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NUMERICAL METHOD EMPLOYING PRECONDITIONED ARTIFICIAL DISSIPATION FOR GAS-LIQUID TWO-PHASE FLOW

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Summary. A stable, time-consistent and high-resolution numerical method for unsteady gas-liquid multiphase flow problems was presented. In this method, the artificial dissipation terms in the upwinding process were reconstructed using a preconditioning matrix to improve the stability and accuracy in complicated multiphase flow computation. A homogeneous gas-liquid mixture flow model, and a second-order Runge-Kutta method as well as a Roe-type flux splitting method coupled with a 3rd-order MUSCL TVD scheme were employed. The presented method is validated using unsteady gas-liquid multiphase flows in a shock tube at arbitrary void fractions, and a very low Mach number flow in a backward-facing step channel. The numerical results show a good agreement with the exact solutions and experimental data. The presented method predicted well the steady and unsteady flows and its effectiveness and stability are also confirmed.

1 INTRODUCTION

Gas-liquid multiphase flows are widely encountered in engineering problems. Accurate prediction and evaluation of such flows is therefore essential for the design of efficient hydraulic machines and devices. However, like cavitating flows, they typically have very complex unsteady flow characteristics, phase changes, turbulence, wide range of sound speeds and local Mach numbers. Since the transient unsteady flow phenomena of such flows are extremely complex, appropriate mathematical descriptions and numerical methods have not yet been established. Therefore, many researchers have devoted great efforts to developing numerical models and analytic methods to accurately predict the mechanism and physical characteristics of multiphase flows [1,2]. In developing numerical methods for solving typical compressible and incompressible flow problems in multiphase flows, artificial compressibility methods and preconditioning methods have been developed [3,4]. Lately, Shin et al. [5]

proposed a homogeneous gas-liquid two-phase flow model. With this model, several cavitating flow problems were solved to investigate and clarify the cavitation phenomena. This method was further developed as a preconditioned dual time-step method to accurately treat unsteady cavitating flows with a wide range of sound speeds and Mach numbers [6].

The purpose of this paper is to improve the previous high-resolution method using a third-order MUSCL TVD scheme [6] into a stable and time-consistent numerical method for solving unsteady gas-liquid two-phase flows. For this, the artificial dissipation terms in the upwind scheme are modified using a preconditioning matrix to provide a stable and accurate treatment of the gas-liquid interface. As numerical examples, unsteady gas-liquid two-phase shock tube flows with arbitrary void fractions and a backward-facing step channel flow with a very low Mach number are computed, and the effectiveness and stability of the proposed method for steady and unsteady gas-liquid multiphase problems are investigated.

2 NUMERICAL METHODS

In this study, the fundamental equations for two-dimensional gas-liquid two-phase flow are the compressible Navier-Stokes equations that express mixture mass, momentum, energy, and gas-phase mass conservation in a curvilinear coordinate system, written as follows:

$$\Gamma^{-1}\frac{\partial \boldsymbol{W}}{\partial t}+\frac{\partial(\boldsymbol{E}_i-\boldsymbol{E}_{\mathrm{v}i})}{\partial \xi_i}=0 \quad \text{with} \quad \boldsymbol{W}=\begin{bmatrix}p\\u\\v\\T\\Y\end{bmatrix},\ \boldsymbol{Q}=J\begin{bmatrix}\rho\\\rho u\\\rho v\\e\\\rho Y\end{bmatrix},\ \Gamma^{-1}=\frac{\partial \boldsymbol{Q}}{\partial \boldsymbol{W}},$$

$$\boldsymbol{E}_i=J\begin{bmatrix}\rho U_i\\\rho u U_i+\partial\xi_i/\partial x\, p\\\rho v U_i+\partial\xi_i/\partial y\, p\\\rho H U_i\\\rho Y U_i\end{bmatrix},\ \text{and}\ \boldsymbol{E}_{\mathrm{v}i}=J\begin{bmatrix}0\\\partial\xi_i/\partial x_j\,\tau_{xj}\\\partial\xi_i/\partial x_j\,\tau_{yj}\\\partial\xi_i/\partial x_j\,u_k\tau_{kj}+\kappa\,\partial T/\partial x_j\\0\end{bmatrix}\quad (i,j,k=1,2)\,. \tag{1}$$

where, p, T, Y and e denote the pressure, temperature, quality of vapor and total energy, respectively. u_i and U_i present physical and contravariant velocity components. τ_{ij} and κ are the stress tensor and the coefficient of thermal conductivity. H is the enthalpy defined by total energy $e=\rho H-p$. J is the Jacobian for the transformation from Cartesian coordinates x_i to curvilinear coordinates ξ_i, and is defined as $J=x_\xi y_\eta - x_\eta y_\xi$. In this study, we solve Eq. (1) using the finite-difference discretization technique with a compressible flow solver.

Gas-liquid multiphase flows contain both compressible and incompressible flow characteristics. To compute such flows, a unified solution method is required that can simultaneously solve compressible and incompressible flows. From a point of view of computation, it is advantageous to modify compressible flow solvers to be able to handle incompressible flows, such as artificial compressibility methods and preconditioning methods [7,8]. In general, these methods have been developed for steady flow problems. In order to compute unsteady flow problems, these methods must be improved into a time-accurate unsteady flow solver. Therefore, in this study, we present a modification of the preconditioning method to solve unsteady problems while maintaining time consistency. That is, the artificial dissipation term in the flux-difference splitting [9] is modified using the preconditioning matrix to increase the numerical stability, whereas the temporal term is treated without the preconditioning matrix to keep the time consistency.

To achieve this, Eq. (1) is modified to preconditioned equations using the preconditioning matrix Γ_p^{-1} rather than the transform matrix Γ^{-1} [6]. In this study, Γ_p^{-1} is obtained by adding the vector $\theta[1, u, v, H, Y]^T$ to the first column of the Γ^{-1}. The value of θ is chosen by Weiss & Smith [10] and Edwards & Liou [11] to be able to handle both compressible and incompressible flows. On the other hand, when using Roe's approximate Riemann solver to enhance the stability during the treatment of gas-liquid interfaces in two-phase media, the flux Jacobian matrix $\boldsymbol{A}_i (= \partial \boldsymbol{E}_i / \partial \boldsymbol{Q})$ of artificial dissipation terms is modified using the preconditioning matrix as $\Gamma_p^{-1} \widetilde{\boldsymbol{A}_\iota} \Gamma$. Here, $\widetilde{\boldsymbol{A}_\iota}$ is a preconditioned flux Jacobian matrix in system $\boldsymbol{W}$ and composed by $\Gamma_p \boldsymbol{A}_i \Gamma^{-1}$. The derivative of flux vectors $\boldsymbol{E}$ of ξ-momentum of $\boldsymbol{E}_i$ in Eq. (1), as an example, is discretized as $(\partial \boldsymbol{E}/\partial \xi)_l = (\boldsymbol{E}_{l+1/2} - \boldsymbol{E}_{l-1/2})/\Delta\xi$, and preconditioned numerical fluxes $\boldsymbol{E}_{l\pm1/2}$ are eventually derived as seen in the following equation.

$$\boldsymbol{E}_{l\pm1/2} = (1/2)\left[\boldsymbol{E}(\boldsymbol{Q}_{l\pm1/2}^L) + \boldsymbol{E}(\boldsymbol{Q}_{l\pm1/2}^R) - (\Gamma_p^{-1}|\widetilde{\boldsymbol{A}}|\Gamma)_{l\pm1/2}(\boldsymbol{Q}_{l\pm1/2}^R - \boldsymbol{Q}_{l\pm1/2}^L)\right] \tag{2}$$

The preconditioned numerical fluxes of η-momentum of $\boldsymbol{E}_i$ can be obtained in the same manner. In the above equation, the $\boldsymbol{Q}_{l\pm1/2}^{L,R}$ are transformed to $\boldsymbol{W}_{l\pm1/2}^{L,R}$, and calculated by applying the 3rd-order MUSCL TVD scheme. For the time integration, a second-order Runge-Kutta method with finite-difference discretization is used. In this way, we can obtain a high-resolution and time-consistent preconditioning method for solving unsteady gas-liquid multiphase flow problems.

3 NUMERICAL RESULTS

3.1 Shock tube flow

The proposed numerical method has been applied in one-dimensional gas-liquid multiphase shock tube problems and evaluated. The computational domain of the shock tube is set as $x =$ [-10 m, 10 m]. Initial conditions are imposed on the left (L) and right (R) sides separated by a diaphragm at $x = 0$ m. In this paper, two test cases are examined as: Case 1 is a classical Riemann problem that was suggested by Sod [12] with pressure of p_L =100 kPa and p_R =10 kPa, while Case 2 is a challenging problem with a large pressure ratio of p_L =100 kPa and p_R =1 kPa. In these two cases, the temperature, velocity, and void fraction are set to $T = 300$ K, $u = 0$ m/s and $\alpha = \alpha_i$, and the initial density ρ of the mixture is given by the equation of state of the gas-liquid multiphase flows.

A comparison of computational results with the exact solutions of Case 1 for ideal gas ($\alpha_i =$ 100%) at $t = 0.01$ s are shown in Fig.1. It can be seen that the results computed by the proposed method using 10,000 grid points (red line) overlap with the exact solutions (black line). The results using a coarse grid of 100 points (symbols) also predicted unsteady shock tube flows fairly well. Moreover, the applicability and stability of the proposed method were examined in two-phase shock tube flow problems. Figure 2 shows computational results of pressure and velocity distributions, and its iteration histories of $|p^{n+1} - p^n|_{max}$ and $|u^{n+1} - u^n|_{max}$ for Case 2 with initial void fractions of $\alpha_i = 10\%$ at time $t = 0.18$ s. Because no exact solutions are available in this two-phase flow case, the results computed by 1st-order upwinding with a very fine grid of 100,000 grid points were referenced as exact solutions (black solid line). As seen in

Fig.2, it can be noticed that the computation result without preconditioning (red solid line) shows serious oscillations near discontinuities in pressure and velocity distributions, whereas the proposed preconditioned ones (symbols) show a significant improvement. From the residual histories, it can be seen that the stability and convergence rate of the computation are greatly improved when applying the preconditioned artificial dissipation term. However, in the problem with a very large pressure ratio, the proposed method still has some oscillations at discontinuities, leaving room for improvements such as tuning the weight parameters of preconditioning method.

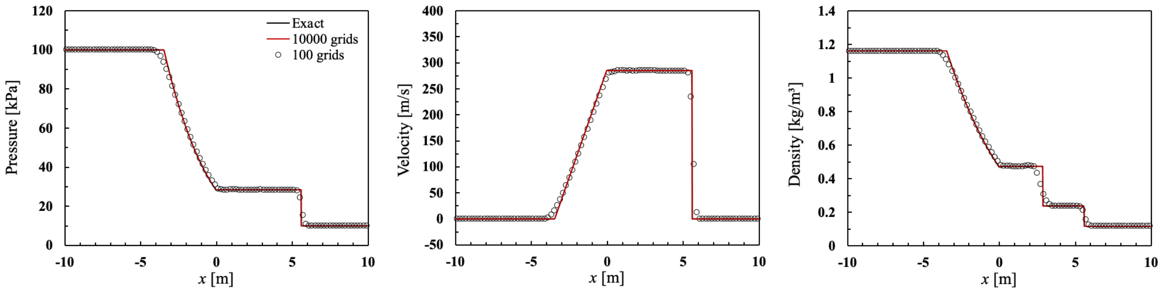


Figure 1: Computational results of pressure, velocity and density distributions for a gas phase shock tube flow with $\alpha_i = 100\%$ (Case 1)

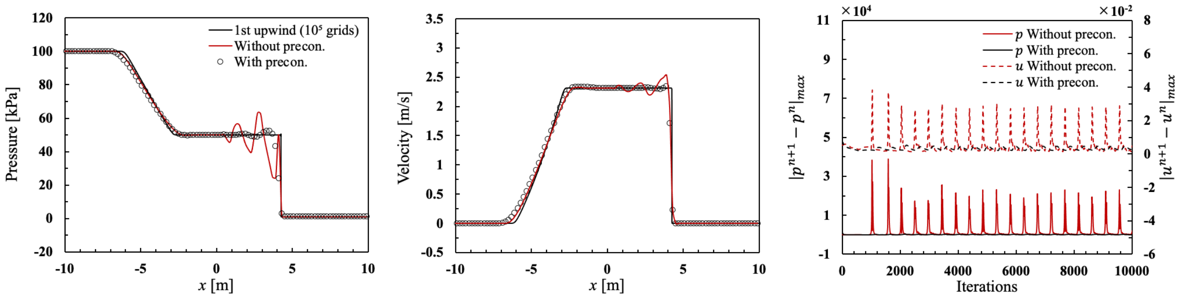
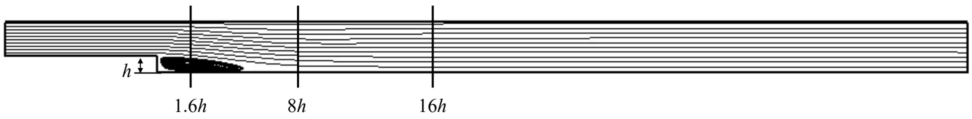


Figure 2: Computational results of pressure, velocity distributions and iteration histories for a gas-liquid two-phase shock tube flow with $\alpha_i = 10\%$ (Case 2)

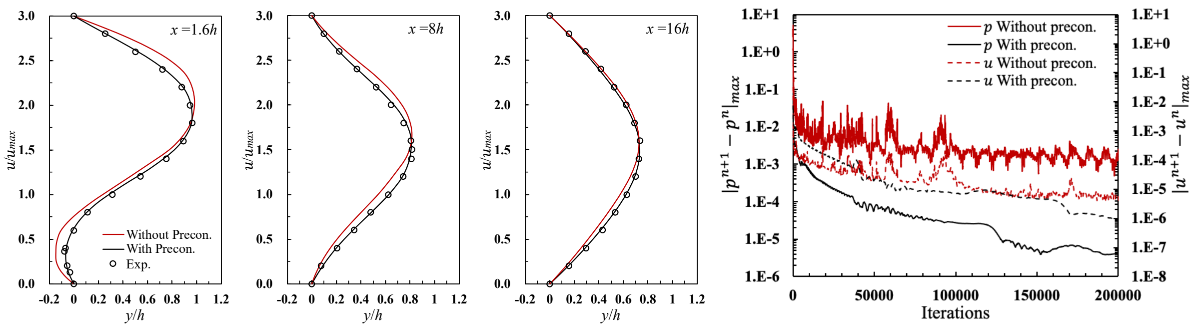
3.2 Backward-facing step channel flow

The proposed method is also validated in a two-dimensional backward-facing step channel flow with gas phase at a low Mach number. The computational domain is a one-side sudden expansion channel with a backward facing step height of h, an inlet width of $d = 2h$ and an expended channel length of $48h$. A body-fitted curvilinear coordinate grid system clustered near the walls was generated with 90×21 grid points. The boundary conditions are set as a Poiseuille flow profile on the inlet, a Dirichlet condition of pressure on the outlet, and no-slip conditions on solid walls. The Reynolds number Re is defined by using the channel inlet width d and the inlet maximum velocity. As a numerical example, a laminar flow with the Reynolds number of Re =300 and inlet maximum Mach number of $Ma = 0.01$ is examined.

The computational results of this step channel flow are shown in Fig.3. As can be observed in Fig.3(a) of the streamlines, a flow recirculates behind the step, and the reattachment occurs on the lower wall of the expanded side. There are three cross sections marked in Fig.3(a), and the velocity profiles at these sections are plotted in Fig.3(b). In each profile of this figure, the streamwise velocity profiles obtained by the non-dimensional time step of 4.2×10^{-4} with preconditioned artificial dissipation terms (black line), and by the time step of 4.2×10^{-5} without preconditioning (red line), as well as the experimental data (symbols) given by Kueny et al. [13] are plotted for a comparison. From these profiles, it can be seen that proposed preconditioned methods are successful in simulating a very low Mach number flow with the separation and recirculation, and the results agree quite well with the experimental data. However, when the same time step was used with and without preconditioning, the no preconditioning one diverged and failed to compute. Even when using a time step as small as 1/10, the results without preconditioning one show a large discrepancy with the experimental values as seen in Fig.3(b). Also, the convergence histories of p (solid line) and u (dotted line) show that the convergence rate and stability with preconditioned stability term are better than the no preconditioning one.



(a) Streamline and cross sections for monitor



(b) Streamwise velocity profiles and convergence histories

Figure 3: Computational results of a backward-facing step channel gas phase flow at $Re = 300$ and $Ma = 0.01$

4 CONCLUSIONS

A stable, time-consistent and high-resolution numerical method for unsteady gas-liquid two-phase flow was presented and applied to one-dimensional shock tube flow and two-dimensional backward-facing step channel flow problems. In this method, the artificial dissipation term in flux difference splitting of upwind scheme was derived using a preconditioning matrix, and finite-difference with a second-order Runge-Kutta method and the 3rd-order MUSCL TVD scheme were used. The computational results obtained by the presented method with preconditioned artificial dissipation terms showed a good simulation for unsteady shock tube flow with a very large pressure ratio, and agreed very well with the exact solutions. In the case of two-dimensional laminar flow computation, the presented preconditioning method successfully calculated a very low Mach number flow with $Ma = 0.01$, and the predicted results agree well with the experimental data. It was also confirmed that the preconditioned stability term can significantly improve the convergence rate and numerical stability for steady and unsteady flow computations compared to those without preconditioning. The applicability and effectiveness of the present method to unsteady two-phase flows with arbitrary void fraction and Mach number were confirmed.

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