) lead to the same design values. If non-linearities are present this is not the case anymore.

In this paper we focus on the load side of this problem (equation (4)). According to the Eurocode 0 option (a) should be chosen if the load effects increase more than the loads (over-linear) and option (b) if they increase less (under-linear), see Paragraph 6.3.2(4) of Eurocode 0[2] . This simplified rule categorizes the non-linearity of structural systems into the under- or over-linear behaviour and chooses the more conservative option. We question this rule in two respects: on the one hand, the categorization can be ambiguous and Eurocode 0 lacks clear definitions of the types. On the other hand, the more conservative choice between option (a) and (b) may be too conservative in some cases, but may also be not conservative enough in other cases. In this paper we investigate these issues on the basis of a case study on a membrane structure.

3 RELIABILITY ASSESSMENT OF NON-LINEARITIES OF AN EXEMPLARY MEMBRANE STRUCTURE

3.1 Problem description

The observed membrane structure is a hyperbolic paraboloid (hypar) which is shown in Figure 1. It is a slightly modified version of the hypar presented in Round Robin Exercise 4[7] (RR4). The structure has a base area of 6×6 m and a height of 2 m (cf. coordinates of edge points in Figure 1). The membrane and its edge cables are fixed at its low and high points. For formfinding the Updated Reference Strategy[8] based on a finite element model is used. Therefor, an isotropic prestress of 3.0 kN/m and a cable force of 30 kN is utilized. The material is modelled with the linear elastic, orthotropic Münsch Reinhardt material law[9] in combination with a stress-strain tensor modification[10] on the basis of the tension field theory to capture wrinkle deformation in case of tension loss in the membrane. As material properties for the membrane the following values are chosen:

- Young's moduli in warp and fill direction: $E_{warp/fill}$=600 kN/m (pre-integrated over the thickness)

- Shear modulus: G=30 kN/m (pre-integrated over the thickness)
- Poissons ratio: ν=0.4

The edge cables have a Young's modulus of 205 kN/mm^2 and a diameter of 12 mm.

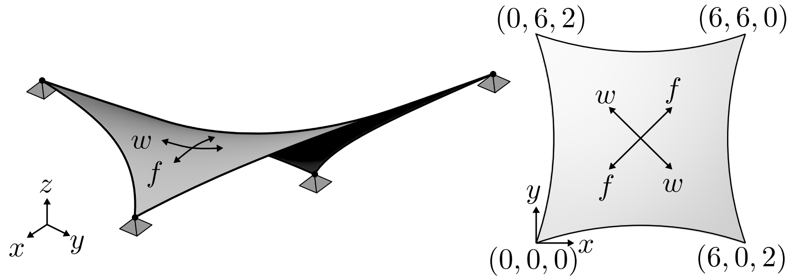

Figure 1: Observed membrane structure with indication of warp (w) and fill (f) direction.

In this case study only the load and the tensile strength are modelled as random variables, whose probability distributions are shown in Table 1. The chosen coefficient of variation (c.o.v.) of the material is based on the assumption that PVC-coated fabrics are used. The literature survey of Stranghöner et al.[11] lists c.o.v. values of 6% - 8% and 12% for joints (page 108) for that material. Zhang et al. investigated the resistance uncertainty of PVC-coated polyesters and evaluated a maximal c.o.v. of 7.2% for the uniaxial tensile strength. De Smedt et al.[6] use a c.o.v. of 10% for the tensile strength of PVC-coated fabrics in their reliability investigations. Based on that we decide to use a c.o.v. of 10%.

Table 1: Chosen distributions of load L and the tensile strength of the membrane M.

	mean	c.o.v.
$L \sim \mathcal{G}$ (Gumbel)	$0.34\ kN/m^2$	0.3
$M \sim \mathcal{LN}$ (Log-normal)	$1.0\ kN/m^2$	0.1

The characteristic values for the design with the partial safety concept are defined as the 98% und 5% fractile, respectively:

$$l_k = F_L^{-1}(0.98) \tag{6}$$

$$m_k = F_M^{-1}(0.05) \tag{7}$$

The utilized partial safety factors are

$$\gamma_F = 1.5 \tag{8}$$

$$\gamma_M = 1.4 \tag{9}$$

with γ_M being taken from the Technical Specification[1] of CEN TC250 WG5.

3.2 Structural design

For the ultimate limit state design the situation is considered in which the maximal stress exceeds the tensile strength of the membrane. Because a snow load is acting in negative z-direction on the membrane, the decisive stress is appearing in warp direction. The progress of the maximal stress in warp direction for an increasing snow load is shown as an action - effect of action graph on the left hand side of Figure 2 (blue line). The computations are performed with a force-controlled[12] geometrically non-linear structural analysis. Thereby the pre-stress is raised completely initially and the snow load is applied step by step. On the right hand side of Figure 2 the stress distribution in warp direction due to design load $l_d = \gamma_F \cdot l_k$ and the position of the maximal stress is shown. It can be seen that the membrane is fully under tension at this stage, i.e., no wrinkling occurs.

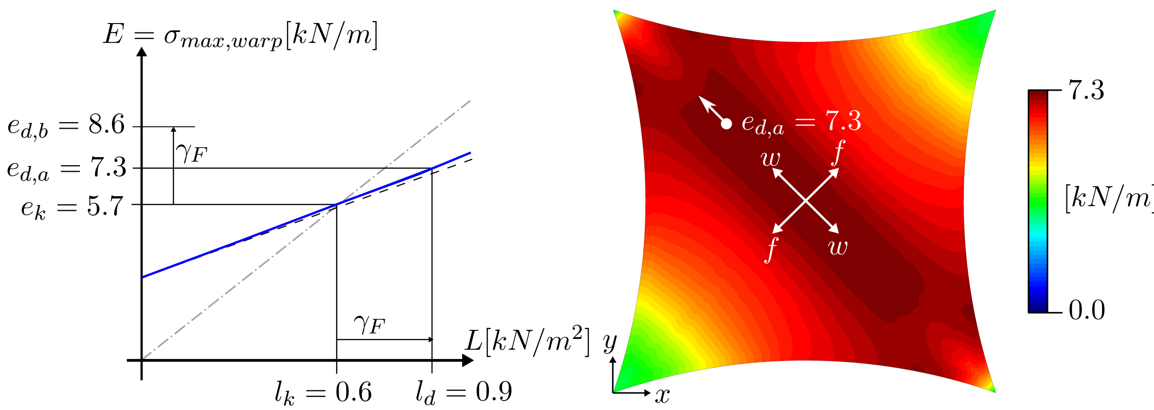

Figure 2: Left: Progress of maximal stress in warp direction of the membrane due to increasing load (blue), its tangent at zero load (dashed) and the line through the origin and the characteristic load effect (dash dotted). Right: Distribution of stress in warp direction due to design load l_d.

The classification of the relation between action and action effect in Figure 2 (left) into the categories over- and under-linear is ambiguous. Based on the pure inspection of the non-linear stress progress (blue graph) in relation to the dash-dotted line through the origin and characteristic point indicates under-linearity. This can be seen as the non-linear action - effect of action relation proceeds under the dash-dotted line if the characteristic load l_k is further increased. On the other side, the observation of second-order properties shows over-linear characteristics as the curvature of the function is convex (compare with linear dashed line). This particularity was already noted by Philipp[13] . Based on the classification introduced by Uhlemann et al.[14] , the behaviour of this limit state function would be under-linear, but Philipp[13] already discussed that the observed under-linearity is mainly achieved by the vertical shift of the action - effect of action relation due to pre-stress. Nevertheless, as the EN 1990 gives no distinct mathematical definition for the crucial functional characteristics for classification, the type of non-linearity for the case under consideration is not defined properly. As a consequence we will not further try to classify the action - effect of action relation into the categories over- and under-linear. Instead, the structural design is performed according to option (a) and (b) (cf. section 2 and Paragraph 6.3.2(4) of Eurocode 0[2]) to study the influence of the design method on the achieved nominal reliability for this membrane structure. Therefor, the design values of the effect of action, i.e., the decisive membrane stress in warp direction is computed according to the rules in equation (4). The results are shown in Figure 2 (left). Based on a linear resistance model the PSF designs

$$(a) \quad t_S\left(\gamma_F \cdot l_k\right) = d \cdot \frac{m_k}{\gamma_M} \qquad \text{or} \qquad (b) \quad \gamma_F \cdot t_S\left(l_k\right) = d \cdot \frac{m_k}{\gamma_M} \tag{10}$$

are obtained, wherein d can be interpreted as the design parameter. The transformation of equation (10) leads to

$$(a) \quad d = \frac{\gamma_M \cdot t_S\left(\gamma_F \cdot l_k\right)}{m_k} \qquad \text{or} \qquad (b) \quad d = \frac{\gamma_M \cdot \gamma_F \cdot t_S\left(l_k\right)}{m_k} \tag{11}$$

as calculation rule for the design d.

3.3 Reliability analysis

Based on the resistance design d (equation (11)), the limit state function can formulated with respect to the random variables for the material M and the load L (see Table 1):

$$g = d \cdot M - t_S(L) = \begin{cases} \frac{\gamma_M \cdot t_S(\gamma_F \cdot l_k)}{m_k} \cdot M - t_S(L) & \text{option (a)} \\ \frac{\gamma_M \cdot \gamma_F \cdot t_S(l_k)}{m_k} \cdot M - t_S(L) & \text{option (b)} \end{cases} \tag{12}$$

According to equation (12) the non-linear function of the static model t_S, i.e., the relation between the load and the maximal stress in warp direction is included twice in the limit state function. Firstly, t_S is part of the computation of the design d and secondly it transforms the load L into the effect of action $S = t_S(L)$. Note that the two occurrences of the non-linear function can have opposite effects on the reliability.

To analyse the reliability of the options (a) and (b), the design d is computed first based on equation (11) with the stress values shown in Figure 2 (left). The resultant reliability indices are then computed with first order reliability analysis (FORM)[15] . They are shown in Table 2. The design d following option (b) is greater than the one following option (a), which −obviously− leads to a higher reliability index.

The limit state surface in the standard normal space of both design options and the design points according to PSF concept and FORM are shown in Figure 3. FORM estimates the reliability indices as the distance of the FORM design point to the origin. Both limit state surfaces are only marginally non-linear, hence the estimation error of FORM is negligible. The differences between the PSF design points and the FORM design points are rather large (in general the PSF concept is calibrated such that its design point is an approximation of the FORM design point). The difference is mainly due to a rather large PSF of the resistance side $\gamma_M = 1.4$.

Table 2: Reliability indices of the membrane according to designs options (a) and (b).

Design option	Reliability index β
(a)	4.96
(b)	5.55

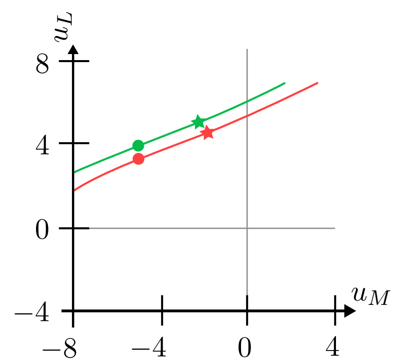

Figure 3: Limit state surface of the membrane designed with option (a) (red) and option (b) (green). The bullet points indicate the design points following the PSF concept. The stars indicate the FORM design points.

3.4 The effect of the non-linearity of membranes on the reliability

The total value of the stress in the membrane is due to pre-stress and stresses caused by the external load. As the principle of superposition is not valid in non-linear analysis, the relation between the snow load and its effects can only be analysed in presence of the pre-stress. Thus, the observed membrane structure includes two challenges in terms of non-linearity, which are (i) a non-linear evolution of the relation between external loads and stress indicated by the curvature of the load - stress graph and (ii) the presence of the of pre-stress in all structural analyses of the membrane leading to a shift of the load - stress relation (cf. blue graph in Figure 2 left).

In order to investigate the influence of the two non-linear effects, the reliability analysis in section 3.3 is repeated for different load - stress relations $t_S(L)$, as shown in Figure 4. Variation 1 is a linearization of t_S for $L > 0$ with the same stress values as the basic membrane model at $L = 0$ and $L = l_k$. Variations 2 and 3 are created by shifting the load - stress graphs of the basic membrane model and Variation 1 to the origin. Thus, Variation 2 is a non-linear and Variation 3 a linear relation between load and stress. For improved differentiation, linear load - stress graphs are shown as orange and the non-linear ones as blue lines in Figure 4.

To analyse the reliability of the Variations 1-3, the design d is recomputed according to equation (11) for design options (a) and (b). Therefor, the design stress values are calculated first based on equation (4). The reliability indices of the different designs are

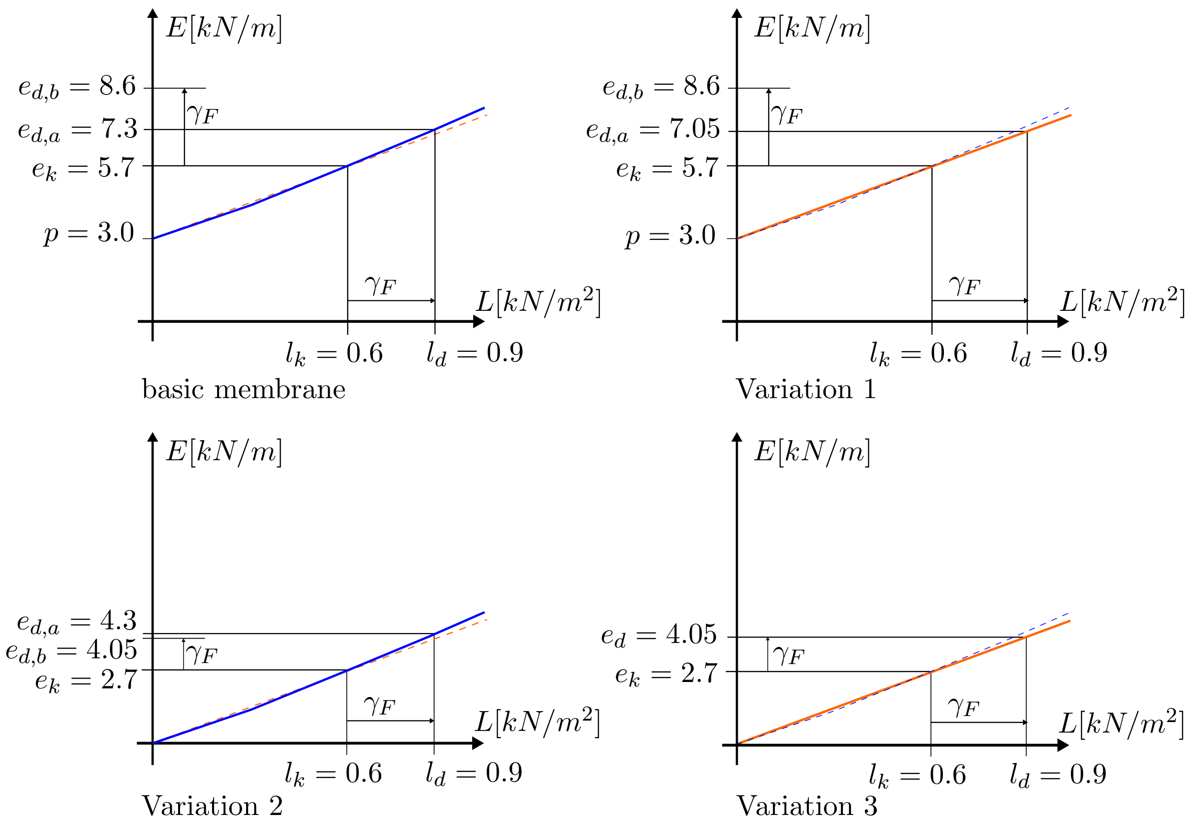

Figure 4: Observed options of linear (orange) and non-linear (blue) action - effect of action relations with and without pre-stress p.

again computed with FORM and are shown in Table 3. Figure 4 visualizes the limit state surfaces of Variation 1-3.

The following remarks can be made based on the numerical investigations:

- The PSF concept is calibrated for linear functional relationships t_S through the origin of actions and action effects. This is the case for Variation 3. In this sense, the reliability index of 4.72 can be interpreted as the target reliability index of the considered membrane structure. Thus, any design variation which leads to a greater reliability index is too conservative and any design variation leading to smaller reliability index is not safe enough.

- Both design options of the basic membrane lead to reliability indices greater than 4.72 (4.96 for design option (a) and 5.55 for design option (b)). Hence, both are conservative.

- The linear stress function t_S of Variation 3 leads to a linear limit state surface of Variation 3 in the original space. If a transformation to the standard normal space is conducted, the limit state surface becomes slightly non-linear (see Figure 5 right). Hence, the non-linearity does not only depend on t_S but also on the utilized distribution types and the probability space.

- Design Variation 1 neglects the convex form of t_S. Its comparison to the basic membrane can be interpreted as an isolated assessment of the non-linear effects of the convex form of t_S. The limit state surfaces of the two design options following Variation 1 (see Figure 5 left) drift apart and away from the origin compared to the limit state surfaces of the basic membrane (see Figure 3). Consequently the reliability indices of the two design options following Variation 1 are increased. Both design options lead to even more conservative designs than the basic membrane, since they are greater than the reliability index of the linear case of 4.72. Moreover, the inclination of the limit state surfaces at their respective FORM design point in the standard normal space changes. Hence, Variation 1 is less sensitive to load variations and more to material variations.

- Design Variation 2 neglects the pre-stressing. Its comparison to the basic membrane can be interpreted as an isolated assessment of the non-linear effects of the pre-stressing. The limit state surfaces of the two design options following Variation 2 (see Figure 5 middle) are much closer to each other and closer to the origin than the limit state surfaces of the basic membrane (see Figure 3). Moreover, the limit state surface of design option (b) is closer to the origin than design option (a) of Variation 2 (for the basic membrane it is the other way around). Consequently, the reliability indices of the two design options following Variation 2 are reduced and their ordering switches. Both design options lead to non conservative designs since they are smaller than 4.72.

- The effects of pre-stresses and of the convex form of t_S on the reliability are counteracting each other. Overall, the effect of the pre-stressing is stronger.

Table 3: Reliability indices β for different load - stress relation variations (see Figure 4) and designs according to option (a) and (b).

	β based on load - stress relations $t_s(L)$ of			
Design option	Basic membrane	Variation 1	Variation 2	Variation 3
(a)	4.96	5.30	4.46	4.72
(b)	5.55	6.20	4.26	4.72

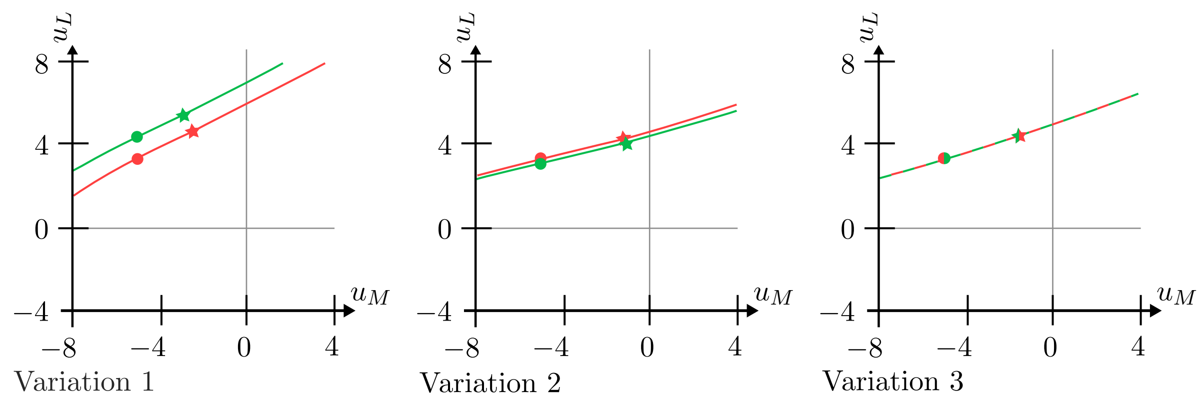

Figure 5: Limit state surface of the membrane designed following option (a) (red) and option (b) (green); whereby t_S follows Variation 1 (left), 2 (middle), 3 (right) (see Figure 4). The bullet points indicate the design points following the PSF concept. The stars indicate the FORM design points.

4 CONCLUSIONS AND OUTLOOK

This contribution shows the impact of the design rules for non-linear limit state functions of EN 1990 (Eurocode 0) on the reliability of an exemplary membrane structure. We discuss that the categorization of the type of non-linearity is not as straight forward as the EN 1990 indicates. In the special case of membrane structures the categorization is especially ambiguous due to the presence of pre-stress.

To investigate the special challenges of the membrane structure in terms of non-linearity several variations of its load - stress relation are used for structural design according to the rules of EN 1990. Reliability analyses of the thereby determined designs indicated that the current approach of the Eurocode 0 can lead to both unsafe structures and too conservative structures.

The presented results are limited to the investigated membrane and the chosen probability distributions. For more general statements and improved rules for codified design, the investigations have to be done on a portfolio of structures, actions and resistances. Furthermore, the influence of pre-stressing has to be studied in a more general setting as well. Even though the contribution covered only one case study, two fundamental problems of the current design rules could be identified which are (i) inconsistent reliability and (ii) ambiguous usage due to unclear categorisation of non-linearities.

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REFERENCES

[1] prCEN/TS, 19102:2021. *Design of tensioned membrane structures (prCEN/TS*

19102:2021-04). CEN - European Committee for Standardization, Brussels, 2021.

[2] Comité Européen de Normalisation (CEN). EN 1990: Eurocode: Basis of structural design (EN 1990:2002 + A1:2005 + A1:2005/AC:2010), 2002/2010.

[3] P. D. Gosling, B. N. Bridgens, and L. Zhang. Adoption of a reliability approach for membrane structure analysis. *Structural Safety*, 40:39–50, 2013.

[4] E. de Smedt, M. Mollaert, M. van Craenenbroeck, R. Caspeele, and L. Pyl. Reliability-based analysis of a cable-net structure and membrane structure designed using partial factors. *Architectural Engineering and Design Management*, pages 1–10, 2020.

[5] Y. Zhang, Y. Lu, Y. Zhou, and Q. Zhang. Resistance uncertainty and structural reliability of hypar tensioned membrane structures with pvc coated polyesters. *Thin-Walled Structures*, 124:392–401, 2018.

[6] E. de Smedt, M. Mollaert, R. Caspeele, W. Botte, and L. Pyl. Reliability-based calibration of partial factors for the design of membrane structures. *Engineering Structures*, 214:110632, 2020.

[7] E. de Smedt, M. Mollaert, L. Pyl, P. Gosling, J. Uhlemann, and J.-C. Thomas. Round robin exercise 4: Reliability analysis of a simple membrane structure: a hyperbolic paraboloid, 2017.

[8] K.-U. Bletzinger and E. Ramm. A general finite element approach to the form finding of tensile structures. *International Journal of Space Structures*, 14(2):131–145, 1999.

[9] R. Münsch and H.-W. Reinhardt. Zur Berechnung von Membrantragwerken aus beschichteten Geweben. *Bauingenieur*, 20:271–275, 1995.

[10] K. Nakashino and M. C. Natori. Efficient modification scheme of stress-strain tensor for wrinkled membranes. *AIAA Journal*, 43(1):206–215, 2005.

[11] N. Stranghöner, J. Uhlemann, F. Bilginoglu, K.-U. Bletzinger, H. Bögner-Balz, E. Corne, N. Gibson, P. Gosling, R. Houtman, J. Llorens, M. Malinowsky, J.-M. Marion, M. Mollaert, M. Nieger, G. Novati, F. Sahnoune, P. Siemens, B. Stimpfle, V. Tanev, and J.-Ch. Thomas. Prospect for european guidance for the structural design of tensile membrane structures: Support to the implementation, harmonization and further developments of the eurocodes. In M. Mollaert, S. Dimova, A Pinto, and St. Denton, editors, *JRC Science and Policy Report*. European Commission, Joint Research Centre, European Union, 2016.

[12] M. A. Crisfield. *Non-Linear Finite Element Analysis of Solids and Structures: Essentials*. John Wiley & Sons, Inc., USA, 1991.

[13] B. F. Philipp. *Methodological Treatment of Non-linear Structural Behavior in the Design, Analysis and Verification of Lightweight Structures*. Ph.D. Thesis, Technical University of Munich, Munich, 2017.

[14] J. Uhlemann, B. Stimpfle, and N. Stranghöner. Application of the semiprobabilistic safety concept of EN 1990 in the design of prestressed membrane structures. In *Proceedings of the EUROSTEEL 2014*, Naples, Italy, Sept. 2014.

[15] R. Rackwitz and B. Fiessler. Structural reliability under combined random load sequences. *Computers & Structures*, 9(5):489–494, 1978.