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PRATT TRUSS CHARASTERISTICS FOR OPITMAL WEIGHT WCCM 2024 CONGRESS

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Abstract. This paper presents a study on the characteristics of Pratt trusses under conditions of optimal or near-optimal weight. Trusses with varying numbers of panels, spans, and heights are selected for analysis. Several characteristics describing truss geometry and internal forces are examined. Four dimensioning approaches are developed to perform calculations and obtain data for analysis. A parametric model of truss geometry is developed and integrated with a finite element calculation algorithm in the Rhino8/Grasshopper software. The data are processed and analyzed using the machine learning software Weka and the statistical analysis software RStudio. Results show correlations between various truss characteristics. This study focuses on truss weight and height-span ratio to find the optimal weight. Based on truss height at optimal weight for each span, other characteristics are analyzed. It is observed that a larger number of panels increases the truss weight but also makes the results more consistent and predictable. The objective of this work is to better understand Pratt truss performance, which can be used to reduce the size of optimization tasks.

1 INTRODUCTION

Research on topology optimizations for structures in most cases focuses on methods and tools to perform the optimization. The goal for optimizations is to find the best solution and less to interpret why a specific solution is the best. This paper discusses the structural behavior and characteristics of specific truss topology that ensures optimal weight.

There are various optimization algorithms available for researchers. Common to most of the methods is that the optimal solution is being gradually searched. The more complex is the problem the more computational resources are needed to find the optimal. Researchers tend to find a method that can accommodate a wide range of conditions and provide the optimal solution. Conversely, we can research how to reduce the complexity of the optimization problems. That would ensure effective use of optimization methods already available and reduce the computational resources needed to solve the problem. One way to reduce the complexity is to understand the problem before optimizing. The very basic principles of system need to be explored that can be done by simply observing it while performing. Machine learning (ML) can ensure the transition from an initial complex problem to a simplified one that can be solved with less computational resources [1]. To apply ML, large amount of data is needed. This issue is addressed in the current research by applying parametric algorithms.

Comparative research of building structure topology performance optimal is not so widespread as topology optimization method development. There is research on Pratt and Scissor performance comparison [2], Fan and Howe timber truss performance analysis [3], research on optimizing and comparing Warren, Howe, and Pratt trusses [4]. Some research focuses on developing current optimization methods to reach better results and by analyzing some truss characteristics besides the optimal truss weight [5].

To analyze the performance of a truss, geometric characteristics and attributes that characterize the truss member forces are needed. Research tasks may require developing specific attributes that can lead to finding a solution or developing the tools needed [6]. There are some attributes introduced in this paper to better evaluate the truss performance. The choice of the available research tools can also determine how successfully goals are met. Grasshopper is a well-known commercial software, that also offers benefits of programming. It allows to create tools to find optimal truss topology and geometry [7]. In the context of this research, Grasshopper can offer combination of parametric geometry, FEM visualization of the results, and data recording. Visual evaluation of the actual results and their sequential change can give another perspective and understanding of the truss performance. The possibility to automate calculation process allows to get large amount of data for ML.

The goal of this research is to find if there is a correlation between truss weight and other characteristics, that could narrow the optimization task and reduce computational resources. It is assumed, that truss height-span ration can be the main attribute to explore. Focus is on geometrical parameters since these are available without any further calculations. This do not exclude attributes that characterize member forces.

2 MODELS AND METHODS

A Pratt truss, supported at upper chord ends, is selected for the research. Values for several variables in Table 1 are set by the test and trial method, to evaluate its effects on the truss. Characteristics for optimal weight are evaluated based on four different member sizing approaches and six variations of each. A total of more than 104 000 trusses are calculated.

Table 1: Main variables for truss calculations

Variable	Unit	Description/ value
1st approach (A1)	-	Constant cross sections for truss members
2nd approach (A2)	-	Sizing, profiles from catalogue
3rd approach (A3)	-	Sizing, custom square hollow profiles
4th approach (A4)	-	Sizing, profiles from catalogue, unified profile for top and bottom chords
Initial cross section	m^2	0.0043, 0.01
Load	kN/m^2	4.0, 6.0
Supports	-	Pinned-simple (XYY), pinned-pinned (XYXY)
Number of truss panels	pcs	4, 6, 8, 10, 16, 20
Truss span	m	$L \in \{x \mid 12 \leq x \leq 25, step=1\}$
Truss height	m	$H \in \{x \mid 1 \leq x \leq 3.55, step=0.05\}$

Sizing approaches according to Eurocode 3[8] are developed to observe truss member dimensioning effects on topology response to externally applied loads. First approach (A1) has no sizing applied, so the topology effects are more visible. Sizing A2 is using profile catalogues with various shapes. Then sizing with custom square profiles is applied (A3) to fit profile more exactly to member load. The last sizing approach (A4) is similar to A2, but the profiles for the upper chord and the bottom chord are unified to whole length as for the largest member. This eases production but increases the truss weight. Truss members in tension are calculated for each case separately. For the truss members in compression there are prepared large catalogs of pre-calculated beams with various length domains and profile combinations. Then exact truss members are compared to the catalog values. When choosing appropriate profile for a compressed member there are two ways to define it in algorithm. First is to choose profiles with the least difference of cross-section resistance and/or buckling resistance to the actual truss member force (maximal utilization). The second way is to pick profile with the least cross section area that pass cross-section resistance and/or buckling resistance tests (least weight). It is well known that maximum the utilization in compression does not guarantee the least cross section area. For better understanding, both cases are to be investigated. In this research, the maximum utilization approach is applied. The least cross section approach will be applied in the future research.

The initial cross section 0.0043 m² is obtained from series of tests and trials, which revealed some outliers for each group of the analyzed trusses at a specific span. These outliers appeared due to the limitations of number precision in the selected software. When performing operations with matrices, the algorithm is not always able to perform calculations correctly. As observed, issues are with floating point numbers that are larger than 10^7 or smaller than10^{-7}. To overcome these issues, unit conversions for the modulus of the elasticity were made in several parts of the algorithm. Second value of the initial cross section selected is set to be considerable larger than first, with no other meaning. Loads are selected larger than typically in designs to express more response of characteristics. Truss supports in design mostly are pinned and simple support without horizontal restraint. For large buildings with multiple trusses in row it can be assumed that the middle trusses have limited horizontal movement in supports, therefore pinned-pinned supports are more representative. Both of the cases are included in this study.

Table 2: Main variables for truss calculations

Variation	Load, kN/m²	Cross section size, m²	Supports
V1	4	0.0043	pinned-simple
V2	4	0.0043	pinned-pinned
V3	4	0.0100	pinned-simple
V4	4	0.0100	pinned-pinned
V5	6	0.0043	pinned-simple
V6	6	0.0043	pinned-pinned

The number of panels is set to be suitable for smaller and larger spans and to explore panel effects on truss characteristics. The truss span range is picked to encompass the minimal and maximal spans of conventional steel structures in industrial buildings. For the initial truss design, experienced engineers frequently use height-span ration 0.125 that would give truss height H=1.5 m and H=3.125 m for structures with the spans L=12 m and L=25 m, respectively. To explore the effects of the height-span ratio, the truss height range is set from 1.0 m to 3.55 m with step 0.05 m. Elevation of the top chord is set constant h=0.5 m for all trusses. It was explored that for current span and height range the effect of the top chord elevation is similar to the effect of the height. Each variation of the sizing approaches includes combinations of parameters as load, type of supports, and initial cross section (Table 2).

By carrying out a general study of the obtained data, it has been decided to examine two of the variations (V1, V2) for each of the approaches in more detail. There are twelve iterations for each case – truss is recalculated after the selected member cross section is applied.

To explore truss behavior in different conditions various characteristics is set and recorded for each calculation. Some characteristics are related to truss topology, for example the total length of all the members (L_{sum}), height-span ratio (H/L), number of panels (p), height (H), and truss weight. Other group of characteristics describe internal force distribution among the members and to the truss supports. There is observed minimum and maximum member forces, maximum deflection, the sum of all the member forces (IFS), the sum of the absolute values of the internal forces (IFS_{abs}) and the support reactions. Beside these characteristics some other ones are developed. It is well known that every new member in truss increases its weight. At some point it may be ineffective and may not add significant value to truss performance. To evaluate how large is the summary force in structure members that is generated from externally applied loads (ELS), a system load transmission coefficient (SLT) is introduced both in the simple summation and the sum of absolute values (1), (2).

$$\mathrm{SLT}=\frac{\mathrm{IFS}}{\mathrm{ELS}} \tag{1}$$

$$SLT_{abs}=\frac{IFS_{abs}}{ELS} \tag{2}$$

The ratio is obtained by dividing structure internal member force regular or absolute sum by externally applied force sum. The idea of the force density method, which connects the member force, and the length is introduced in this research. The force density method is a well-known form finding tool for cable net structures. It is based on force equilibrium at mesh nodes. This research focuses on the analysis of Pratt truss topology, instead of finding an optimal one. Characteristics are introduced that are inverse of member length divided by member force. For wider exploration of how this parameter can characterize a truss, several forms are introduced: regular sum (FD_{sum}), absolute sum ($FD_{sum,abs}$), average value (FD_{avg}), and average of absolute values ($FD_{avg,abs}$).

This research involves large number of calculations. To achieve high performance of truss calculations Rhino8/Grasshopper[9] software is used. At first parametric truss geometry code is developed using visual programming and Python scripts. Then finite element method (FEM) is introduced using the very same visual programming principles and connected to the parametric truss geometry script. That allows to feed parametrical truss geometry and other variables as a task for FEM to solve. Visualization of calculation results is developed showing deformations, tension and compression members and members with force close to zero or zero, and proportional members sizes. For truss member sizing a separate code is introduced. It is designed to perform both tension and compression steel member sizing calculations according to Eurocode 3 and to select appropriate profile from catalogue. For the approach A4 there is another code introduced, that evaluates which segment of top or bottom chord has the largest profile. Then this profile is applied by the algorithm to all the top or bottom chord members and the member forces are recalculated accordingly. The truss characteristics are prepared for recording and analysis. A list of variables for each calculation is developed and slider animation is used to execute each of the calculation cases. The execution time for each calculation can vary from less than a second for a truss with four panels and pinned-simple supports to 80 seconds for truss with 20 panels and pinned-pinned supports. While slider animation executes each calculation, Grasshopper code is generating visualization for each calculation results. Visual observations of sequential truss results by increasing its height and span gives another dimension of understanding the truss behavior. When calculations for specific variation of sizing approach is finished, the recorded data are exported and prepared for analysis in the machine learning software Wekka 3.8.6 [10] and statistical analysis software R-studio [11]. Wekka is used to visualize data, for clustering, to select attributes and for classification based on truss weight. R-studio is used to perform regression analysis.

3 RESULTS

The truss calculations provide large amount of data. Research on the data is done gradually and using different methods. At first, the data from each truss member sizing approach and variations are inspected visually. Then the data are analyzed using machine learning algorithms to select, cluster and classify attributes. Finally, mathematical statistical analysis is performed on the data.

3.1 Understanding the data

Strong correlation of truss weight and other characteristics are observed in A1 dimensioning approach where all members have the same cross section (Figure 1 a). The results of other sizing approaches showed visually weaker correlation or no visible correlation (Figure 1 b, c, d). The A1 excludes member dimensioning effects, and the data reflects truss weight dependence on the characteristics of the truss topology. Truss weight correlation with summary member length (L_{sum}), height-length ratio (H/L), number of panels (p) can be described by linear functions. When sizing is applied, this correlation can be described with unimodal function. Finding minimum of the function will give optimal weight and appropriate value of H/L, p or L_{sum}.

To find and rank the truss characteristics that best describe truss weight, the attribute evaluator CfsSubsetEval is used in combination with the BestFirst search method. The attribute selection results show significant differences between A1 and the rest of the three approaches. The best three attributes that describe truss weight are truss height (H), number of panels (p) and total sum of truss member length (L_{sum}) in the A1 approach. When sizing for truss members is applied, the best three attributes slightly differ for each case. The first attribute is the number of truss panels (p), second is the sum of externally applied load (ELS) and the last of top three attributes is vertical support reaction of the truss (support_y). The attribute ranking is summarized in Table 3.

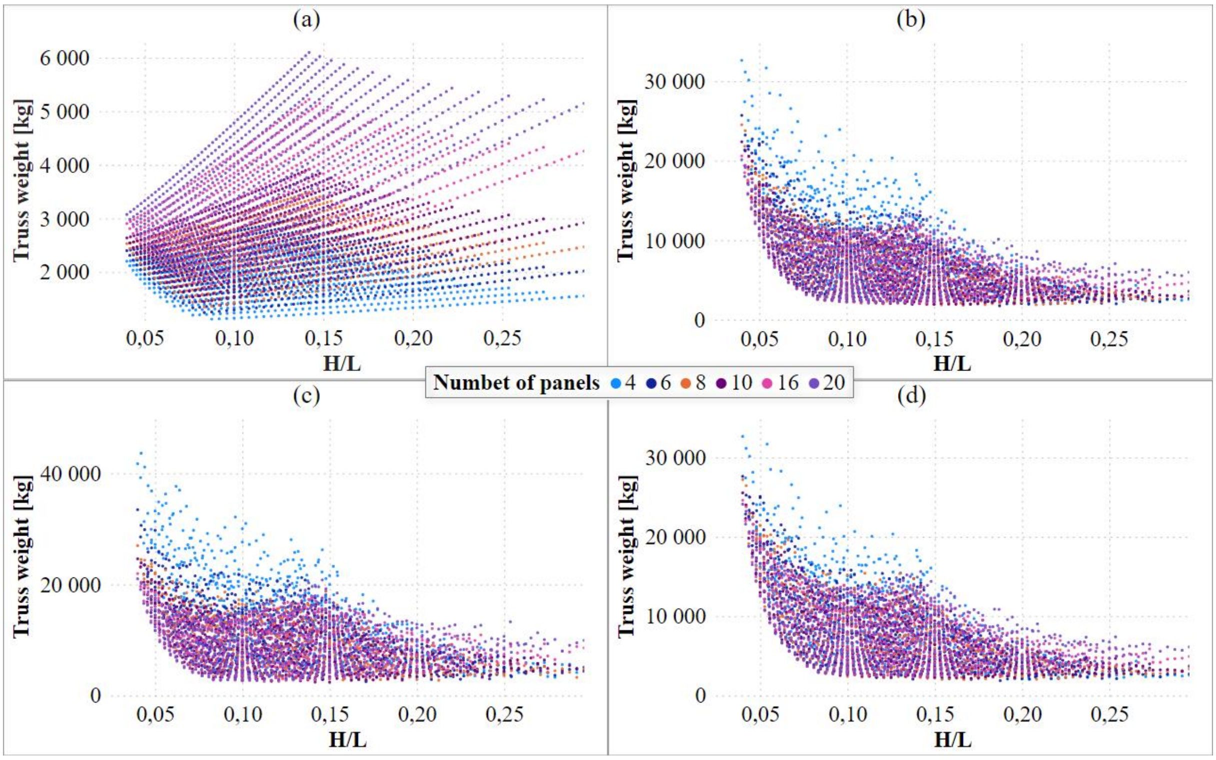

Figure 1: Truss weight and height-span ratio correlation. (a) Approach A1, (b) Approach A2, (c) Approach A3, (d) Approach A4

When the truss calculation results are split by number of panels and this attribute is excluded from the evaluator search list, the top three attributes describing truss weight is height (H), total sum of truss member length (L_{sum}) and sum of externally applied load (ELS). If the member sizing is applied, the total sum of truss member length (L_{sum}) is ranked as the first attribute. The second attribute is sum of externally applied load (ELS) and the third is vertical support reactions of the truss (support_y).

Table 3: Ranking truss weight describing attributes ranking using the CfsSubsetEval evaluator.

Approach	Variation	Attributes by rank		
		1	2	3
A1	V1	Height	Panels	L_{sum}
A1	V2	Height	Panels	L_{sum}
A2	V1	Panels	ELS	Vert. supp. reaction
A2	V2	ELS	Vert. supp. reaction	Min member force
A3	V1	Panels	ELS	$FD_{sum,abs}$
A3	V2	Panels	ELS	Vert. supp. reaction
A4	V1	L_{sum}	ELS	Vert. supp. reaction
A4	V2	ELS	Vert. supp. reaction	Vert. supp. reaction

The difference in truss supports, member sizing and approach are clearly visible when clustering is applied to all data. A simple expectation maximization (EM) algorithm is used, and three clustered instances retrieved. Results of trusses, with supports XY and Y, are added to cluster2 (50% of all data). The remaining results are divided as follows: cluster0 (26%), cluster1 (24%). Inclusion in cluster0 or cluster1 mainly depends on truss span and height. Division in clusters are indicative and can differ applying other algorithms.

3.2 Correlation of truss characteristics

Several Weka classification tree algorithms show good results on predicting truss weight from entire data volume but involved large number of available characteristics. This leads to large, more complex models with limited practical application.

To simplify the models within certain error, the number of attributes is reduced, and the data are split up by sizing approaches. Based on the ranking of the truss characteristics and clustering results, attributes, related to truss topology, supports, and sizing approaches are selected to predict truss weight.

The best results for truss weight prediction are found by M5P algorithm. Weight predictions summarized in Table 4, showing the characteristics selected by the algorithm from those provided. Externally applied load sum is not selected by algorithm in any case.

Table 4: Truss weight prediction by M5P algorithm.

Approach	Variation	r	RRSE	RAE	Truss characteristics				
					L	H	p	ELS	H/L
A1	V1	1	0.90%	0.83%	•	•	•	-	-
A1	V2	1	0.90%	0.83%	•	•	•	-	-
A2	V1	0.98	18.71%	14.63%	•	•	•	-	•
A2	V2	0.98	19.10%	14.66%	•	•	•	-	•
A3	V1	0.97	23.63%	20.77%	•	•	•	-	•
A3	V2	0.97	25.67%	22.98%	•	•	•	-	•
A4	V1	0.98	19.22%	15.50%	•	•	•	-	•
A4	V2	0.98	18.54%	14.76%	•	•	•	-	•

When researching individual span and number of panels for sizing approaches A2, A3 and A4, it is evident that weight fluctuates as truss height is uniformly increased (Figure 2). These fluctuations depend on sizing method, which in this research select the cross-section with the maximum utilization. It can be observed that this approach increases the truss weight prediction error for compression members. It is also notable, that including other profiles than square hollow sections (A2, A4) give smaller truss weight compared to custom catalogue that includes only hollow square profiles (A3). From this perspective, previously described ML model RRSE values are understandable and acceptable to find the truss height H or H/L for minimum regression weight.

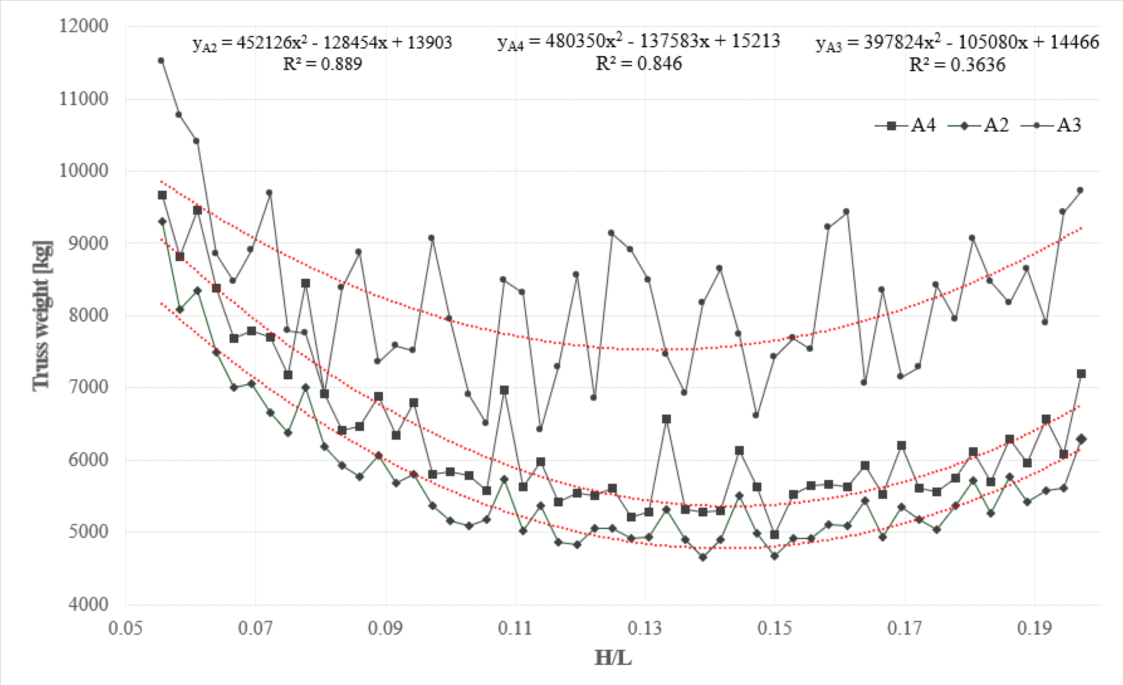

Figure 2: Truss weight and height-span ratio correlation for 18 m span and 8 panels, using sizing approaches.

By continuing to evaluate separate cases based on truss span, number of panels and sizing approach, quadratic regression functions for truss weight and height can be found for each case. The minimum of the function will give the optimal weight with a certain error rate and the appropriate truss height. This specific truss height is used to perform new series of truss calculations and get corresponding characteristics to analyze. The regression optimal weight (ROW) obtained, tends to be lowest truss weight for each group of panels and truss spans calculated initially. It can be observed that as the span increases within each group of certain number of panels, ROW is getting larger compared to minimum within the group. The trusses with 4,6 and 8 panels shows uneven ROW results, which differ the most from minimal values within each span group. In contrast, the trusses with 10, 16, 20 panels exhibit more consistent results, being evenly distributed and staying close to the group minimum. The same is observed for other truss characteristics obtained at ROW case. The successive change of the truss height-span ratio reveals that with an increasing number of panels, the regression optimal H/L ratio is decreasing. The average H/L ratio for each group decreases from 0.166 to 0.115 if the number of panels increases from 4 to 20. The minimum value in all cases is decreasing and tends to approach 0.1 (Figure 3). In each group by number of panels, the smallest H/L ratio is for larger truss spans.

The following compares various parameters characterizing the truss, obtained in the case of the optimal truss mass, with other results in the respective group. Such comparisons enhance the understanding of truss performance in the case of optimal weight. It is possible to observe whether, in the optimal case, the parameter tends to approach the group's minimum, maximum, or average value. The largest compression force in truss members for ROW cases tends to be closer to the average value of each group of span and number of panels. Conversely, it can be said that the average compression force in a span group corresponds to the truss with the lowest weight. Similar situation can be observed for the maximum tension force in trusses. The regression optimal results tend to have the average tension force among the remaining results in the group. A similar correlation can be observed for regression optimal weight and maximum deflection.

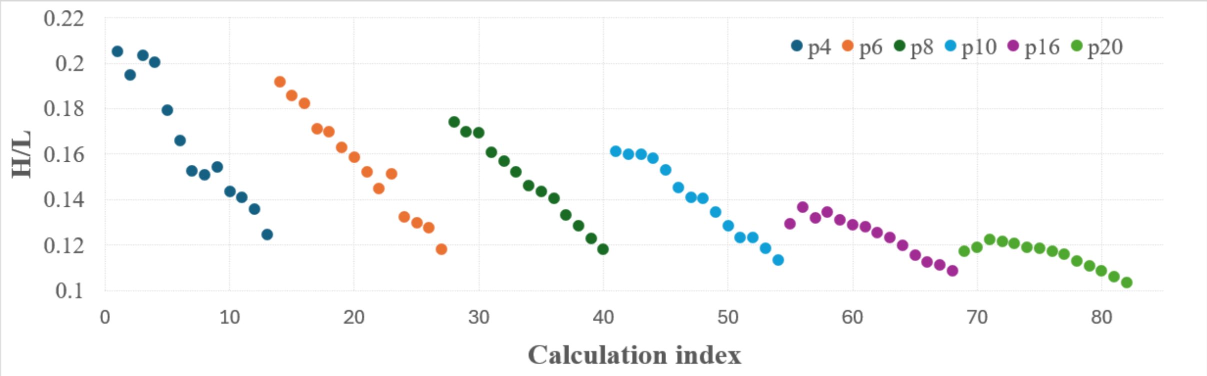

Figure 3: Sequential regression optimal height-span ratio by number of truss panels

For trusses with 4,6 and 8 panels, the maximum deflection of ROW cases tends towards the smallest value for a specific span group. For trusses with 10,16 and 20 panels, the maximum deflection is around the average value of each truss span. Truss member force sum (IFS) and absolute sum of member forces (IFS_{abs}) for regression optimal case with 4,6 and 8 panels shift from average towards the smallest value of the group. For trusses with 10, 16 and 20 panels, the value is around average of each specific span. Other characteristics that were explored for regression optimal is SLT and SLT_{abs}. It represents the ratio between the sum of internal forces and the sum of externally applied load. With increasing number of panels, the regression optimal SLT are more uniform (Figure 4).

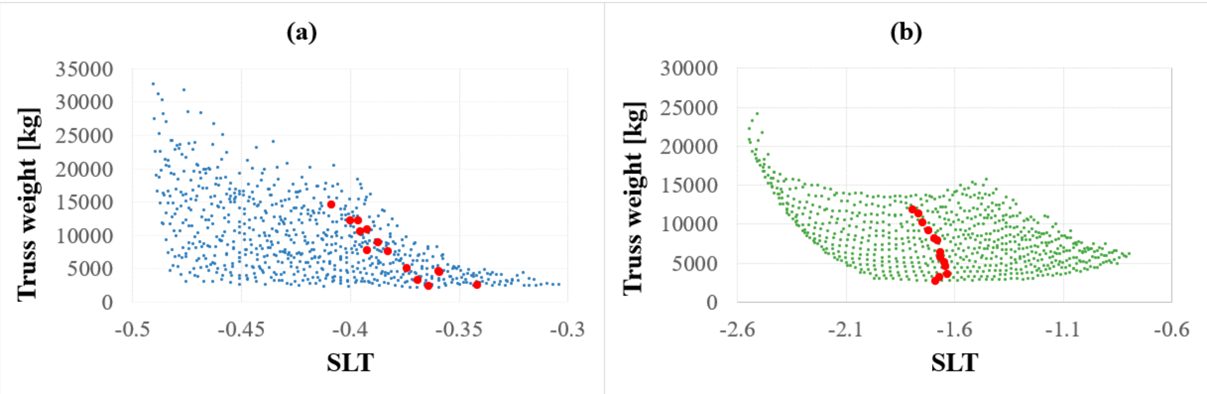

Figure 4: Truss weight and system load transmission coefficient with ROW values. (a) 4 panels, (b) 20 panels

The average values for different number of panels are as follows: SLT_{p4}=-0.38, SLT_{p6}=-0.74, SLT_{p8}=-0.97, SLT_{p10}=-1.14, SLT_{p10}=-1.51, SLT_{p20}=-1.70. All the SLT values are negative, which means that in the regression optimal trusses the sum of compression forces is larger than the sum of tension forces. The absolute summary load transmission coefficients are as follows: $SLT_{abs,p4}$=6.24, $SLT_{abs,p6}$=10.01, $SLT_{abs,p8}$=13.76, $SLT_{abs,p10}$=17.78,

$SLT_{abs,p10}$=30.95, $SLT_{abs,p20}$=40.92. The trusses with the same span but with more panels consist of more members and consequently more material is used, to transfer the same external load. If there was a system consisting of one member and an external load would be applied along its axis, then the externally applied load would be equal to the absolute member force value, resulting in SLT=1. Adding each extra member to the system would increase the SLT value according to specifics of the topology.

The regression optimal weight relation to other truss characteristics can be described. It should be noted that ROW value is not necessarily the optimal solution. It is the point around which, within a certain range, an optimal weight can be found. Respectively, the corresponding truss characteristics of ROW are not specific values, but rather reference points. Regression optimal reference points for characteristics, in context with rest of the data, shows its behavior when reaching optimal performance for minimum weight. Similar approach can be taken for other truss characteristic optimizations. This allows to reduce the initial optimization task and computational resources to find the optimal solution.

4 CONCLUSIONS

This paper introduces the idea of understanding truss topology performance in context of optimization tasks. Correlation has been observed between various truss characteristics. The knowledge of truss performance at regression optimal weight allows to narrow down the range of variables for optimization tasks. There can be explored other truss characteristics, using the same approach.

- The correlation between truss weight and the height-span ratio can be described by a quadratic function to find the regression optimal value within a certain error rate. The R-squared value varies depending on the dimensioning approach, the number of truss panels and the span.
- Narrowing the range of truss height-span ratio around regression function minimum will reduce the optimization problem size and the amount of computational resources needed to find the optimum. The range size should be clarified by performing additional data analysis.
- When dimensioning truss members based on maximal utilization, the total truss weight mainly depends on the sizing results of the compressed members. Regression analysis for optimal weight cases shows that the sum of compression forces in the truss is larger than the sum of tension forces.
- The number of truss panels significantly affects the truss performance. A larger number of panels increases the truss weight, but it also makes the results more consistent and predictable.

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