Ductile-Fragile Transition: A Novel Comparison Between Fracture Mechanics of Materials and Framed Structures

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Abstract

This paper deals with the issue of structural collapse considering an analogy between ductile-brittle transition of materials, taking into account the current literature which also considers the number of fragility and the stress intensification factor in the presence of crack, with extension on a larger scale involving framed structures subjected to increasing vertical loads.

It is evaluated the ductile-fragile transition in relation to concrete frames with different structural hierarchy (2x2, 5x5, 11x11).

Keywords: Structural Mechanics, Fracture Mechanics, Materials, Robustness

1 Introduction: Structural Collapse And Fracture Mechanics

In modern civil engineering problems the concept of structural resistance is often explained as the design limit beyond which the structure does not show problems with reference to the serviceability limit state and the ultimate limit state

It is also necessary to start analyzing the single structural element to determine formulations that can be extended to larger and more complex structures subject to different types of loads.

The modern study of fracture mechanics and the response to structural collapse of buildings and infrastructures certainly derives from experiments conducted by A.A. Griffith (1893-1963), when he first published in 1920 a groundbreaking scientific work on strength and fracture, deposited with the Royal Society.

Griffith's work was mainly concentrated on materials with typically fragile behavior such as glass but, thanks to the importance of its deductions, it was also extendable to other structures, even those of considerable size such as airplanes and ships.

It is in fact irrefutable that, at the beginning of the 20th century, a large number of structures collapsed, due to the fact that the structural models of the time did not take into account, in addition to the shape of the elements, also the characteristics and geometric dimensions.

An absolutely enlightening concept in this sense was introduced at the time of Griffith, when it was understood that it was necessary to consider the concept of concentration of stresses, as soon as a change of shape or a crack occurs in a material or structure, with consequent weakening and predisposition to the propagation of the collapse in these positions.

In this regard, considering for example a simple structural element, such as a slab, subject to traction, it can be highlighted that the isostatic lines approach each other (or local intensification of the stress level) where there is a narrowing of the section or there is a discontinuity or crack [1].

In this paper, it is therefore analyzed the ductile-brittle transition, first of the materials, keeping in mind the stress concentration aspects and determining the limit load in addition to which a change in the structural response is determined.

2 Ductile-Fragile Transition Of Materials

Having introduced in the previous Paragraph the key concept of modern Fracture Mechanics such as the search for points or areas of the material and structure where there is an intensification of the stresses due to a narrowing of the local section or the presence of a crack, it is possible to move on to analyze in more detail the ductile-brittle transition of the materials which certainly integrates, in addition to the correspondence between stress and deformation, also the presence and propagation of cracks within the material, with consequent variation in the overall response to the internal stresses of the analyzed element. There are two main types of materials in the structural field: ductile and brittle.

In the first case it is possible to include all those materials which, subject to increasing load, show a trend for large linear sections and an increasing deformation response, up to failure. In the second case, however, the material subjected to the same load, gradually increasing, shows an initially linear trend up to a certain point beyond which, as the load increases, there is a progressive unloading of internal stress (to a failure) to which follows a deformation response, with behavior defined as negative work hardening. This trend defined as "snap-back" occurs when, upon reaching a vertex point of the stress-strain diagram, a first crack opens in the material with consequent dissipation of energy in the hardening section with a negative slope. It has been demonstrated, from laboratory tests, that this hardening section has an increasingly negative slope as the size of the specimen increases, until it reaches a positive snap back slope which is a characteristic of unfortunately sadly known catastrophes in civil engineering. The aforementioned energy is defined as fracture energy and is the integral of the stress-strain curve underlying the negative hardening branch and is, together with strength, an intrinsic characteristic of the material. While the greater or lesser ductility of a material or a structure depends on the geometric dimensions.

As the size increases, as seen before, there is a greater propensity to pass ductile-brittle. The materials that have a ductile behavior also have a compressive and tensile yield stress of the same order of magnitude. On the contrary, it occurs that materials with less ductile behavior have resistances of different orders of magnitude to compression and traction, as in the case of concrete [2].

The transition from ductile to brittle collapse in materials, considering for example a beam subject to central concentrated load P, depends on a dimensional effect and in particular on the length of the crack a_0. For crack lengths $a < a_0$ the collapse propation occurs in a ductile manner, while for $a > a_0$ the brittle collapse precedes the plastic collapse. The characteristic length can be expressed through the following relationship which takes into account the stress intensification factor in the vicinity of the crack:

$$a_0 = \frac{1}{\pi}\frac{K_I^2}{\sigma_P^2} \qquad (1)$$

where K_I is the stress intensification factor, while σ_P is the ultimate resistance of the material (in this case tensile strength).

In particular, with reference to [2] it is possible to deduce the following relationship which links the various geometric characteristics of the beam, with crack width *a*, ultimate plastic load P_{max} and tensile strength σ_P:

$$\frac{P_{max}L}{\sigma_P t H^2} = \left(1 - \frac{a}{H}\right)^2 \quad (2)$$

where *L* is the length of the beam, *H* is the height and *t* is the thickness.

3 Analogies With Progressive Collapse Of 2D Frames

2D framed structures are examined in Figure 1, which are composed of reinforced concrete with a high plastic strain and rotation capacity, as an application of the suggested methodology. The structures are composed of varying numbers of structural cells (n^2), despite having the same overall size. The frames can be understood as a hierarchical reorganization of the frame with n = 11 when n = 2 and n = 5 are used. This is achieved by using a primary structure composed of fewer but larger structural elements. In order to achieve this, we proportionately set the cross sectional area and reinforcement of the beams and columns to their respective lengths, *L* and *H*. In this manner, for varying n, the structural elements' slenderness remains constant. Now refer frame's hierarchical level is referred as *1/n*. Details on the parameters of the materials, cross sections, and design strategy can be found in [6, 7]. The frames are subjected to the sudden removal of beams and columns within a damage area, dotted in Figure 1. Details of damages scenarios can be found in [3].

With the scope of this this paper it is treated the general case of the CB damage scheme.

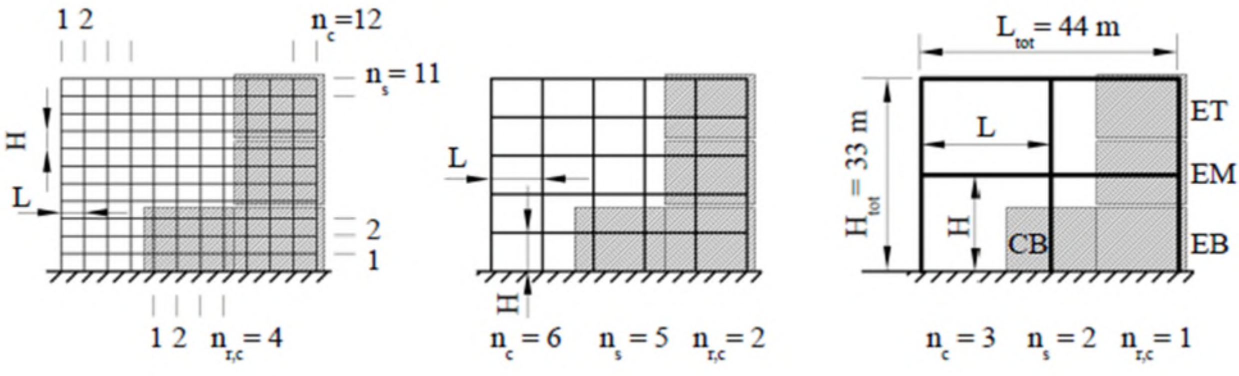

Figure 1 Studied frames and damage positions (dashed area). B = bottom, T = top, C = central,

E = external.

It is considered f_c = σ_P 35MPa: very strong columns inducing a collapse with bending mode mechanism [3].
Returning to the analogy with materials treated in equation [2] of Paragraph 2, this formulation can also be extended for the introduced frame structures.
In particular:

	2x2	5x5	11x11
P_{max} [kN]	433125	173250	78750
L [m]	22	8,8	4
σp [kN/m^2]	35000	35000	35000
t [m]	1	1	1
H [m]	16,5	6,6	3
a [m]	0	0	0

Table 1 Parameters of intact structures

	2x2	5x5	11x11
P_{max} [kN]	54141	15593	3986
L[m]	22	8,8	4
σp [kN/m^2]	35000	35000	35000
t [m]	1	1	1
H [m]	16,5	6,6	3
a [m]	8,3	2,6	1,1

Table 2 Parameters of damaged structures

With reference to the parameters inserted in Table 1 and Table 2 it is thus possible to diagram in Figure 2 the variation of the dimensionless ultimate plastic load P_{max} with respect to the width of the damage zone a, as the hierarchy of the structures considered varies. With the same size of the damaged area, the ultimate plastic load P_{max} is greater in the hierarchical structure, 2x2, rather than in the more heterogeneous and less hierarchical ones, 5x5, 11x11.

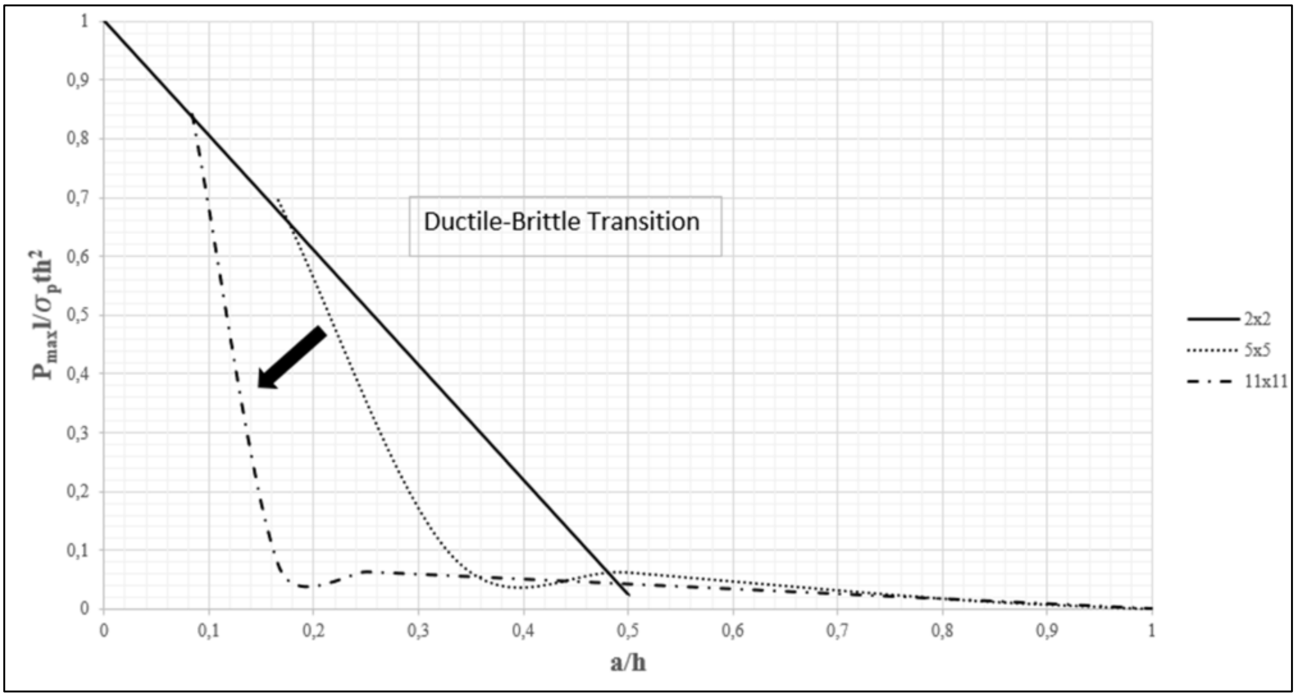

Figure 2 Transition from Ductile to Brittle behaviour in studied frames with damage CB

In the graph of Figure 2 it can be also highlighted that the transition between ductile and brittle is more evident between the 2x2 and 5x5, 11x11 structures, compared to the ductile-brittle transition between the 5x5 and 11x11 structures, Confirmation of the more fragile behavior of less hierarchical structures can be detected by introducing the *fragility number s*, in analogy with fracture mechanics of materials, the expression of which is reported in equation (3) with reference to [1]:

$$s = \frac{K_{IC}}{\sigma_P \sqrt{2H}} \quad (3)$$

Where KIC, is the stress intensification factor, already determined in equation (1).

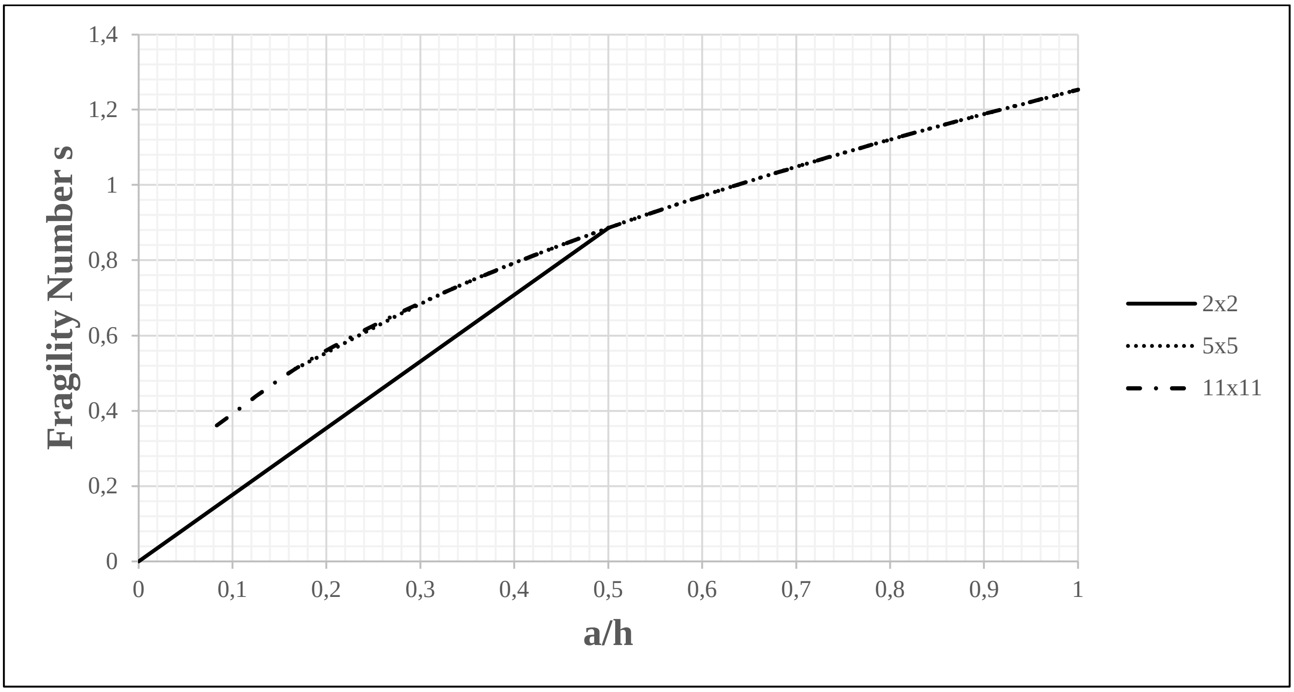

Figure 2 Transition from Ductile to Brittle behaviour in studied frames with damage CB

As written before, the confirmation that the number of fragilities increases, in reference to the structures analysed, going from the more hierarchical structures to the less hierarchical structures is highlighted in Figure 3.

4 Conclusion And Final Remarks

In this paper, the analogy between materials and reinforced concrete frame structures, having the same overall dimensions but different structural hierarchy, was analyzed. In principle, an equivalence has been established between the fracture of materials or simple structures and the collapse of buildings, thus considering extending the typical formulations of Fracture Mechanics to civil frame structures. The use of the same equations allowed us to confirm a good adaptation even to the case of a larger scale, compared for example to a concrete specimen in the laboratory. What was in fact obtained is that, confirming the references in [3], [6], [7] the more hierarchical structures are more resistant than the less hierarchical and more heterogeneous ones with reference to the ultimate collapse load and have a lower number of fragility as they are more ductile.

5 References

[1]. "Strutture sotto sforzo", James E. Gordon, Zanichelli Editore.

[2]. "Meccanica dei Materiali e della Frattura", A. Carpinteri, Pitagora Editrice Boloogna.

[3]. M. E., C. A., C. B.M. and A. Calvi, PROGRESSIVE COLLAPSE AND ROBUSTNESS OF FRAMED STRUCTURES: SIMULATION AND DESIGN, Alessandro Calvi's personal collection (2024).

[4]. A. Calvi, The Twin Towers Collapse, Event Analyses and Structural Lessons, Master's Degree Thesis, Alessandro Calvi's personal collection (2024).

[5]. BS Eurocode 1, ”Actions on structures - Part 1-7: General actions - Accidental actions”, EN 1991-1-7, undated (c.2004).

[6]. E. Masoero, “Progressive collapse and robustness of framed structures”, PhD Thesis, Politecnicodi Torino, Italy, 2010.

[7]. E. Masoero, F.K. Wittel, H.J. Herrmann, B.M. Chiaia, J. Eng. Mech. - ASCE, 136 (8), 987 995, 2010.