Determination of the S-wave propagation velocity of ballast by Spectral Analysis of Surface Waves

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ABSTRACT

The Laboratorio de Geotecnia-CEDEX uses some software and tools for studying different phenomena and performance of track sections. For this purpose, it is usually necessary to determine the S-wave (shear elastic wave) velocity of the different layers in the track section which typically are, from bottom to top: natural ground, embankment, form layer, sub-ballast and ballast.
The Laboratorio de Geotecnia-CEDEX has experimental S-wave propagation velocity (Vs) values of embankments, form layers and sub-ballast layers, obtained from several campaigns where the Spectral Analysis of Surface Waves (SASW) was applied on the different layers during the construction of several track sections.
To complete the studies and to obtain Vs values for the ballast layer, two campaigns were carried out in the CEDEX Track Box (CTB), a railway testing facility where it is possible to test complete railway sections on a 1:1 scale; and a field measurement campaign on an in-service railway track. Due to the discontinuous nature of the ballast layer carrying out tests to obtain Vs presents serious problems. To avoid these problems, a new procedure was developed to obtain these values using the SASW method by installing sensors on top of the sleepers.
Through the interpretation of measurements taken with the SASW technique on the ballast, the dispersion curve is obtained, and from it, the values of Vs are calculated. The results are presented in this article and are also compared with values found through a literature review obtained or estimated by other authors.

Keywords: ballast; SASW; dispersion curve; S-wave velocity.

1. Introduction

The Laboratorio de Geotecnia-CEDEX, a Laboratory that is part of CEDEX (public center for technical assistance and R+D+i of the Spanish Ministries of Transport and Environment), uses tools and computer software to study different phenomena and behaviors of railway sections. Often, it is necessary to model these structures, and among various parameters, the propagation velocities of S-waves (Vs) in different layers of materials need to be introduced. These layers, generally starting from the deepest, include natural ground, embankment, form layer, sub-ballast, and ballast.

Between 2010 and 2012, the Laboratorio de Geotecnia-CEDEX conducted several testing campaigns during the construction of various high-speed railway sections. The geophysical technique of Spectral Analysis of Surface Waves (SASW) was applied to determine the range of Vs values in the embankment, form layer, and sub-ballast. Due to the lack of values obtained experimentally in the Laboratorio de Geotecnia-CEDEX for the ballast layer, two measurement campaigns were carried out with SASW on the ballast layer of the CEDEX Track Box (CTB), a railway testing facility where it is possible to test complete railway sections on a 1:1 scale (Estaire et al. 2017). One campaign took place in November 2019, and another in October 2020. Later, in 2022, a measurement campaign was conducted to obtain Vs values in a high-speed railway section already in operation, where obtaining Vs values for the ballast layer was possible.

The Vs values obtained, both in the CTB and in the field campaign, have been compared with those provided by other authors, compiled after a literature review; the summary of such comparison is also included in this article.

Obtaining reliable Vs values from the ballast layer is more challenging than from other layers because the signals obtained during the test are generally not as clear as in the rest of the layers. This may be because the ballast layer is on the verge of being considered a continuous medium, given the relationship between the size of the particles (about 5 to 10 cm) and the thickness of the layer (about 50 or 60 cm in total) (Anbazhagan et al. 2010; Anbazhagan et al. 2011; Stark et al. 2018). The number, distribution, size, and shape of the ballast particles influence the number and strength of contacts between these particles, and this, in turn, is decisive in the value of the wave propagation speed through this particle structure.

Finally, it should be noted that the SASW method has been used instead of the MASW (Multichannel Analysis of Surface Waves) method, another widely used prospecting technique based on the analysis of surface waves for obtaining Vs. This choice is because the SASW provides greater resolution than MASW for studying thin surface layers, even though fieldwork is slower and very laborious. Additionally, this decision is influenced by the high level of expertise in the Laboratorio de Geotecnia-CEDEX in the application of the SASW method, which has been used for a long time for various applications and on different types of materials and grounds.

2. Basic principle of the Spectral analysis of surface waves

The Spectral Analysis of Surface Waves (SASW) involves determining the propagation velocity of transverse seismic waves, or S-waves (Vs), through the analysis of Rayleigh surface seismic waves. These waves are generated by the interaction of longitudinal seismic waves, or P-waves, with the S-waves. Rayleigh waves have a slightly lower velocity than S-waves, they propagate along the free surface of the ground and their amplitude exponentially decreases with depth (Foti 2005). They are dispersive, therefore a phase velocity (V_F) can be defined for each frequency or wavelength (λ), and the representation $V_F(\lambda)$ is called "dispersion curve". From the values of V_F, the values of Vs can be obtained by using Eq. (1), which relates V_F to Vs through a factor dependent on the Poisson's ratio (ν) of the ground (Sheriff, 1995). Additionally, it is assumed that the depth (z) is a fraction of the wavelength, i.e., the ratio λ/z is a constant. A value of 4 is recommended in sites where stiffness increases rapidly with depth, a ratio of 2 where stiffness remains reasonably constant, and a proportion of 3 as a generally reasonable value (Gazetas 1982).

From Vs, the dynamic shear modulus (G) of the ground for small deformations, on the order of 10^{-6}, can be determined using Eq. (2), given the known or estimated density (ρ) as well (Kramer, 1996).

$$V_S = \frac{1}{(0.874+0.197\,\nu-0.056\,\nu^2-0.0276\,\nu^3)} \cdot V_F \qquad (1)$$

$$G = \rho \cdot {V_S}^2 \qquad (2)$$

The execution of the test can be summarized as follows (Fig. 1): a multifrequency disturbance is generated on the surface by a source, which is simultaneously recorded by two sensors installed at a known distance "d." The phase difference "Θ" between both signals is determined from these recordings, resulting in the so-called phase curve (phase as a function of frequency). From this curve, along with the value of the sensor separation, the wavelength (λ) of the signal for each frequency (f) can be determined, and consequently, the corresponding phase velocity (Eq. (3)). The obtained values are attributed to the ground beneath the midpoint between the sensors, i.e., at d/2 (Nazarian et al. 1984; Foti et al. 2017).

Longer wavelengths of surface waves will penetrate deeper into the ground during their propagation, allowing the determination of S-wave velocities at greater depths in the it (Fig. 1).

$$\lambda = \frac{360^0}{\theta} \cdot d \qquad\qquad V_F = \lambda \cdot f \qquad (3)$$

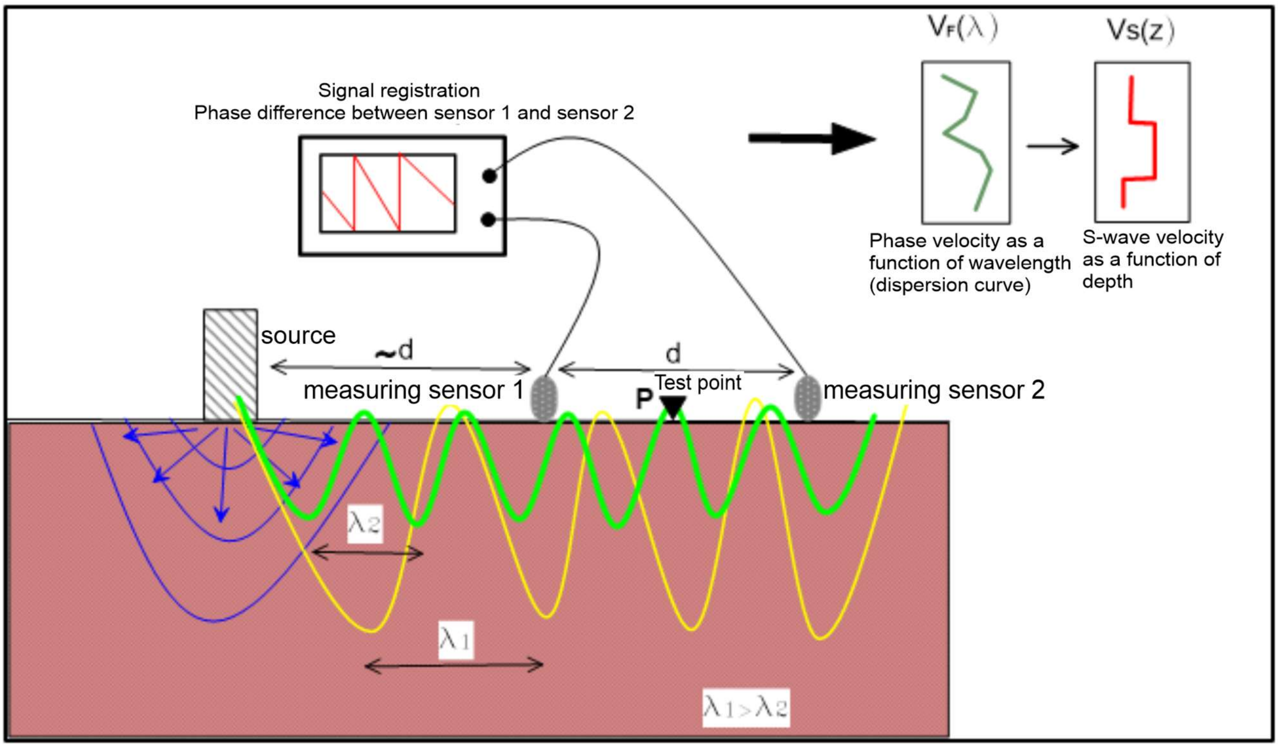


Figure 1. Diagram of the execution of the Spectral Analysis of Surface Waves (SASW) method to obtain the dispersion curve $V_F(\lambda)$ and the velocity curve Vs(z), which will be associated with the test point P.

Any device or system capable of generating waves on the ground within a suitable frequency range for its characteristics and the study objectives can be used as a source. Vibrators can be employed, allowing the fixation of a frequency range and signal amplitude. Alternatively, mass drop or striking with hammers on a plate or directly on the ground can also be used.

As measurement sensors, geophones or accelerometers are employed depending on the frequency range of interest, providing a good response within that range.

3. Literature review

A literature review was conducted to search for publications where experimental values of Vs of ballast layers were obtained or estimated to compare our results with those obtained by other authors. The publications found have been grouped based on whether the Vs values for the ballast layer were obtained through field tests or estimated for the definition of a railway track layer model used in a numerical method.

The techniques used in the field tests to obtain experimental Vs values are the SASW, MASW, and the Cross-Hole method. Table 1 compiles the references where experimental values are obtained, along with the obtained values, a comment regarding the validity of that value, and the technique used.

On the other hand, Table 2 presents the references found in which the Vs value is estimated, along with that value and a comment on its validity.

4. CEDEX Track Box

CEDEX Track Box (CTB) is a facility measuring 21 meters in length, 5 meters in width, and 4 meters in depth. Its main objective is to test complete sections of both conventional and high-speed railway tracks at a 1:1 scale, subjecting them to the passage of trains carrying passengers or goods at speeds of up to 420 km/h. Fig. 2 provides an overview of the installation.

In one week of operation at the CTB, the facility can replicate the effects that train traffic would have on a real track over the course of a year, making it a significant advantage. Additionally, it can simulate the effects of train approach, passage, and departure.

The loads on the track are generated using three pairs of servo-hydraulic actuators, and the simulation of effects resulting from potential wheel imperfections on the rails is also possible through two piezoelectric actuators.

The mechanical response of the tested railway line section is obtained through measurements of displacements, velocities, accelerations, and pressures. These measurements are taken using many transducers and measuring devices installed on the section (Estaire et al. 2017).

Table 1. Values of Vs for ballast layer found in the literature obtained through field tests.

Reference	Vs (m/s)	Comment about Vs value	Method
Anbazhagan et al. (2010)	140	Clean ballast	MASW
	135 – 150	Ballast contaminated with a contamination percentage < critical value	
	~ 100 – 135	Highly contaminated ballast	
Anbazhagan et al. (2011)	150	Contaminated ballast at 2.25%	MASW
	175	Contaminated ballast at 11.50%	
Hwang and Park (2014)	50 – 60	Depth < 0.1 – 0.2 m from the surface	SASW
	200 – 220	Depth > 0.3 m from the surface	
	100 – 150	Ballast on expansion joints between viaduct beams	
Hong et al. (2017)	80 – 150	Depth > 0.3 m from the surface	Cross - Hole
Stark et al. (2018)	140 - 290	Obtained in different track sections	SASW

Table 2. Values of Vs for the ballast found in the literature considered in numerical methods models.

Reference	Vs (m/s)	Comment about Vs value
Madshus and Kaynia (2000, 2001)	250	Value for a rock-type embankment (upper layer ballast + lower layer sand)
	150	Reduced value to account for the nonlinear behavior of the material
Alves Costa et al. (2015) **Mezher et al. (2016)**	170	Used in numerical models
Sayeed and Shahin (2016, 2018)	200	Used in numerical models

Figure 2. Overview of the CEDEX Track Box.

5. Work carried out in CEDEX Track Box

The measurement campaigns were carried out in November 2019 and October 2020. The railway section available in the CEDEX Track Box (CTB), on which the measurements were conducted, has a ballast layer of about 0.6 m thickness.

Two accelerometers were used as measurement sensors, and after various tests, they were fixed on the sleepers because they provided better-quality measurements compared to placing them on the ballast due to better contact surface. The measured waves not only propagate through the ballast layer and other deeper layers but also through the sleepers in the area near the sensors. Despite this, it is considered that the wave travel through the sleeper is negligible compared to the total, considering the arrangement and separation between sensors and the range of wavelengths that will be considered in the test.

On the other hand, various devices were tested to find a source capable of generating signals of the highest quality within the desired frequency range. Finally, hammer strikes directly on the sleeper were used, also on a metal screw supported on the sleeper, and on some ballast particles. In the 2019 campaign, some measurements were also made using a vibrator as a source. With all these methodologies, some acceptable phase records could be obtained, as well as others entirely unusable for interpretation. Therefore, it cannot be concluded which option is better for its use as a source.

The devices for the measurements were positioned as follows: the accelerometers were fixed on two adjacent sleepers with 0.5 m between them, and the sleeper next to each side was chosen as the positioning point for the source, with the distance to the nearest sensor being approximately 0.5 m (Fig. 3).

Fig. 4 shows a photo of a moment during the measurements, featuring the accelerometers used and the source, which in this case consisted of hammer strikes on a screw supported on the sleeper.

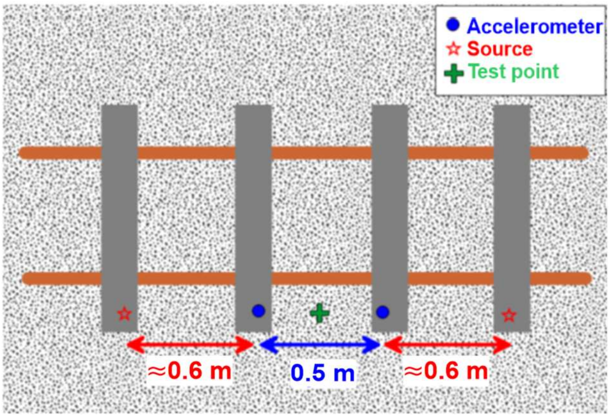


Figure 3. Diagram of the placement points of the measurement devices on the track and their relative separation.

Figure 4. Photo during the test. The accelerometers are highlighted in blue, and the source is indicated in red, with the relative separations between them.

5.1. Results obtained in CEDEX Track Box

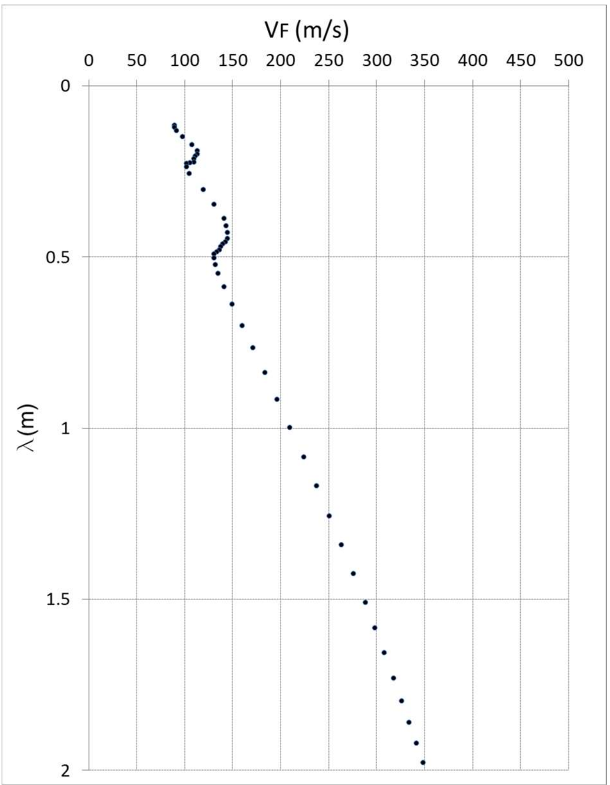
Among all the recorded measurements, those whose phase curves allowed a better interpretation were selected. After processing, the experimental dispersion curve covering a specific range of wavelengths is obtained. The dispersion curves of all selected measurements are represented together to obtain the overall dispersion curve, which can be considered as an average of all individual experimental curves. From it, the corresponding layer models were obtained.

Fig. 5 includes the dispersion curve obtained from the measurements taken during the November 2019 CTB campaign, and the representation of the interpreted layer model. Table 3 lists the layers defined in this model. The first 0.60 m corresponds to the ballast thickness where two sub-layers were interpreted. The shallowest one with a Vs value lower than the deepest one. These sub-layers are shaded in Table 3.

Fig. 6 includes the dispersion curve of the October 2020 CTB campaign and the interpreted layer model. Table 4 lists the layers defined. Again, two sub-layers were obtained in the ballast with a low-velocity layer first (shaded in Table 4).

In both campaigns conducted at the CTB, similar results were obtained. A first sub-layer with a Vs of 110 m/s is followed by another with a greater thickness and a Vs of 220 m/s. That is, a first sub-layer of looser ballast followed by another more compacted one. The thicknesses obtained for both sub-layers are slightly different in each campaign. The thickness of the looser ballast sub-layer in the 2020 campaign is greater than in the 2019 campaign. This may indicate a more altered ballast after almost a year of track testing in the CTB.

Then, the third layer (defined as a half-space) corresponds to the sub-ballast layer.



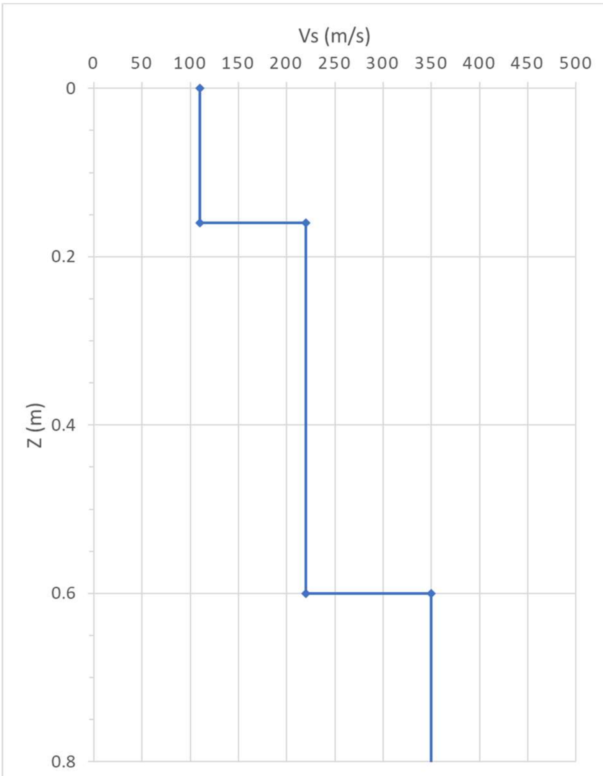


Figure 5. Dispersion curve obtained in the November 2019 CTB campaign (left graph) and the corresponding layer model defined (right graph).

Table 3. Values of the layer model of the November 2019 CTB campaign. The sub-layers of the ballast layer have been shaded.

Layer	Thickness (m)	Vs (m/s)
1	0.16	110
2	0.44	220
3	∞ (half-space)	350

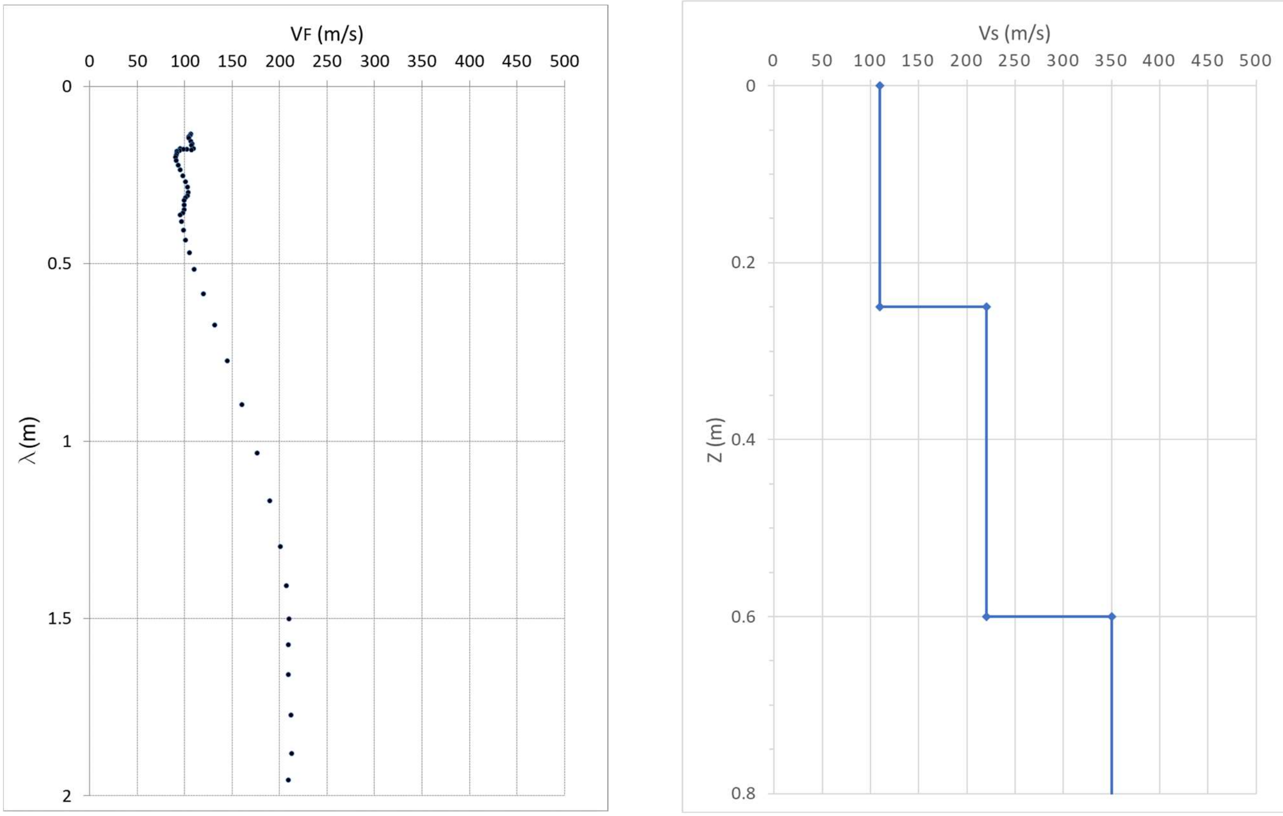


Figure 6. Dispersion curve obtained in the October 2020 CTB campaign (left graph) and the corresponding layer model defined (right graph).

Table 4. Values of the layer model of the October 2020 CTB campaign. The sub-layers of the ballast layer have been shaded.

Layer	Thickness (m)	Vs (m/s)
1	0.25	110
2	0.35	220
3	∞ (half-space)	350

6. Field campaign on an in-service track

In May 2022, a series of SASW tests were carried out to determine the total dispersion curve of a railway section at a point on the Madrid-Barcelona high-speed line. This work presents only the results obtained for the ballast layer. Measurements were conducted by placing accelerometers on the surface of the sleepers and striking a small metal rod directly onto some ballast particles or by striking directly on the sleepers near the sensors. The measurements were taken with sensor spacings of 0.60 m and 1.20 m, i.e., the distance between one or two sleepers (see Fig. 7).

6.1. Results obtained in the field campaign

Various points were tested, but only in three of them the measurements allowed a confident interpretation. For the wavelengths analyzed in these tests, in addition to the ballast layer, it was also possible to characterize deeper layers (Tijera et al. 2024). Fig. 8 presents the dispersion curves interpreted at each point (P3, P4 and P5) and the associated average dispersion curve. Fig. 9 presents the graph of the layer models defined at each point and the average layer model.

Figure 7. Photo during the test. Accelerometers placed on two adjacent sleepers (0.60 m) and the hammer used as a source are observed.

Focusing only on the ballast layer, which is the subject of the present study, we observe that a layer with a total thickness of 0.60 m was interpreted. It was subdivided into two sub-layers, one with a thickness of 0.14 m and Vs equal to 115 m/s; and another of 0.46 m and Vs of 225 m/s.

Then, the third layer, represented as a half-space, would be the sub-ballast layer.

In Table 5, the numerical values of thickness and Vs velocity for the layer model are included. The layers defining the ballast layer have been shaded.

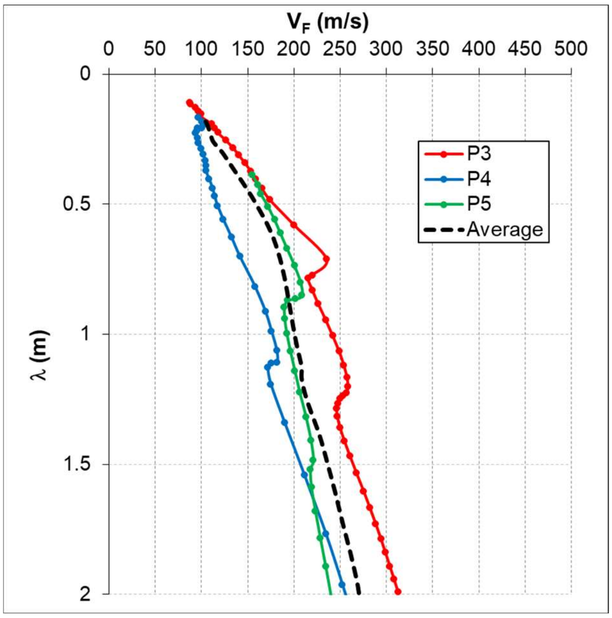


Figure 8. Dispersion curves obtained in the three points interpreted (P3, P4 and P5) and the associated average dispersion curve.

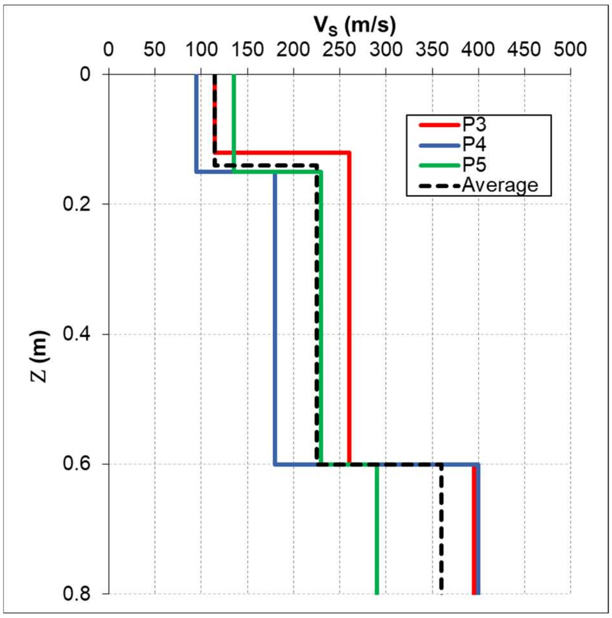


Figure 9. Layer models defined at each point and the average layer model.

Table 5. Layer model obtained for the ballast and beginning of the sub-ballast in the field campaign. The layers defining the ballast have been shaded.

Layer	Thickness (m)	Vs (m/s)
1	0.15	115
2	0.45	225
3	∞ (half-space)	360

7. Conclusions

The main objective of the work described in this article was to obtain a range of shear wave velocity (Vs) values for the ballast layer of the train tracks to complete the experimental values already available for other layers. Based on the presented results, it can be concluded that the objective has been achieved, and we have obtained a Vs value for the ballast based on experimental field measurements conducted by the Laboratorio de Geotecnia-CEDEX. This value, along with the Vs values already available for the other layers of the railway section, is crucial in defining railway section models used in computer tools for studying track behavior.

To achieve this, the geophysical technique based on surface waves SASW (Spectral Analysis of Superficial Waves) has been employed. The application of this technique, widely used in the Laboratorio de Geotecnia-CEDEX, had to be fine-tuned for the study of ballast due to the unique characteristics of this material and the configuration of the track.

The results obtained from three campaigns have been presented, two conducted in the Cedex Track Box (CTB) and one on an in-service track section. The results obtained are very similar. In all three campaigns, the same structure of the ballast layer was identified, consisting of a first sub-layer with a low Vs value followed by another with a higher Vs value with a total thickness of 0.60 m. The first low-velocity sub-layer corresponds to the upper part of the ballast, located above the base of the sleepers, so it seems logical that it is looser and less confined than the part of the ballast layer that is situated below the base of the sleepers. The thicknesses of these sub-layers vary among different models, although in all of them, the superficial low-velocity layer has a slightly smaller thickness than the high-velocity layer. A summary of the velocities and thicknesses obtained is shown in Table 6.

If we compare the values of Vs with those obtained by other authors, we see that in cases where the same geophysical technique has been used, the values are of the same order as obtained for the compacted ballast sub-layer (Hwang and Park 2014, Stark et al. 2018). In Table 6 these Vs values obtained by other authors with SASW are included. In other cases, the values obtained in our campaigns are somewhat higher, but it should be noted that the ballast characterized in these publications was under special conditions or different geophysical techniques were used (Anbazhagan et al. 2010, Anbazhagan et al. 2011, Hong et al. 2017). Regarding the comparison with values estimated in numerical models found in the literature, it is confirmed that the values obtained for the high-velocity layer are within the range considered in these publications (Alves Costa et al. 2015, Mezher et al. 2016, Madshus and Kaynia 2000, 2001, Sayeed and Shahin 2016, 2018).

In future works, the SASW technique aims to be applied again in other track locations to complete these results, and there is also an interest in developing laboratory tests to determine Vs values of the ballast under various confinement conditions and dynamic loading.

Table 6. Summary of the Vs velocities and thicknesses of the sub-layers obtained for the ballast layer in the three campaigns, and the Vs values obtained by other authors in field campaigns with SASW.

CTB November 2019		
Layer	**Thickness (m)**	**Vs (m/s)**
1	0.16	110
2	0.44	220
CTB October 2020		
Layer	**Thickness (m)**	**Vs (m/s)**
1	0.25	110
2	0.35	220
Field campaign (2022)		
Layer	**Thickness (m)**	**Vs (m/s)**
1	0.15	115
2	0.45	225
Literature review (SASW field tests)		
Reference	**Comment**	**Vs (m/s)**
Hwang and Park (2014)	Depth > 0.3 m from the surface	200 – 220
Stark et al. (2018)	In different track sections	140 - 290

Acknowledgements

The field data on which this work is based were obtained under an R+D+i agreement between CEDEX and ADIF (Spanish railway infrastructure administrator). The authors would like to thank all the ADIF staff who have collaborated on this work.

The authors would also like to thank the colleagues of Laboratorio de Geotecnia-CEDEX Inés Crespo Chacón, who conducted the literature review of which a summary was included in this article; and Fernando Pesquera for his collaboration in CTB measurement campaigns.

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