14th World Congress on Computational Mechanics (WCCM)
ECCOMAS Congress 2020)
Virtual Congress: 11-–15 January 2021
F. Chinesta, R. Abgrall, O. Allix and M. Kaliske (Eds)

A TOPOLOGICAL DERIVATIVE-BASED HYDRO-MECHANICAL FRACTURE MODEL

J. M. M. LUZ FILHO[1], A. A. NOVOTNY[2]

[1] Laboratório Nacional de Computação Científica
25651-075
jmorvan@lncc.br

[2] Laboratório Nacional de Computação Científica
25651-075
novotny@lncc.br and https://novotny.lncc.br/

Key words: Hydraulic Fracture, Topological Derivative, Francfort-Marigo Damage Model, Biot's Theory

Abstract. The present work aims to introduce a hydraulic fracture model based on the concept of the topological derivative. This well stimulating technique also known as fracking was proposed by [1] in 1949 and has been widely used in oil and gas industry [2]. Overall, the fracking technique consists in significantly increase the production surface area of a porous medium from a pre-existing fault in such a way that the gas trapped in the rock formation might be collected in the surface. Furthermore, the fracking process has been the subject of many recent works due to its environmental impacts and economical aspects.

In this context, we present an extension of the hydro-mechanical fracturing model proposed by [3] in pursuit of a scenario closer to reality by taking into account the initial stress state (*in situ* stress) and the fluid inertia during the propagation process. The present model results from adapting the Francfort-Marigo damage model [4] to the context of hydraulic fracture together with Biot's theory [5]. We consider a two dimensional idealization in which the fracturing process is activated by a non constant pressure field distributed over the whole domain. A shape functional given by the sum of the total potential energy of the system with a Griffith-type dissipation term is minimized with respect to a set of ball-shaped inclusions by using the topological derivative concept. Then the associated topological derivative is used to construct a topology optimization algorithm designed to simulate the nucleation and propagation process. Finally, some numerical examples depict the role of the *in situ* stress in the fracture propagation, specific crack path growth (allowing kinking and bifurcations) and also the applicability of the methodology here proposed.

1 INTRODUCTION

The topological sensitivity analysis provides an scalar function, known as topological derivative, which measures the sensitivity of a given shape functional with respect to a infinitesimal singular perturbation, such as the insertion of holes for instance. In this work, the concept of the topological derivative is applied in the context of fracking. The fracking is a well stimulating technique that consists in greatly increase the production surface area of a porous medium in such a way that the gas trapped in the rock formation might be collected in the surface. In this work, we propose a hydro-mechanical fracturing model that results from adapting the Francfort-Marigo damage model along with Biot's consolidation theory. The work is organized as follows: Biot's model is presented in Section 2; The proposed hydro-mechanical fracturing model is presented in Section 3 and the associated topological derivative formula is presented in Section 4; In Section 5 a simple topological optimization algorithm is proposed and some numerical results are presented in Section 6. Finally, some conclusions and remarks are made in Section 7.

2 BIOT'S MODEL

In 1941 M. A. Biot proposed a model to describe soil behavior under load, a phenomenom known as consolidation [5] under an Eulerian approach which was later extended to an Lagrangian one by [6]. In other words, Biot's model provides the mechanical behavior of a porous matrix where the fluid is free to flow through its connected porous.

2.1 FULLY COUPLED MODEL

Let $\Omega \in \mathbb{R}^n$ $(n = 2, 3)$ be the domain occupied by a linear elastic and fluid saturated porous matrix. The Eulerian and Lagrangian porosity are defined as:

$$\phi_e = \frac{dV_p}{dV}, \quad \phi_l = \frac{dV_p}{V_0}, \tag{1}$$

where V_p and V_0 represent respectively the volume of connected porous in the current and reference configuration and V denotes the total volume of the poroelastic sample. That is, $V = V_p + V_s$, where V_s is the volume of the solid phase of the referred sample. The mass balance for fluid phase may be written as follows

$$\frac{\partial m}{\partial t} + \text{div}(\rho_f v_D) = 0, \tag{2}$$

in which ρ_f denotes the fluid density and v_D is Darcy's velocity, defined as

$$v_D = \phi_e \left(v_f - \frac{\partial u}{\partial t} \right), \tag{3}$$

where v_f is the fluid velocity. In addition, the term m in (2) is the so called Lagrangian mass content of fluid proposed in [6], given by

$$m = \rho_f \phi_l. \tag{4}$$

It is important to emphasize that the referential is fixed to the solid in this approach. By denoting with the subscript zero the reference value of each quantity, we can write the following constitutive law for the mass content of fluid

$$m = m_0 + \rho_{f_0} \alpha \text{div}(u) + \frac{\rho_{f_0}}{M}(p - p_0), \tag{5}$$

where p denotes the porepressure, u is the solid phase displacement, α represents the Biot-Willis coefficient and M is known as the non-drained Biot's modulus. In addition, the last two terms are defined as

$$\alpha = 1 - \frac{K_b}{K_s} \quad \text{and} \quad \frac{1}{M} = \frac{\alpha - \phi_l}{K_s} + \frac{\phi_l}{K_f}, \tag{6}$$

in which K_b, K_s, K_s are, respectively, the bulk modulus of the porous matrix, solid and fluid. Similarly, we can also write a constitutive law for the porosity, given by

$$\phi_l = \phi_{l_0} + \alpha \text{div}(u) + \frac{1}{N}(p - p_0), \tag{7}$$

where $1/N = (\alpha - \phi)/K_s$. Now, taking the derivative with respect to time of equation (5), we have

$$\frac{\partial m}{\partial t} = \rho_{f_0} \alpha \text{div}\left(\frac{\partial u}{\partial t}\right) + \frac{\rho_{f_0}}{M}\frac{\partial p}{\partial t}. \tag{8}$$

By combining (2) and (8), it follows that

$$\rho_{f_0} \alpha \text{div}\left(\frac{\partial u}{\partial t}\right) + \frac{\rho_{f_0}}{M}\frac{\partial p}{\partial t} + \text{div}(\rho_f v_D) = 0. \tag{9}$$

Finally, a linearization is applied to the mass flux by taking $\rho_f = \rho_{f0}$, which gives us the equation that governs the hydrodinamic system, namely

$$\frac{1}{M}\frac{\partial p}{\partial t} + \text{div}(v_D) + \alpha \text{div}\left(\frac{\partial u}{\partial t}\right) = 0. \tag{10}$$

Now, to obtain the geomechanical governing equation, we part from the effective stress principle [5], given by

$$\sigma_t = \sigma_e - \alpha p \text{I}, \quad \text{with} \quad \sigma_e = \sigma_0 + \sigma, \tag{11}$$

where σ_t denotes the total stress, σ_e is the effective stress, σ_0 represents the *in situ* stress and σ is given by the following constitutive law

$$\sigma = \lambda \text{div}(u)\text{I} + 2\mu \nabla u^s \tag{12}$$

where λ and μ are known as Lamé's coefficients and ∇u^s is the strain tensor, namely

$$\nabla u^s = \frac{1}{2}\left(\nabla u + \nabla u^T\right). \tag{13}$$

From the conservation of linear momentum (when neglecting acceleration and gravitational effects), we have that

$$\text{div}(\sigma_t) = 0. \tag{14}$$

Therefore, by combining equations (11) and (14), we have

$$\text{div}(\sigma) = \alpha \nabla p - \text{div}(\sigma_0). \tag{15}$$

Finally, we have the Biot's problem in its fully coupled formulation, that consists in finding the fields $u(x,t)$ and $p(x,t)$, where $x \in \Omega$ and $t \in (0,T)$, such that

$$\begin{cases} \text{div}(\sigma) = \alpha \nabla p - \text{div}(\sigma_0) & \text{em } \Omega \times (0,T), \\ \dfrac{1}{M}\dfrac{\partial p}{\partial t} + \alpha \text{div}\left(\dfrac{\partial u}{\partial t}\right) - \text{div}(k \nabla p) = 0 & \text{em } \Omega \times (0,T). \end{cases} \tag{16}$$

2.2 ALTERNATIVE FORMULATION

The fully coupled Biot's problem formulation demands that geomechanical and hydrodynamical to be solved simultaneously, which is extremely expensive from the computational point of view. The use of sequential formulations, where each equation is solved separately and an iterative process is introduced, is a commonly used alternative in order to tackle this problem. Among these iterative formulations, the fixed-stress algorithm is a very efficient method due to its robustness and unconditional stability [7].

The idea lies on rewriting the term $\alpha \mathrm{div}\left(\frac{\partial u}{\partial t}\right)$ from the hydrodynamical equation (9) in terms of the total stress. By taking the trace of the total stress given by (11), we have

$$\mathrm{tr}(\sigma_t) = \mathrm{tr}(\sigma_0) + (3\mu + 2\lambda)\mathrm{div}(u) - 3\alpha p. \tag{17}$$

Then, we define the average total stress as

$$\overline{\sigma}_t = \frac{1}{3}\sum_{i=1}^{3} \sigma_{ii} = \frac{1}{3}\mathrm{tr}\sigma_t, \tag{18}$$

which allows us rewriting (17) as

$$\overline{\sigma}_t = \overline{\sigma}_0 + \frac{3\lambda + 2\mu}{3}\mathrm{div}(u) - \alpha p, \tag{19}$$

where $\overline{\sigma}_0 = 1/3\mathrm{tr}(\sigma_0)$. Now, taking the derivative with respect to time, it follows that

$$\mathrm{div}\left(\frac{\partial u}{\partial t}\right) = \frac{\alpha}{K_b}\frac{\partial p}{\partial t} + \frac{1}{K_b}\frac{\partial \overline{\sigma}_t}{\partial t}, \tag{20}$$

where the bulk modulus of the porous matrix is given by $K_b = (3\lambda + 2\mu)/3$. Finally, by combining (20) with the hydrodynamical equation (9), we have

$$\beta^* \frac{\partial p}{\partial t} + \mathrm{div}(v_D) + \alpha \mathrm{div}\left(\frac{\partial u}{\partial t}\right) = 0, \tag{21}$$

where,

$$\beta^* = \left(\frac{1}{M} + \frac{\alpha^2}{K_b}\right) \tag{22}$$

represents the compressibility of the system, taking into account the solid grain and fluid compressibilities (through the term $1/M$) and the drained contribution α^2/K_b. The alternative formulation of Biot's problem consists in finding the fields $u(x,t)$ and $p(x,t)$, such that

$$\begin{cases} \mathrm{div}(\sigma) = \alpha \nabla p - \mathrm{div}(\sigma_0) & \text{em } \Omega \times (0,T), \\ \beta^* \dfrac{\partial p}{\partial t} - \mathrm{div}(k\nabla p) = -\dfrac{\alpha}{K_b}\dfrac{\partial \overline{\sigma}_t}{\partial t} & \text{em } \Omega \times (0,T). \end{cases} \tag{23}$$

The formulation above is solved with the aid of the iterative fixed-stress algorithm. Generally speaking, the main idea consists in fixing the total average stress when solving the hydro-dynamical problem, followed by the geomechanical solution obtained with the previously calculated pressure field [8]. More details about the fixed stress split algorithm may be found in [7, 8].

3 HYDRO-MECHANICAL FRACTURING MODEL

The hydro-mechanical fracturing model proposed in this work comes from adapting the Francfort-Marigo damage model to the fracking context together with Biot's consolidation theory. We assume a two-dimensional idealization and also that the reservoir is composed by the pressurization well and the region to be fractured. The initial stress state (*in situ*) is also taken into account. The proposed model is defined in the following scenario: we consider a porous satured matrix (isotropic and linearly elastic) subject to a proppant flux. The matrix is represented by a geometrical domain $\Omega \in \mathbb{R}^2$, containing the subdomain $\omega \in \Omega$ which represents the geological fault.

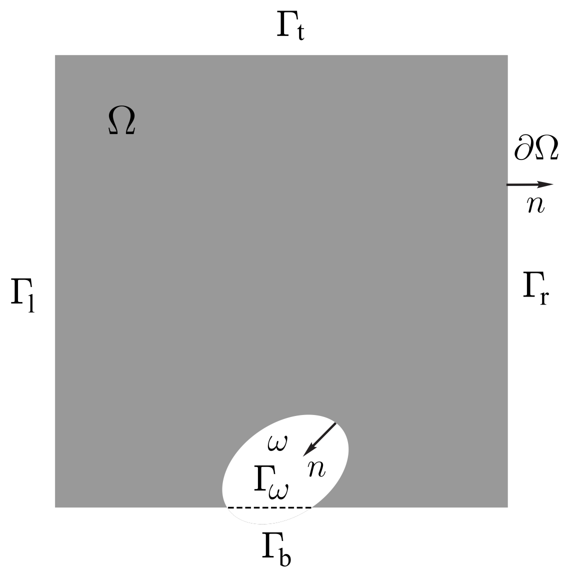


Figure 1: Saturated block containing a geological fault.

The damaged region is characterized by the parameter ρ, given by

$$\rho = \rho(x) := \begin{cases} 1, & \text{se } x \in \Omega \backslash \overline{\omega}, \\ \rho_0, & \text{se } x \in \omega, \end{cases} \tag{24}$$

with $0 < \rho_0 \ll 1$. The region $\Omega \backslash \overline{\omega}$ represents the undamaged porous medium, whereas ω is the geological fault.

Then the task is to minimize the shape functional proposed by Francfort-Marigo $\mathcal{F}_\omega(u)$ with respect to ω

$$\mathcal{F}_\omega(u) = \mathcal{J}(u) + \kappa|\omega|, \tag{25}$$

where κ is an energy release parameter and $\mathcal{J}(u)$ is the total potencial energy, given by

$$\mathcal{J}(u) = \frac{1}{2}\int_\Omega \sigma(u) \cdot \nabla u^s - \int_\Omega \alpha p \operatorname{div}(u) + \int_\Omega \sigma_0 \cdot \nabla u^s, \tag{26}$$

in which u is the displacement field and σ_0 represents the *in situ* stress. Furthermore, u is solution of the following variational problem: Find $u \in \mathcal{U}$, such that

$$\int_\Omega \sigma(u) \cdot \nabla \eta^s = \int_\Omega \alpha p \operatorname{div}(\eta) - \int_\Omega \sigma_0 \cdot \nabla \eta^s, \quad \forall \eta \in \mathcal{V}, \tag{27}$$

where α is known as the Biot-Willis coefficient and $p = p(x)$ is the pressure field acting on the porous matrix, which is solution of the following variational problem: Find $p \in \mathcal{P}$, such that

$$\int_\Omega \beta^* \frac{\partial p}{\partial t} \varphi + \int_\Omega k \nabla p \cdot \nabla \varphi = - \int_\Omega \frac{\alpha}{K_b} \frac{\partial \overline{\sigma}_t}{\partial t}, \quad \forall \varphi \in \mathcal{Q}, \tag{28}$$

where k is the permeability coefficient given by

$$k = k(x) := \begin{cases} k_r, & \text{se } x \in \Omega \backslash \overline{\omega}, \\ k_f, & \text{se } x \in \omega, \end{cases} \tag{29}$$

with $k_r \ll k_f$. Futhermore, the stress tensor $\sigma(u)$ is defined as

$$\sigma(u) = \rho \mathbb{C} \nabla u^s. \tag{30}$$

As the porous matrix is considered to be isotropic, then the stress tensor may be written in terms of the Lamé's coefficients, which, under the plane strain assumption are given by

$$\mu = \frac{E}{2(1+\nu)}, \quad \lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}, \tag{31}$$

where E denotes the Young's modulus and ν is the Poisson's ratio. The set $\mathcal{U}$ and the space $\mathcal{V}$ are defined as

$$\mathcal{V} = \mathcal{U} := \left\{ \varphi \in H^1(\Omega; \mathbb{R}^2) : \varphi_{|\Gamma_b} = 0, \quad \varphi \cdot n_{|\Gamma_r \cup \Gamma_l} = 0 \right\}, \tag{32}$$

while the set $\mathcal{P}$ and the space $\mathcal{Q}$ are given by

$$\mathcal{P} := \left\{ \varphi \in H^1(\Omega) : \varphi_{|\Gamma_t} = 0, \quad \varphi_{|\Gamma_w} = \overline{p} \right\}, \tag{33}$$

$$\mathcal{Q} := \left\{ \varphi \in H^1(\Omega) : \varphi_{|\Gamma_t} = 0, \quad \varphi_{|\Gamma_w} = 0 \right\}, \tag{34}$$

where $\overline{p}$ is the prescribed pressure on Γ_ω at a given time, namely

$$\overline{p} = \overline{p}(t) = p_0 + rt, \tag{35}$$

where p_0 is the reference pressure ($p_0 = 0$ in $t = 0$) and r is the increment pressure rate over Γ_ω. Finally the minimization problem is defined as

$$\underset{\omega \subset \Omega}{\text{Minimize}} \ \mathcal{F}_\omega(u), \text{ subject to (27)}. \tag{36}$$

4 TOPOLOGICAL DERIVATIVE FORMULA

The topological sensitivity analysis where only mechanical properties are considered sensitive to the topological perturbation is developed in [3] on section 3.1. Although in [3] considers Biot's quasi-static formulation, when only mechanical properties are sensitive to the topological perturbation it is possible the adapt the topological derivative formula associated to the quasi-static case to the transient scenario. Such is possible because we consider that the permeability is not sensitive to the topological perturbation, which introduces the hypothesis that crack propagation and fluid percolation are phenomenons with completely different time scales. This hypothesis is pretty reasonable once crack propagation in brittle materials is characterized to happen catastrophically so the pressure field is not sensitive to this phenomenon. More precisely, we consider crack propagation to be instantaneous and the pressure field is frozen during this process. Therefore, the topological derivative formula associated is given by the following result.

Theorem 1. *The topological derivative of the shape functional (25), with respect to the nucleation of a small circular inclusion with different mechanical properties to the matrix, is given by the sum*

$$D_T\mathcal{F}_\omega(x) = D_T\mathcal{J}(x) + \kappa_\delta D_T|\omega|(x) \quad \forall x \in \Omega, \tag{37}$$

where $\kappa_\delta D_T|\omega|(x)$ is given as follows

$$\kappa_\delta D_T|\omega|(x) = \begin{cases} +\kappa_\delta, & se\ x \in \Omega\backslash\overline{\omega}, \\ -\kappa_\delta, & se\ x \in \omega, \end{cases} \tag{38}$$

and the term $D_T\mathcal{J}(x)$ is given by the result previously obtained in section 3.1 of [3], namely

$$D_T\mathcal{J}(\hat{x}) = \mathbb{P}_\gamma\sigma(u)(\hat{x}) \cdot \nabla u^s(\hat{x}) + (1-\gamma^\alpha)\frac{1+a}{1+a\gamma}\alpha p(\hat{x})\mathrm{div}(u)(\hat{x}) - \frac{(1-\gamma^\alpha)^2}{2\rho\mu(1+a\gamma)}\alpha^2 p(\hat{x})^2. \tag{39}$$

In addition, the polarization tensor $\mathbb{P}_\gamma$ is given by

$$\mathbb{P}_\gamma = -\frac{1}{2}\frac{1-\gamma}{1+b\gamma}\left((1+b)\mathbb{I} + \frac{1}{2}(a-b)\frac{1-\gamma}{1+a\gamma}\mathrm{I}\otimes\mathrm{I}\right), \tag{40}$$

with the coefficients a and b defined as

$$a = \frac{\lambda+\mu}{\mu} \quad and \quad b = \frac{\lambda+3\mu}{\lambda+\mu}. \tag{41}$$

The terms γ and γ_α in (40) *represent the contrasts which affect the elasticity tensor $\mathbb{C}$ and the Biot's coefficient, respectively. For more details see [3].* —

5 TOPOLOGY OPTIMIZATION ALGORITHM

The topology optimization algorithm proposed is based on the fact that the value of the associated shape functional might decrease by introducing an inclusion inside a region ω^* where the topological derivative is negative,

$$\omega^* := \{x \in \Omega : D_T\mathcal{F}_\omega(x) < 0\}. \tag{42}$$

Thus, the topology optimization algorithm is designed in order to nucleate inclusions whose size is compatible with the previously damaged region through the parameter $\beta \in (0,1)$. More precisely, $\beta = 0$

represents a nucleation in a single point (the mininum topological derivative point) and $\beta = 1$ would nucleate all the negative topological derivative region. Therefore, we define the following quantity

$$D_T \mathcal{F}_\omega^* := \min_{x \in \omega^*} D_T \mathcal{F}_\omega(x), \tag{43}$$

which allows us to establish the region to be nucleated $\omega^\beta \subset \omega^*$ as

$$\omega^\beta := \{x \in \omega^* : D_T \mathcal{F}_\omega(x) \leq (1-\beta) D_T \mathcal{F}_\omega^*\}, \tag{44}$$

where $\beta \in (0,1)$ is chosen in such a way that $|\omega^\beta| \approx \pi\delta^2/$ (and $|\omega^\beta| \leq \pi\delta^2/4$). Thus the size of the inclusion is related to the width of the initial damage δ.

The steps of the proposed algorithm are summarized in 1.

Algorithm 1: Damage evolution algorithm.

Input : Ω, ω, δ, N, p_0, r, T
Output: Optimal topology ω^*

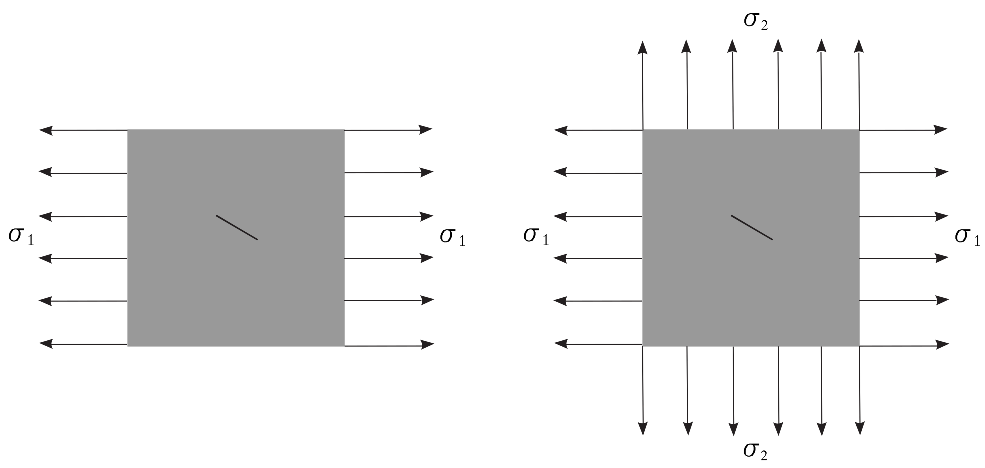
for $i = 1 : N$ **do**
  solve geomechanical (27) and hydrodynamical (28) problems via fixed-stress split;
  evaluate the topological derivative $D_T \mathcal{F}_\omega$ according to (37);
  compute the threshold ω^* de (42);
  while $|\omega^*| \geq \pi\delta^2/4$ **do**
    intensify the mesh at the crack tip;
    solve the geomechanical problem (27);
    evaluate the topological derivative $D_T \mathcal{F}_\omega$;
    compute the threshold ω^* from (42);
    compute the threshold ω^β from (44);
    nucleate a new inclusion ω^β inside ω^*;
    update the damaged region: $\omega \leftarrow \omega \cup \omega^\beta$;
    solve geomechanical problem (27) and evaluate $D_T \mathcal{F}_\omega$;
    compute the threshold ω^*;
  end while
end for

6 NUMERICAL EXPERIMENTS

We consider here a series of examples, in all of them the reference domain Ω represents one block of the reservoir containing a geological fracture with length h and width δ. Furthermore, the region to be fractured is represented by the distribution of elastic material while the geological fracture is identified by a compliant material. We also assume that the structure is under plane strain condition and total intensity of the prescribed pressure is incremented along with time with rate r, as shown in (35), until it reaches the maximum value $p = 8$ MPa. The total pressurization time $T = 24$ h is divided into $N = 200$ increments to solve the fixed-stress algorithm. Finally, linear triangular finite elements are used in the discretization of the hydro-mechanical coupled system.

6.1 IN SITU STRESS

We consider here a block of $8 \times 8\ m^2$, with a centered geological forming an angle of 30° with the horizontal axis. This example is proposed by [9] to depict the effect of the *in situ* stress on fracture propagation. Two different scenarios are considered, in the first the block is subject to uni-axial load while in the later bi-axial load is considered, as shown in the Figure 2.



(a) Uni-axial stress. (b) Bi-axial stress.

Figure 2: Types of loads.

As mentioned before, the algorithm proposed in this work is based on the topological derivative concept while the approach in [9] is based on the method known as phase field. The parameters considered in these examples are summarized in Table 1, where l denotes the diameter of the inclusion, α^m and α^f represents Biot's coefficient at the matrix and the fault respectively and γ^f represents the contrast on the permeability. The results obtained (left side) are compared with the ones presented by [9] (right side), see Figures 3 and 4. Qualitatively, one observes that in both cases the crack reorients itself along the vertical axis. As expected, the reorientation takes place within a smaller area when the block is under uni-axial stress. The same critical pressure was observed in both cases, namely $p_c = 1.04$ MPa.

Table 1: In Situ stress - parameters.

Parâmetro	Valor	Parâmetro	Valor
h	0.8 m	E	10 GPa
δ	0.025 m	ρ_0	10^{-5}
l	(2/3)δ	ν	1/3
p	8 MPa	k	1 mD
α^m	0.1	α^f	1.0
γ^f	10^3	N	200

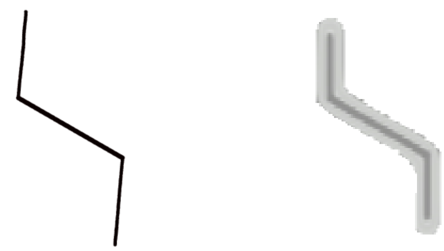


Figure 3: First example: Zoom in the damaged region after propagation.

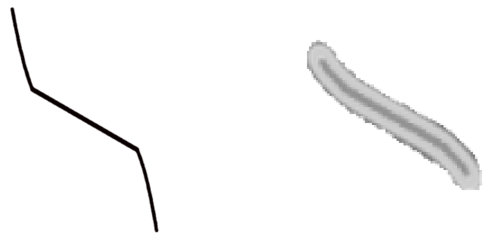


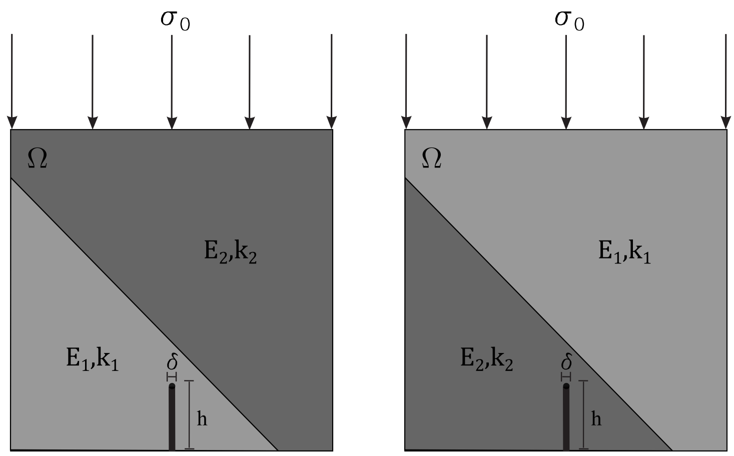
Figure 4: Second example: Zoom in the damaged region after propagation.

6.2 Stratified heterogeneous medium

In this example, we consider a $5 \times 5\ m^2$ stratified block composed by two layers with different permeability and Young's modulus, both corrupted with White Gaussian Noise (WGN) of zero mean and standard deviation τ. Therefore, E and k are replaced by $k_\tau = k(1 - \tau_p s)$ and $E_\tau = E(1 - \tau_e s)$, where $s : \Omega \to \mathbb{R}$ is a function assuming random values within the interval (0,1) and $\tau_p = 0.5$ and $\tau_e = 2.0$. The spatial distribution of both layers is shown in Figure 5, where $E_2 = 2 \times E_1$ and $k_2 = k_1/2$. Furthermore, we consider a preexisting geological fault located at the center of the bottom side, immediately above the pressurization well. All the data used to perform this example is summarized in Table 2.

Table 2: Stratified heterogeneous medium: parameters.

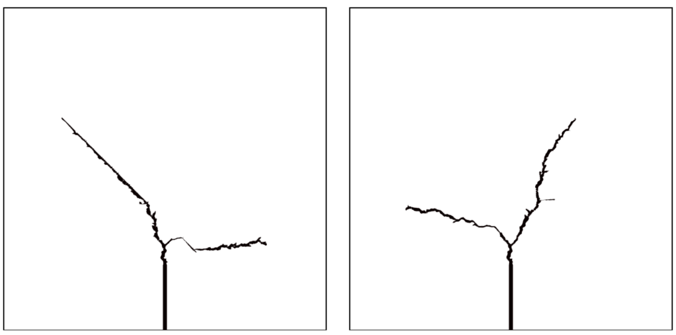
Parameter	Value	Parameter	Value
h	1.0 m	E_1	17 GPa
δ	0.0625 m	ρ_0	10^{-4}
l	(2/3)δ	ν	0.3
p	8 MPa	κ_s	60.0 J/m
k_1	40 mD	α^m	0.75
γ^f	10^6	α^f	1.0
N	200	σ_0	3.5 MPa



(a) Case 1. (b) Case 2.

Figure 5: Stratified medium.

By taking into account the data presented in Table 2 it was observed that for a given constant pressure on Γ_ω, the pressure field reaches the steady state after about 15 hours. The critical pressure that activates the geological fault in case 1 is $p_c = 4.64$ MPa, while in case 2 is $p_c = 6.00$ MPa. As we assumed the increment pressure rate $r = 2/3$ MPa/h, this means that the fracture propagation process starts at $t = 7$ h and $t = 9$ h, for cases 1 and 2 respectively. The total prescribed pressure $p = 8$ MPa on Γ_ω is reached after 12 h. Thus we assure that the propagation process takes place within the transient state in both cases. The obtained results are presented in Figure 6. Note that, due to heterogeneity of the medium, we can observe kinking and bifurcations phenomena. In both cases one bifurcation takes place in the first few steps of the propagation process. In case 1, the left branch propagates in the interface of both layers, as the right branch propagates in the horizontal direction crossing the interface between them. In case 2, the left branch tends to propagate in the horizontal direction, while the right branch propagates in direction of the layer with smaller rigidity. The same noise background was considered in both cases.



(a) Final damage: case 1. (b) Final damage: case 2.

Figure 6: Final damage after propagation.

7 CONCLUSIONS

The present work aimed to extend the model proposed in [3] by considering Biot's problem fully coupled formulation, which was solved with the aid of the fixed-stress split algorithm. The results obtained to depict the effect of the *in situ* corroborated with a known benchmark example that uses a different method. In the stratified heterogeneous medium example, the algorithm was capable of capturing kinking and bifurcations phenomena, which is expected from the physical point of view. Finally, it is important to emphasize that the present model is limited to two-dimensional domains, so the three-dimensional case is a subject for future works.

REFERENCES

[1] J. B. Clark, A hydraulic process for increasing the productivity of wells. *Trans Am Inst Mining Metall Eng* (1949) **186**(1):1–8.

[2] A. Cheng, *Poroelasticity.* Springer (2016).

[3] M. Xavier, N. Van Goethem and A. A. Novotny. Hydro-mechanical fracture modeling governed by the topological derivative method. *Computer Methods in Applied Mechanics and Engineering* (2020). **365**.

[4] G. Francfort and J. J. Marigo. Revisiting brittle fracture as an energy minimization problem. *Journal of the Mechanics and Physics of Solids*. (1993) **12**(2):149–189.

[5] M. A. Biot, General theory of three-dimensional consolidation. *Journal of Applied Physics.* (1941) **12**(2):155–164.

[6] O. Coussy. Poromechanics. John Wiley & Sons (2004).

[7] J. Kim; H. Tchelepi and R. Juanes. Stability, accuracy and efficiency of sequential methods for coupled flow and geomechanics. *SPE Journal* (2011). **186**(2):249–262.

[8] A. Settari and D. Walters. Advances in coupled geomechanical and reservoir modeling with applications to reservoir compaction. *SPE Journal* (2001). **6**(3):334–342.

[9] C. Chukwudozie; B. Bourdin and K. Yoshioka. A variational approach to the modeling and numerical simulation of hydraulic fracturing under in-situ stresses. *Proceedings of the 38th Workshop on Geothermal Reservoir Engineering.* 2013.