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ON THE NONLINEAR BEHAVIOR OF MASONRY CLOISTER VAULTS: A CASE STUDY IN PIACENZA, EMILIA ROMAGNA, ITALY

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Summary. A simple and rapid approach to analyse the failure hazard of historical masonry vaults is presented, using a paradigmatic case study from a massive urban aggregate in Piacenza (Emilia Romagna, Italy). The curved structure under examination is a cloister vault with a considerable span, over which a wall was constructed in the key position. The urgency of a stability assessment to prevent failure is evident. The geometrical characteristics and some material information were obtained through in situ surveys. To assess its stability and determine the risk of failure a 2D finite element model is proposed, in which the bricks are discretized using 4-noded elastic elements in plane stress, and the mortar joints are characterized by (i) orthotropic shell elements coupled with (ii) ductile cutoff bars. The backfill is considered in the model by using a system of equivalent forces reproducing both soil self-weight and horizontal earth pressure. Additionally, a possible retrofitting intervention using Carbon Fiber Reinforced Polymer (CFRP) is tested and it is modelled with elastic perfectly ductile cutoff bars placed at the intrados of the vault and connected to each node of the mesh, ensuring a perfect bond between substrate and reinforcement. After a thorough analysis of the results, including force-displacement curves and a detailed examination of the failure mechanisms, an assessment of the cloister vault is provided.

1 INTRODUCTION

The Italian historical architectural heritage is characterized by masonry arches and vaults, which can encounter challenges under gravitational forces. These structural features define numerous ancient buildings, including castles, palaces, and churches. According to ICOMOS guidelines, a thorough examination is essential to evaluate the stability of arches and vaults to prevent potential failures. Achieving this understanding necessitates a combination of diverse methodologies, such as historical research, non-destructive testing, and sophisticated structural analyses [1]. The latter are crucial for investigating how curved structures behave when subjected to excessive gravitational forces or concentrated loads. While these arches and vaults are engineered to support vertical loads through their form, they may occasionally face stability issues.

Following Heyman's groundbreaking studies on arches [2], limit analysis swiftly emerged as the benchmark for accurately estimating the collapse load of brick-made curved structures

[3-8]. The strength of limit analysis lies in its proficiency in simulating common failure modes associated with such structures, characterized by the formation of flexural hinges in which cracks develop according to a defined flow rule. Moreover, shear forces are generally not a fear, compressive failure is rare, and actual deformation before collapse is minimal—all attributes that align with the classic limit analysis theorems, whether static or kinematic. Nevertheless, limit analysis has its shortcomings; it fails to consider the propagation of damage, post-peak softening behaviour, predictions of displacement, or the structural condition before failure, which complicates efforts to prevent masonry arches and vaults from failing.

Consequently, the nonlinear finite element (FE) method is deemed the primary tool for a broad spectrum of nonelastic analyses, including both static and dynamic ones [9]. Various methodologies recorded in the literature include simplified micro-modelling, which overlooks mortar thickness and regards it as an interface between neighbouring blocks [10], and macro-modelling, in which masonry is treated as a fictitious homogeneous material [11] with assigned mechanical properties derived through different approaches. Homogenization can be accomplished using several methods, including FE-based strategies [12] and semi-analytical techniques [13].

The accuracy of the analyses performed in whatever FE software is significantly enhanced through heterogeneous approaches, which model masonry's composite nature by using distinct mortar and brick elements [14]. Since masonry consists of bricks (or stones) and mortar, it is ideal to represent each material component individually. Currently, this is predominantly conducted through complex and costly software, such as Abaqus, making this methodology less accessible to practitioners and designers [15].

To address the considerable limitations associated with heterogeneous methods, this paper introduces a comprehensive practical methodology for analysing the structures of arches and vaults. This approach consists of the following phases: (i) historical and critical study: begin with a thorough historical and critical understanding of the case study through literature or archival research [1]; (ii) non-destructive in-situ testing: conduct non-destructive tests in situ to achieve at least a level of knowledge LC1, following the Italian regulation [16], which help to determine the quality of the infill and the geometric characteristics of the curved structure under examination; (iii) nonlinear finite element analysis: perform nonlinear finite element analyses using straightforward methodologies, such as the one proposed by two of the authors in [17] and [18], where nonlinearities are concentrated in the mortar joints and the bricks are discretized using 4-noded elastic elements. These joints are modelled with the simplest finite element, i.e. cutoff bars, which are truss elements with predefined tensile and compressive strength limits. These can exhibit either elastic perfectly brittle or elastic perfectly ductile behaviour. Additionally, in the absence of precise information regarding the mechanical parameters of the masonry and the backfill, it is recommended to conduct a series of sensitivity analyses, including varying the coefficient of earth pressure [19]; (iv) consolidation interventions: possible retrofitting improvements are encouraged for example introducing intrados and extrados reinforcements employing composite materials such as FRP (Fiber Reinforced Polymer) or FRCM (Fiber Reinforced Cementitious Matrix), equipped with anchoring elements (such as anchor spikes) to prevent debonding. Simplified modelling approaches can also be employed for reinforcement, utilizing cutoff bars. These trusses can either perfectly overlap with the substrate mesh nodes guaranteeing a perfect bond between substrate and reinforcement [18] or be placed in parallel to simulate the nonlinear debonding phenomena at the interface between the strengthening system and the support [20, 21, 17]. It has been demonstrated in [20] that two elastic perfectly brittle cutoff bars in parallel are necessary to simulate the debonding of an FRP composite material from the substrate, while three trusses can simulate the detachment of an FRCM from the support [21]. For the latter, two cutoff bars are assumed to be elastic perfectly fragile, while the third one exhibits ductile behaviour after the elastic phase to account for the residual tensile strength typical of FRCMs. The advantage of using such a method is the fact that the approaches, either that referred to FRP or that related to FRCM, are directly implemented in commercial FE software.

The aforementioned approach is applied to a real case study by referring to a cloister vault, characterized by a big span, placed in the ex-monastery of Santa Maria della Pace in Piacenza (Emilia Romagna, Italy). The building was hit by the Emilia Romagna earthquake in 2012 [22] and its vulnerability has already been studied by [23]. However, the most critical part from a static point of view is the vault object of investigation. The curved structure is a paradigmatic case study since it is loaded by a wall located exactly in the middle of the vault.

2 THE CASE STUDY OF THE CLOISTER VAULT OF SANTA MARIA DELLA PACE AND IN SITU SURVEYS

The case study analyzed in this paper regards a masonry cloister vault built in a massive urban aggregate located in Piacenza (Emilia-Romagna, Italy), and called Santa Maria della Pace. This building was structurally analyzed in its complexity by [23], but the most critical part from a static point of view is represented by the cloister vault present at ground level and highlighted in Figure 1. This curved masonry structure is characterized by significant spans (16 x 8 m) and it is loaded by a wall constructed on the first floor, marked in Figure 1, which is placed exactly in the middle of the vault, as represented by the section AA reported in the same figure. For further information related to the history of the building, the reader is referred to [24].

To obtain sufficient information about the geometry and the materials that characterize the vault object of study, surveys were conducted by the authors during multiple site visits. These surveys were then cross-referenced with the geometric data found in the documentation supplied by the building's owner [25]-[28]. Additionally, minor destructive tests were performed following Italian regulations, as required to achieve the LC1 level of knowledge [16]. The purpose of these tests was to gain a more thorough understanding of the identified construction characteristics, which were considered highly important for structural safety. Specifically, the study focused on the cloister vault, which was analysed through tests thanks to which was possible to obtain information about (i) the thickness of the vault at the keystone and at the haunches, (ii) the stereotomy of masonry i.e. herringbone bond texture, (iii) the type and thickness of the backfill. Here, for the sake of brevity, some details regarding in situ surveys are missed, but the reader can see [24].

In Figure 2 the resulting technical drawing of the vault is shown with some pictures related to in situ inspections.

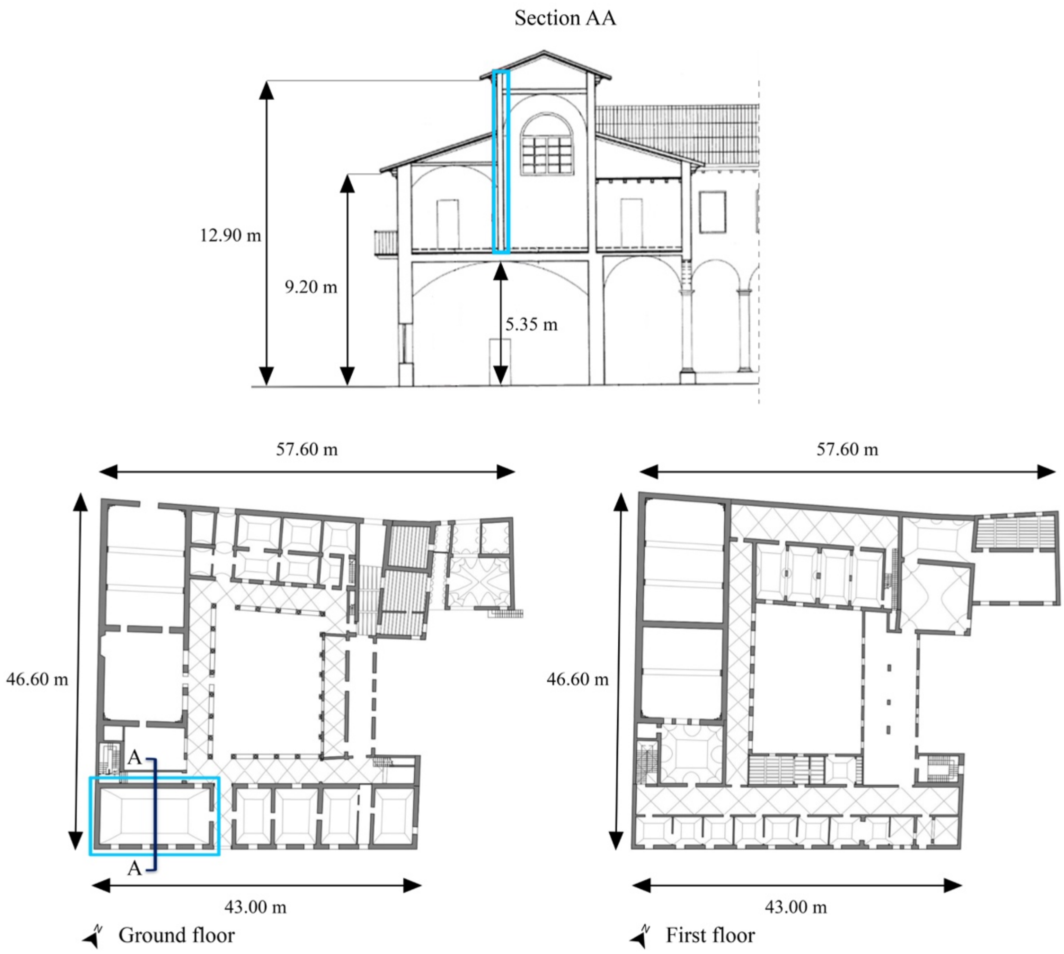

Figure 1: Cloister vault object of study. Cross section of the vault highlighting the wall positioned on the keystone of the vault on the first floor. Perspective of the vault from the ground level.

In Figure 2 some photos related to the surveys are shown, whereas in Figure 3 the technical drawing of the vault derived from the information obtained is depicted.

Figure 2: In situ surveys of the cloister vault.

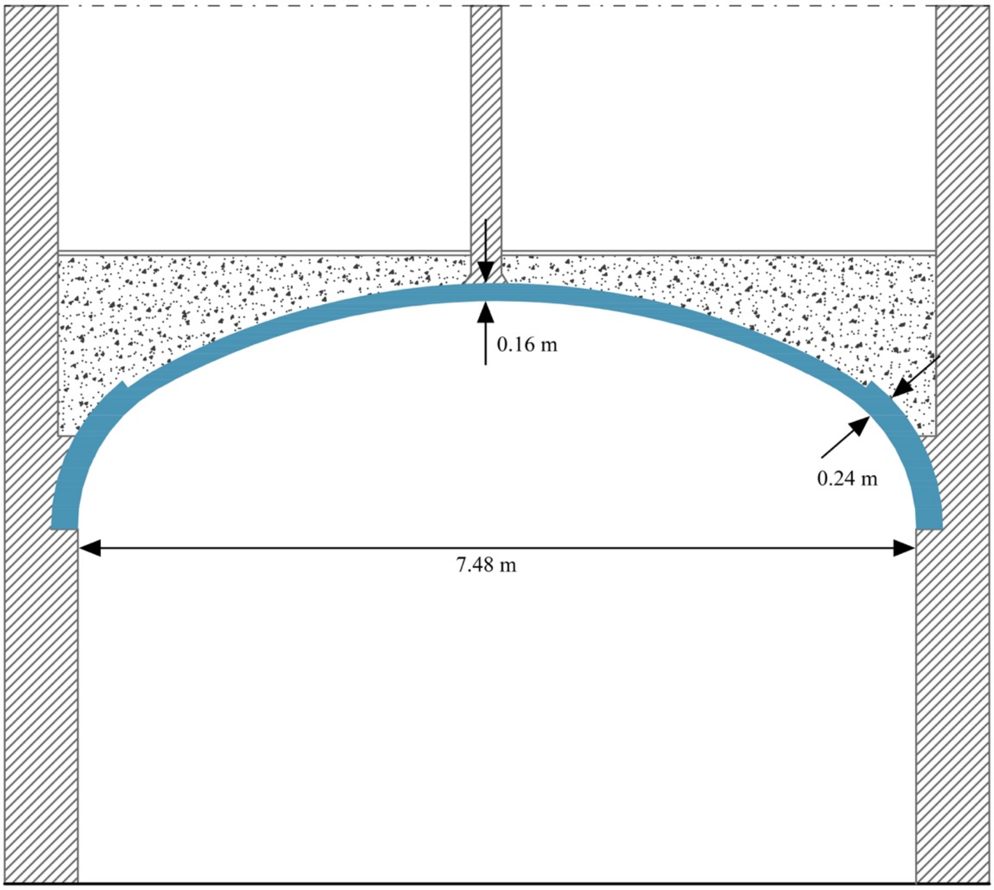

Figure 3: Cross section of the vault obtained from on-site surveys.

3 NONLINEAR FINITE ELEMENT MODEL

The finite element (FE) analyses are conducted by first applying gravity loads and subsequently imposing an increasing vertical displacement at the location of the wall placed in the middle of the vault, as illustrated in Figure 4. The nodes are constrained in the out-of-plane direction, an out-of-plane thickness of 10 cm is assumed, and finally, only half of the arch is considered because of symmetry reasons.

To efficiently model the vault in a manner suitable for designers, a commercial finite element (FE) software is used to simulate the nonlinear behaviour. This approach utilizes a methodology previously developed by two of the authors in [17], incorporating the following finite elements:

- Bricks are meshed with quadrilateral structural shell elements subjected to 2D loads (see Figure 4) for which the following parameters need to be set:
  - E_b, that is the elastic modulus of the blocks assumed equal to 6500 MPa;
  - ρ_b, that is the density of the blocks assumed equal to 1800 kg/m^3;
  - ν_b, that is the Poisson's ratio assumed equal to 0.2.
- Mortar Joints are modeled with orthotropic shell elements coupled with ductile cutoff bars, as represented in Figure 4. 4-noded elements are tuned through the following parameter:

- G_m, that is the shear modulus of the mortar assumed equal to $250\ MPa$.

Whereas the mono-dimensional elements are characterized by:

- $f_{t,m}$, that is the tensile strength of the mortar assumed equal to $0.05\ MPa$;
- $f_{c,m}$, that is the compressive strength of the mortar assumed equal to $1\ MPa$.

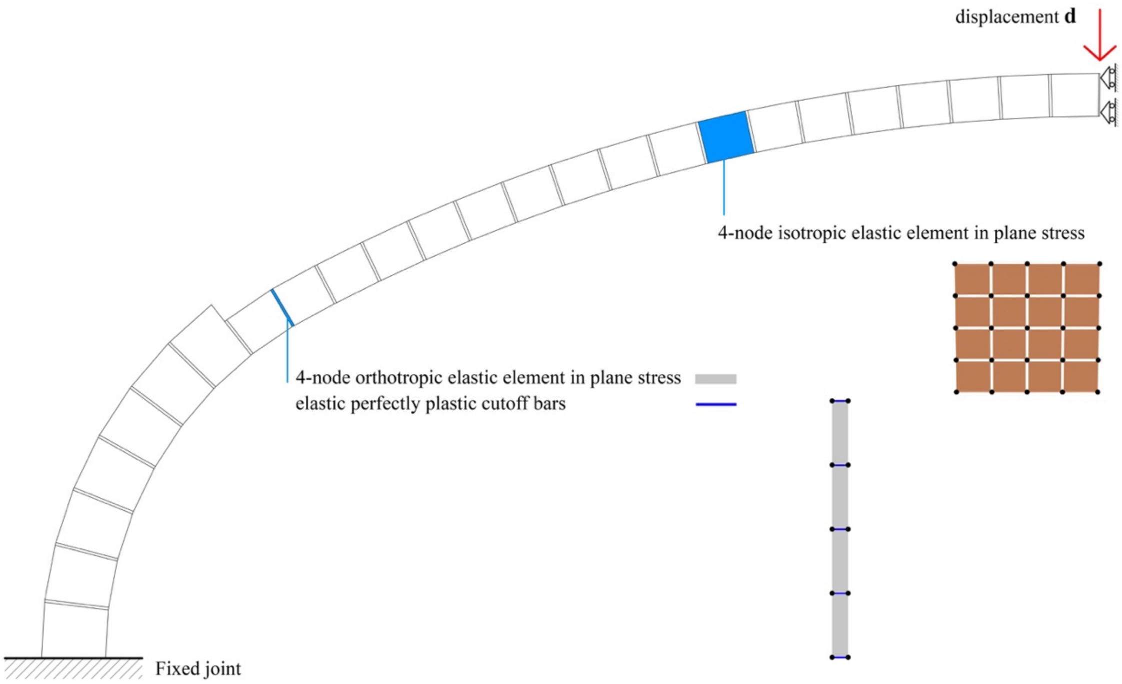

Figure 4: Mesh proposed for the FE model.

The model proposed by the author to assess the stability of the cloister vault, located in the former monastery of Santa Maria della Pace, considers not only the arch with spandrels but also the contribution provided by the backfill reproduced using equivalent forces, as shown in the zoom reported in Figure 5. To do so, classic geotechnics is involved following the procedure proposed by [19]. So, in pursuit of such an approach, the vertical component is calculated using the following formula:

$$A_f t_f \gamma_f = F_{f,v} \tag{1}$$

Where:

- A_f is the area of the backfill above each block;
- t_f is the out-of-plane thickness of the filling equal to that of the vault ($10\ cm$);
- γ_f is the specific weight of the backfill and it is assumed equal to $16\ kN/m^3$;
- $F_{f,v}$ is the vertical force applied to each block.

Whereas the horizontal component is calculated as follows:

$$\sigma_f t_f l_b = (k_i \gamma_f h_f) t_f l_b = F_{f,h} \tag{2}$$

Where:

σ_f is the horizontal pressure applied to each block;
l_b is the length of each block;

k_i is the coefficient of earth pressure and can be at rest (k_o) or passive (k_p);
h_f is the distance between each block and the top part of the backfill;
$F_{f,h}$ is the horizontal force applied to each block.

Since experimental tests that could prove the earth pressure of the filling are not at disposal, a sensitivity analysis is performed considering different coefficients of earth pressure at rest (k_o) and passive earth pressure (k_p), as illustrated in Figure 5. It is possible to notice that the higher the coefficient, the higher the ultimate capacity of the structure, so the collapse load increases from $35.8\ kN/m$ (referred to $k_o = 0.7$) to $66.0\ kN/m$ (referred to $k_p = 2.8$). Additionally, when examining the force-displacement curves shown in Figure 5, the initial rigid segment observed in all simulations is attributed to the increase in horizontal forces, which in turn raises the vertical forces to support the vault. These rigid segments are independent of the collapse loads.

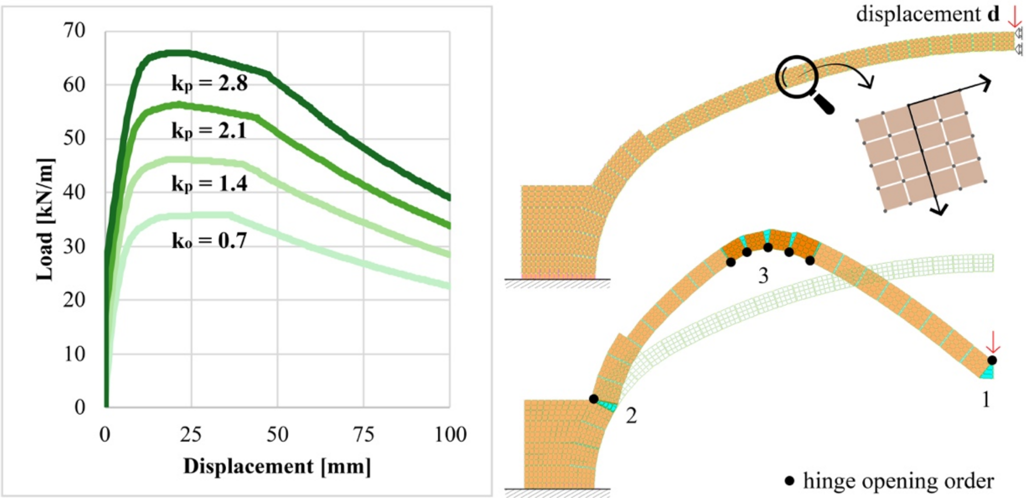

Figure 5: Force-displacement curves related to the model proposed for the unreinforced case by changing the coefficient of earth pressure and the relative deformed shape found at collapse.

3.1 CFRP reinforcement applied

The approach proposed in this paper considers a possible retrofitting intervention which relies in the application of CFRP (Carbon Fiber Reinforced Polymer) strips at the intrados of the cloister vault, as illustrated in Figure 6, for which the mechanical properties assumed are that from [29]. The reinforcement is modeled in the commercial software using elastic perfectly ductile cutoff bars, directly connected to each node of the support (see the zoom provided in Figure 6), and therefore assuming a perfect bond between the substrate and the strengthening system. In this context, a simplification has been introduced which considers only the typical tensile failure of the reinforcement [30, 18]. This approach avoids the need to account for potential debonding phenomena, which would require a more complex discretization, as discussed by two of the authors in [31, 17, 21].

Additionally, regarding the vault's out-of-plane dimension of $10\ cm$, $3\ cm$ (or 30%) are covered by CFRP, as shown in Figure 6.

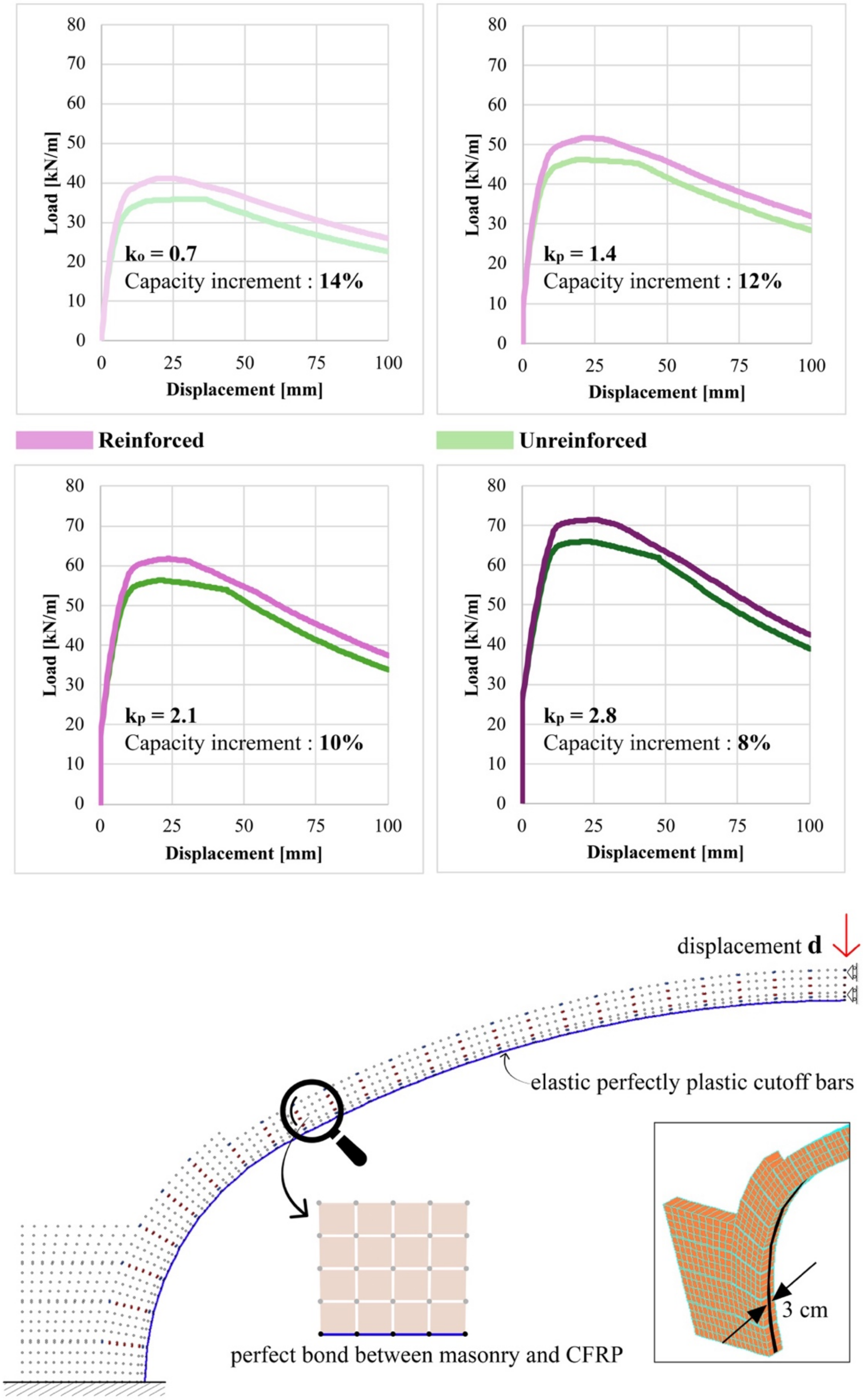

Figure 6: CFRP strips applied at the intrados of the vault discretized with cutoff bars connected to mesh's nodes and force-displacement curves related to the reinforced model compared with those of the unreinforced case.

In Figure 6 in addition, a comparison between unreinforced versus reinforced cases is provided. It is possible to notice that the lower the coefficient of earth pressure, the higher the contribution provided by the reinforcement, with the capacity increasing from 14% (corresponding to $k_o = 0.7$) to 8% (related to $k_p = 2.8$).

In conclusion, the addition of $30\ cm$-wide FRP strips per meter along the intrados, while a preferred intervention due to its minimal invasiveness and ease of application, does not appear to significantly enhance load-bearing capacity enough to recommend its usage, as indicated by the comparative results in Figure 6.

4 CONCLUSIONS

A general and simple approach to prevent the collapse of historical masonry vaults subjected to vertical loads has been presented. To achieve this goal, a protocol consisting of the following phases has been proposed: (i) historical and critical analysis, which is obtained through the study of literature and archival documentation; (ii) in-situ non-destructive tests to obtain at least a level of knowledge LC1, sufficient to gather information on the geometric characteristics of the structure and the type of infill used; (iii) nonlinear finite element analysis using a simple method suitable for any designer. This involves concentrating the nonlinearities in the mortar joints by using elastic perfectly ductile cutoff bars, while the bricks remain elastic; (iv) finally, reinforcement proposals such as the introduction of innovative composite materials positioned on the intrados or extrados of the vault, modeled with a simple strategy i.e. with truss elements. These can either be connected to each mesh's node, assuming perfect adhesion between the substrate and reinforcement, or placed in parallel concerning the interface to account for the debonding phenomenon.

This protocol has been applied in the paper to a real case study, i.e. the cloister vault in the former monastery of Santa Maria della Pace in Piacenza (Emilia Romagna, Italy). This curved structure is very particular because a $25\ cm$ thick wall was built directly on its keystone. The authors have conducted a historical and critical analysis of the structure, performed in-situ non-destructive tests obtaining a level of knowledge LC1, and implemented nonlinear finite element analyses. In particular, the model proposed has been characterized by the following characteristics: the bricks have been modeled with four-node elastic elements; the mortar joints have been discretized by coupling orthotropic shell elements with elastic perfectly ductile cutoff bars, where nonlinearities are concentrated; the analyses have been conducted under a displacement control numerical strategy, with the displacement applied at the vault keystone to simulate the overlying wall; all the nodes have been constrained in the out-of-plane direction; finally an out-of-plane thickness of the vault has been kept equal to 10 cm. Within this approach, a consolidation intervention has been proposed employing CFRP (Carbon Fiber Reinforced Polymer) placed at the intrados of the vault and modeled using ductile cutoff bars assuming a perfect bond between substrate and reinforcement.

Possible future developments of the approach include: (i) introducing trusses in the mortar joints that allow for mode 2 failure; (ii) attempting an innovative reinforcement system more compatible with masonry, such as FRCM (Fiber Reinforced Cementitious Matrix), including anchoring systems to prevent debonding phenomena; (iii) compacting (or reinforcing) the infill; (iv) extending the procedure to 3D modeling by implementing a code that automatically converts the mortar joints from brick elements to truss ones to reduce the computational burden.

REFERENCES

[1] Roca P., Cervera M., Gariup G., and Pelà L. 2010. "Structural analysis of masonry historical constructions. Classic and advanced approaches." *Arch Computat Methods Eng*, 17, 299-325.
[2] Heyman J. 1969. "The safety of masonry arches." *Int J Mech Sci*, 43, 209–24.
[3] Foraboschi P. 2014. "Resisting system and failure modes of masonry domes." *Engineering Failure Analysis*, 44, 315-337.
[4] Aita D., Bruggi M., Garavaglia E. 2024. "Collapse analysis of masonry arches and domes considering finite friction and uncertainties in compressive strength." *Engineering Failure Analysis*, 163, 108462.
[5] Galassi S. 2023. "An alternative approach for limit analysis of masonry arches on moving supports in finite small displacements." *Engineering Failure Analysis*, 145, 107004.
[6] George J., Menon A. 2022. "Kinematic approach for scour analysis of masonry arch bridges." *Engineering Failure Analysis*, 141, 106703.
[7] Milani E., Milani G., Tralli A. 2008. "Limit analysis of masonry vaults by means of curved shell Finite Elements and homogenization." *Int J Solids Struct*, 45, 5258-5288.
[8] O'Dwyer D. 1999. "Funicular analysis of masonry vaults." *Computers and Structures*, 73, 187–197.
[9] Rossi M., Calderini C., Roselli I., Mongeli M., De Canio G., Lagomarsino S. 2020. "Seismic analysis of a masonry cross vault through shaking table tests: the case study of the Dey Mosque in Algiers." *Earthquakes Struct*, 18, 57-72.
[10] Gaetani A., Bianchini N., Lourenço P.B. 2020. "Simplified micro-modelling of masonry cross vaults: stereotomy and interface issues." *Int. J. Masonry Res. Innovat*, 6 (1), 97-125.
[11] Milani G., Milani E., Tralli A. 2009. "Upper bound limit analysis model for FRP-reinforced masonry curved structures. Part II: Structural analyses." *Computers and Structures*, 87, 1534-1558.
[12] Rekik A., Gasser A. 2016. "Numerical homogenization model for effective creep properties of microcracked masonry." *Int. J. Solids Struct*, 102–103, 297–320.
[13] Bertolesi E., Milani G., Lourenço P.B. 2016. "Implementation and validation of a total displacement non-linear homogenization approach for in-plane loaded masonry." *Computers and Structures*, 176, 13–33.
[14] Calderon L., Gagliardo R., Portioli F. 2020. "LiABlock_3D: a software tool for collapse mechanism analysis of historic masonry structure." *Int. J. Architect. Herit.*, 14 (1), 75-94.
[15] Milani G., Valente M., Fagone M., Rotunno T., Alessandri C. 2019. "Advanced non-linear numerical modeling of masonry groin vaults of major historical importance: St John Hospital case study in Jerusalem." *Engineering Structures*, 194, 458-476.
[16] Ministero delle Infrastrutture e dei trasporti. 2018. Aggiornamento delle "Norme tecniche per le costruzioni", *Gazzetta ufficiale della Repubblica Italiana*, Roma.
[17] Pingaro N., Milani G. 2023. "Simple non-linear numerical modelling of masonry arches reinforced with SRG using elasto-fragile and elasto-ductile truss finite element." *Engineering Structures*, 293, 116637.
[18] Gandolfi A., Pingaro N., Milani G. 2024. "Simple nonlinear numerical modeling for unreinforced and FRP-reinforced masonry domes." *Buildings*, 14 (1), 166.
[19] Gelfi P. 2002. "Role of horizontal backfill passive pressure on the stability of masonry

vaults." *Internationale Zeitschrift für Bauinstandsetzen und Baudenkmalpflege 8*, Jahrgang, 6, 573-590.
[20] Milani G. 2022. "Simple model with in-parallel elasto-fragile trusses to characterize debonding on FRP-reinforced flat substrates." *Composite Structures*, 296, 115874.
[21] Pingaro N., Calabrese A.S., Milani G., Poggi C. 2023. "Debonding sawtooth analytical model for SRG-strengthened joints subjected to direct shear and experimentally tested." *Composite Structures*, 319, 117113.
[22] Acito M., Buzzetti M., Chesi C., Magrinelli E., Milani G. 2023. "Failures and damages of historical masonry structures induced by 2012 northern and 2016–17 central Italy seismic sequences: Critical issues and new perspectives towards seismic prevention." *Engineering Failure Analysis*, 149,107257.
[23] Acito M., Buzzetti M., Cundari G.A., Milani G. 2023. "General methodological approach for the seismic assessment of masonry aggregates." *Structures*, 57, 105177.
[24] Pingaro N., Buzzetti M., Milani G. 2024. "Advanced numerical modeling to predict historical masonry vaults failure: A paradigmatic case study.". *In press*
[25] Silva F., Restauro scientifico e cambio destinazione d'uso della chiesa di S. Maria della Pace e locali annessi posto in via Scalabrini N° 17/29, Relazione storica 1994, Piacenza.
[26] E. Strade s.r.l., Restauro dell'antica sede degli Ospizi Civili. Via G.B. Scalabrini N 19-21-23 Realizzazione di N 2 centri socio-riabilitativi residenziali e relativi servizi. Ricostruzione storica e progetto 1994 Piacenza.
[27] Ospizi Civili di Piacenza Perizia de lavori da eseguirsi nell'ex-Convento della Pace pel traslocamento delle Orfane 1820 Piacenza.
[28] Ospizi Civili di Piacenza Progetto pel Concentramento di due Orfanotrofi femminili 1856 Piacenza.
[29] Grande E., Milani G. 2008. "Modelling and analysis of FRP-strengthened masonry panels." *Engineering Structures*, 30 (7), 1842-1860.
[30] Pingaro N., Milani G. 2024. "A novel analytical model for fiber reinforced cementitious matrix FRCM coupons subjected to tensile tests." *Composite Structures*, 327, 117666.
[31] Pingaro N., Milani G. 2024. "Non-linear 1D 16-DOF finite element for Fiber Reinforced Cementitious Matrix (FRCM) strengthening systems." *Computers and Structures*, 300, 107422.