16th World Congress in Computational Mechanics (WCCM)
21-26 July 2024, Vancouver, Canada
A. Korobenko, M. Laforest, S. Prudhomme, R. Vaziri (Eds)

OPTIMAL DESIGN OF GRAPHENE-REINFORCED COMPOSITES USING SHUNTED PIEZOELECTRIC SYSTEMS FOR OPTIMAL VIBRATION ATTENUATION

MARIA-STYLIANI DARAKI[1], GEORGIOS A. DROSOPOULOS[2,3], GEORGIA A. FOUTSITZI[4] AND GEORGIOS E. STAVROULAKIS[1]

[1] School of Production Engineering and Management, Technical University of Crete
Kounoupidiana, Chania, GR-73100, Crete, Greece
mdaraki1@tuc.gr; gestavroulakis@tuc.gr, http://www.comeco.tuc.gr

[2] Discipline of Civil Engineering, University of Central Lancashire
Preston campus, PR1 2HE, UK
gdrosopoulos@uclan.ac.uk

[3] Discipline of Civil Engineering, University of KwaZulu-Natal
Durban, 4041, South Africa
drosopoulosg@ukzn.ac.za, https://secm.ukzn.ac.za/

[4] Department of Informatics and Telecommunications, University of Ioannina
Arta, GR-47100, Greece
gfoutsi@uoi.gr

Key words: Vibration suppression, Piezoelectric shunted system, Graphene nanocomposites, Optimization, Dynamic response, Finite element analysis.

Abstract. Smart structures exploit the synergy between several coupled physical phenomena to produce materials and structures with enhanced properties. Such structures incorporate integrated sensors and actuators, mechanical and electronic components and control. The design of smart structures is a multidisciplinary challenge, which is of high importance for viability and resilience of industry. Usage of piezocomposites with integrated nonlinear shunted circuits for vibration suppression improves effectiveness and accuracy of many high-value products. The finite element method is widely used to simulate the mechanical response of composite materials and multi-physics problems. In this work a numerical investigation is conducted on small scale beams aiming to improve their vibration response by applying piezoelectric shunted circuits. Finite element models are developed in MATLAB, simulating graphene-reinforced nanocomposite piezoelectric beams with shunted circuits under vibration excitations. Piezoelectric materials are applied to the beams to allow for the interaction between electric charge and mechanical deformation. In addition, shunted circuits, which are paired with piezoelectric elements, are used to provide damping (vibration suppression) of one or more critical eigenfrequencies. To derive the optimal vibration response, a particle swarm optimization (PSO) algorithm is adopted. Optimization is then aimed to minimize the vibration amplitude as well as optimize the mechanical and electrical parameters of the investigated system. Results illustrate the accuracy and reliability of the present finite element formulation for the considered frequency response analysis of the piezoelectric composite beams.

1 INTRODUCTION

Composite lightweight structures are extensively used in several applications in civil, mechanical, aerospace, marine and automotive engineering. Various components from wind turbine blades to aerospace engineering applications are manufactured using those materials. The need arises, therefore, to investigate optimal solutions that maximize their mechanical response. To address this issue several numerical methods aim to evaluate the response and failure of composite structures [1].

In several efforts a coupling is attempted between mechanical and electrical properties, aiming to control chosen parameters of those structures. Piezoelectric materials are then used to offer an electro-mechanical transformation, that converts an electric field to mechanical strain or produces an electrical charge due to mechanical actions.

One of the investigated aspects is the passive vibration control of structures subject to vibration loads [2, 3]. In those cases, shunted piezoelectric systems connected to electric circuits can be used to account for vibration suppression [4-7]. In [8] a shunted piezoelectric system is adopted to investigate vibration suppression in wind turbines. In [9] a machine learning approach is proposed to evaluate vibration suppression in composite beams.

To further enhance the mechanical properties of composites the concept of introducing a small content of nanoreinforcement is investigated in recent studies. Graphene nanoplatelets and carbon nanotubes are among the nanoreinforcement materials adopted. A finite element laminate model is used in [10] aiming to optimize the nature frequency of graphene and fibre reinforced rectangular laminates. This study is extended in [11] to skew plates. In [12] a multi-objective optimization scheme is used to evaluate the optimal vibration response and cost for hybrid graphene-fibre reinforced laminates.

A finite element model is proposed in this study, to evaluate vibration suppression within an optimization context. A graphene-fibre reinforced laminate beam is simulated and shunted piezoelectric patches are attached to the model for vibration attenuation. Design variables are the resistance R and the inductance L of the shunted system. The vibration suppression is then recorded when graphene reinforcement is used as well as when no nanoreinforcement is assigned.

2 THEORETICAL FORMULATION OF THE PROPOSED SHUNTED PIEZOELECTRIC FINITE ELEMENT MODEL

A finite element model for a cantilever laminate beam is developed. Two piezoelectric patches are attached to the top and bottom surfaces of the beam. The electrodes are connected in series to a passive electrical circuit composed of a resistor R and an inductor L. The elastic and the piezoelectric layers are thin, indicating that plane stress conditions can be considered. The details for the geometry of the beam, the layer configuration, the piezoelectric patches and the boundary conditions are shown in Figure 1.

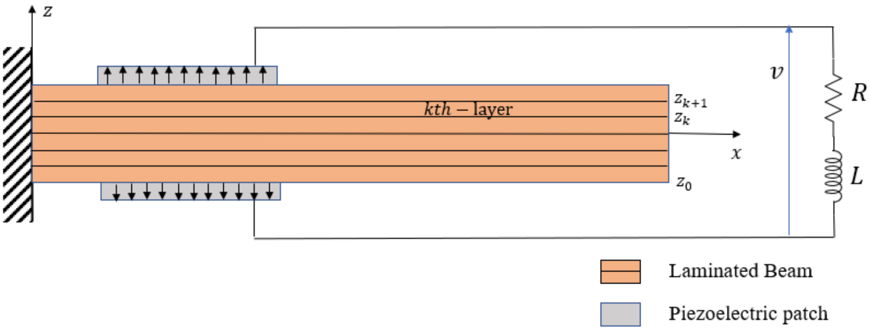

Figure 1: The cantilever beam with piezoelectric patches.

Applying Hamilton's principle and using Kirchhoff's voltage law, the coupled electromechanical equations for the piezocomposite beam under consideration are given by

$$\boldsymbol{M}\ddot{\boldsymbol{d}} + \boldsymbol{K_o}\boldsymbol{d} + C_p^{-1}(\boldsymbol{\Theta}_1 + \boldsymbol{\Theta}_2)q = \boldsymbol{F}_m \tag{1}$$

$$LC_p\ddot{q} + RC_p\dot{q} + 2q + (\boldsymbol{\Theta}_1^T + \boldsymbol{\Theta}_2^T)\boldsymbol{d} = 0 \tag{2}$$

where

$$\boldsymbol{K}_o = \boldsymbol{K}_u + C_p^{-1}(\boldsymbol{\Theta}_1\boldsymbol{\Theta}_1^T + \boldsymbol{\Theta}_2\boldsymbol{\Theta}_2^T) \tag{3}$$

In the above equations, $\boldsymbol{M}$ is the mass, $\boldsymbol{d}$ denotes the displacement vector, $\boldsymbol{K}$ is the stiffness matrix, $\boldsymbol{\Theta}_1$ (resp. $\boldsymbol{\Theta}_2$) is the electromechanical coupling matrix for the upper (resp. lower) piezoelectric patch, q is the output charge generated by each piezoelectric layer, C_p is the capacitance of the piezoelectric patches and $\boldsymbol{F}_m$ is the vector of mechanical forces. In this work, the simulation is carried out in Matlab by transforming Equations (1)-(2) into frequency domain.

3 EFFECTIVE MATERIAL PROPERTIES

The cantilever beam which is investigated in this study is a three-phase graphene and glass fibre reinforced composite laminate. To develop the corresponding finite element model, effective material properties need to be considered. First, effective material properties are calculated for the graphene reinforced matrix using the Halpin-Tsai model and the rule of mixture [10-12]. Then, the effective material properties for the graphene-fibre reinforced laminate are determined using micromechanics equations [10-12]. It is noted that the material of the piezoelectric patches is assumed to be homogeneous.

4 FORMULATION OF THE OPTIMIZATION PROBLEM

Optimal values of the shunt circuit parameters are investigated in this article resulting in maximum vibration suppression for the considered cantilever piezoelectric beam. Minimization of the maximum amplitude of the frequency response function $f(\boldsymbol{x})$ over the frequency band of interest is implemented. With $\boldsymbol{x}$ the design variables, which for this study are the resistance R and the inductance L, are denoted. Upper limits, R_U, L_U and lower limits, R_L, l_L, are also considered. Relation (1) briefly provides the mentioned optimization scheme. For the solution of this optimization problem particle swarm optimization is applied in Matlab.

$$\begin{aligned} & minimize\ f(\boldsymbol{x}) \\ st\quad & R_L \leq R \leq R_U \\ & L_L \leq L \leq L_U \end{aligned} \tag{4}$$

5 RESULTS AND DISCUSSION

An 8-layered cantilever composite beam with length 170mm, width 20mm, thickness 2mm, two piezo type PIC 151, and one passive resonant shunted circuit RL for damping the second critical eigenfrequency is adopted. The material properties of the beam are provided in Table 1.

Table 1: Material properties for the considered composite laminate.

Parameter	GPL	Matrix	Glass fibres	PZT
Young's modulus $E_{11} = E_{22} = E$ [GPa]	1010	3	72.4	66.7
Poisson's ratio, $v_{12} = v$	0.186	0.34	0.20	0.31
Shear modulus G_{12} [GPa]	$E/(2(1+v))$	$E/(2(1+v))$	$E/(2(1+v))$	25.46
Density, ρ $[Kg/m^3]$	1060	1200	2400	8500
Piezoelectric constant $\tilde{e}_{31}$ $[C/m^2]$	-			-14
Dielectric constant $\bar{\xi}$ [nF/m]	-			$2068\varepsilon_0$

$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m is the free space permittivity

Design variables are the electric parameters resistance R, and inductance L, while the graphene and fibre contents as well as the fibre angle are considered as constants. The values of those constant parameters are given in Table 2.

Table 2: Constant parameters during the optimization process.

Values of System Parameters	
Fibre volume content, V_F	30% per layer
Graphene nanoplatelets volume content, V_{GPL}	1% per layer
Fibre angles, θ	[0/90/0/90/0/90/0/90]

The optimal design variables as obtained from the optimization process are provided in Table 3. The best fitness, the amplitude reduction (dB) and the number of iterations that arise from this optimization simulation are -73.6090, 39.8763 and 26~150, respectively.

Table 3: Optimal design variables from optimization with non-zero graphene reinforcement.

Optimal design variables	
Resistance, R	10265.8 Ohm
Inductance, L	30 H

According to the frequency response diagram shown in Figure 2, vibration attenuation is achieved from this process.

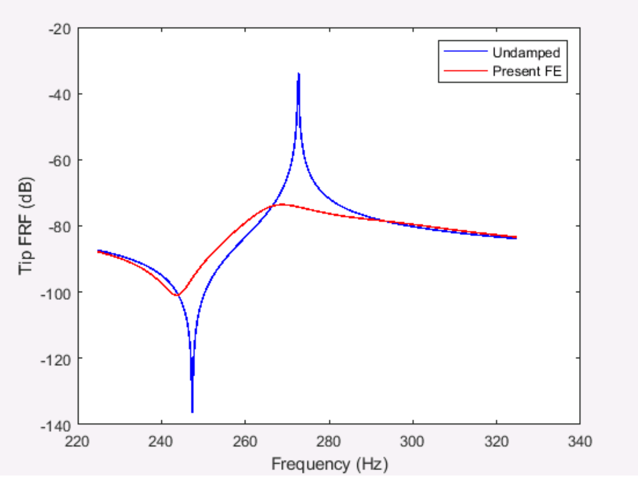

Figure 2: Frequency Response Diagram around the second eigenfrequency for non-zero graphene reinforcement.

To evaluate the influence of graphene reinforcement on vibration suppression, a second optimization simulation is conducted, considering zero graphene content, while the remaining parameters, namely, the fibre content and stacking sequence, are the same as in the previous simulation. The optimal design variables in this case are shown in Table 4.

Table 4: Optimal design variables from optimization with zero graphene reinforcement.

Optimal design variables	
Resistance, R	30000 Ohm
Inductance, L	30 H

The corresponding frequency response diagram is provided in Figure 3. In addition, the best fitness, the amplitude reduction (dB) and the number of iterations are for this case equal to −57.5271, 35.5699, and 21~150, respectively.

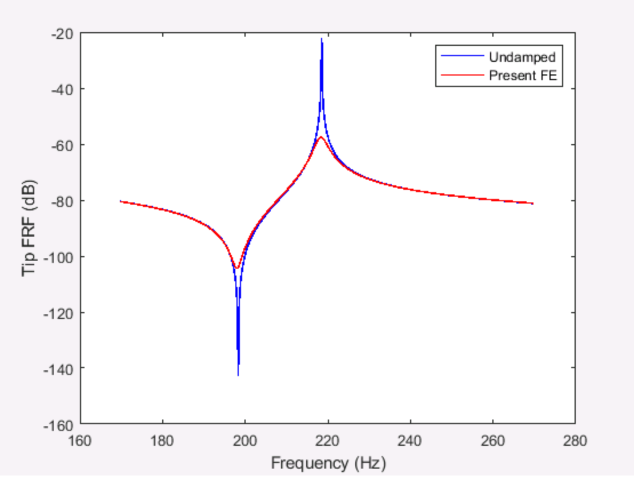

Figure 3: Frequency Response Diagram around the second eigenfrequency for zero graphene reinforcement.

A comparison between Figures 2 and 3 indicates that a significant improvement in vibration attenuation is achieved as is shown in Figure 2, where non-zero graphene reinforcement is assigned. Those results highlight the beneficial influence of adding a small content of nanoreinforcement in the investigated problem.

6 CONCLUSIONS

An optimization formulation is proposed in this article, to investigate vibration suppression for composite piezoelectric beams. A finite element model for a graphene-fibre reinforced cantilever beam with piezoelectric patches and shunted circuits is developed. Micromechanics equations are used to derive the effective material properties. For the solution of the optimization problem a particle swarm optimization scheme is used. Design variables are the resistance R and the inductance L. Results indicate the proposed approach can achieve vibration attenuation. It is also shown that introducing graphene nanoreinforcement can improve the expected outcome. The study can be extended by solving problems with more design variables, investigating the influence of those new design variables on the optimal vibration response.

REFERENCES

[1] Drosopoulos, G.A. and Stavroulakis, G.E. *Non-linear Mechanics for Composite Heterogeneous Structures*. CRC Press, Taylor & Francis, (2022).

[2] Moheimani, S.R. and Fleming, A.J. *Piezoelectric Transducers for Vibration Control and Damping*. Springer Science & Business Media: Berlin/Heidelberg, Germany, (2006).

[3] Preumont, A. *Vibration Control of Active Structures: An Introduction*. Springer: Berlin/Heidelberg, Germany, (2018).

[4] Marakakis, K., Tairidis, G.K., Foutsitzi, G.A., Antoniadis, N.A. and G.E. Stavroulakis. New optimal design of multimode shunt-damping circuits for enhanced vibration control. *Signals*. (2022) **3**(4):830-856.

[5] Tairidis, G.K. Vibration control of smart composite structures using shunted piezoelectric systems and neuro-fuzzy techniques. *Journal of Vibration and Control*. (2019) **25**(18): 2397-2408.

[6] Thomas, O., Ducarne, J. and Deü, J.F. Performance of piezoelectric shunts for vibration reduction. *Smart Mater. Struct.* (2011) **21**:015008.

[7] Deghboudj, S., Boukhedena, W. and Satha, H. Free Vibration analysis of symmetric laminated composite thin rectangular plate and passive control with attached patches. *J. Fail. Anal. Prev.* (2021) **21**:1240–1251.

[8] Daraki, M.S, Marakakis, K., Alevras, P., Foutsitzi, G.A. and Stavroulakis, G.E. Passive shunted piezoelectric systems for vibration control of wind turbine towers: A feasibility study. *Energies.* (2024) **17**(10):2420.

[9] Drosopoulos, G., Foutsitzi, G., Daraki, M.S. and Stavroulakis, G.E. Vibration suppression of graphene reinforced laminates using shunted piezoelectric systems and machine learning. *Signals.* (2024) **5**(2):326-342.

[10] Jeawon, Y., Drosopoulos, G.A., Foutsitzi, G., Stavroulakis, G.E. and Adali S. Optimization and analysis of frequencies of multi-scale graphene/fibre reinforced nanocomposite

laminates with non-uniform distributions of reinforcements. *Engineering Structures.* (2021) **228**:111525.

[11] Jeawon, Y., Drosopoulos, G.A., Foutsitzi, G., Stavroulakis, G.E. and Adali, S. Optimization of graphene/fibre reinforced cantilever skew laminates for maximum fundamental frequency via non-uniform distribution of reinforcements. *Thin-Walled Structures*. (2023) **189**:110903.

[12] Drosopoulos, G.A., Gogos C. and Foutsitzi G. Multi-objective optimization for maximum fundamental frequency and minimum cost of hybrid graphene/fibre-reinforced nanocomposite laminates. *Structures.* (2023) **54**:1593-1607.