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KEY COMPONENTS FOR EFFICIENT HIGH RE FLOW SIMULATIONS AROUND AN AIRFOIL USING LBM

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Summary. Accurately simulating the noise induced by typical flows of many technical applications is a challenging task requiring a trade-off between computational costs and the degree of modeling used in the simulation. A method well suited for this is the Lattice Boltzmann Method (*LBM*). Especially advances in wall-modeled (*WM*) large-eddy simulations (*LES*) for *LBM* enabled the application of this method to the field of high Reynolds number (Re) flows, which are omnipresent. The interaction of the individual models (components) in the *LBM* context needs to be further investigated to understand their influence on each other. Previous research on that topic has explored parts of it. In this work, we summarize and analyze recent studies of the group from a meta-perspective. From that, a broader overview and new insights are gained, allowing for better combinations of state-of-the-art key components for *WMLES* in high Re flows and to efficiently tailor them to the target case using the massively parallel *LBM* solver *Musubi*.

1 INTRODUCTION

Many flows in technical applications are characterized by a high Reynolds number (Re). For example, the flow around wind turbine blades typically exhibit a Re in the range of several millions. At the same time, noise reduction in technical flows is increasingly important, as a rising awareness of noise-induced health hazards drive the desire for less obtrusive technical devices. In the case of wind turbines the flow-induced noise mainly originates from the outer part of the rotor blades [1]. We can further identify the dominant humanly notable noise to be emitted by the trailing edge. Aeroacoustic simulations can help to investigate the causes and mechanisms of that noise, as done by Stahl [2], and thereby contribute to design noise reduction strategies. Stahl investigated the mechanism of flow-induced noise generation at the trailing edge with special consideration of its bluntness by means of experiments and simulations for two different Re using the Lattice Boltzmann Method (*LBM*). These Re are in the order of a million, which would result in a total of a trillion elements for a simulation where all scales are resolved (*DNS*). If we then keep in mind, that the computational costs scale with Re and are dependent on the resolved scales, a *DNS* resolving all scales is neither feasible nor economically viable. Thus, modeling is indispensable for these kind of applications. With emerging advances in wall-modeled (*WM*) large-eddy simulations (*LES*) for *LBM*, a large range of modeling options becomes available. The interaction of the individual models or components in the *LBM* context needs to be further investigated to understand their influence on each other. For a fair comparison, different state-of-the-art key components for *WMLES-LBM* simulations of high Re flows were implemented in *Musubi*. For example: advanced collision schemes based on stability enhancing strategies including regularization [3] and recursive regularization [4] in hybrid [5] or projected [6] fashion as well as the promising collision scheme operating in cumulant space [7, 8] were implemented. Their node-level performance was then investigated in [9] to determine the potential for optimization to reduce the time-to-solution. Besides, the solvers communication patterns were investigated [10] and the scalability was improved by using non-blocking communication for health checks [11]. The aim of this work is to examine the interaction of the components by analyzing different investigations [12–15] from a meta-perspective and to tailor them to our target case of the flow around an airfoil with as little modeling as possible.

The paper is organized as follows: in Section 2, the *LBM* solver *Musubi* is presented. Subsection 2.1 gives a brief introduction to *LBM* and specifies the used components. The results of several investigations [12–15] are discussed in Section 3. All the findings are summarized and related to each other from a meta-persepctive to conclude next steps from that in Section 4.

2 LATTICE BOLTZMANN SOLVER MUSUBI

Musubi [16] is an open-source, multi-level parallel *LBM* solver maintained and extended by DLR. It is part of the *APES* software framework, which offers pre- and post-processing tools for simulations on large-scale parallel computing systems. The frameworks centerpiece is the library *TreElM* [17]. It provides a basis for massively parallel mesh-based simulations. For that, it uses an octree discretization in conjunction with the Morton [18] space-filling curve. Which allows for a simple partitioning in *MPI* [19] parallel computations on distributed systems. The mesh for these computations can be either created internally, for simple setups, or externally using the frameworks mesh generator *Seeder* [20]. Due to the octree discretization, Seeder can easily generate multi-level meshes. The interpolation between these different levels of resolution is done in *Musubi*. In recent works, *Musubi* has been investigated and extended to support high Re flow simulations [11–15, 21, 22].

2.1 Lattice Boltzmann Method

We provide a brief description of *LBM* with respect to the aspects discussed in this work. For a detailed introduction, the reader is refered to the book of Krüger et al. [23]. *LBM* is based on the Lattice Boltzmann equation. It simplifies the original Boltzmann equation by discretizing it in space, time, and velocity space and can be written as:

$$f_i\left(\boldsymbol{x}+\boldsymbol{c}_i\Delta\mathrm{t}, t+\Delta\mathrm{t}\right) = f_i\left(\boldsymbol{x}, t\right) + \Delta\mathrm{t}\cdot\Omega_i(\boldsymbol{x}, t). \tag{1}$$

f_i is the discrete Probability Density Function (PDF) of particles streaming from one lattice node $\boldsymbol{x}$ to an adjacent one $\boldsymbol{x}+\boldsymbol{c}_i\Delta\mathrm{t}$ with a lattice velocity of $\boldsymbol{c}_i$ during one time step $\Delta\mathrm{t}$. Thereby, the directions to move in are restricted to the lattice. The nodes of the lattice result from the velocity space discretization which is indicated by the lattice stencil *DdQq*. d is the number of spatial dimensions and q is the number of discrete velocities $\boldsymbol{c}_i$. The most common ones for *3D* are the D3Q19 and D3Q27.

The collision operator $\Omega_i(\boldsymbol{x}, t)$ leads to a local redistribution of the PDFs, such that mass and momentum are conserved. The oldest and simplest collision scheme is BGK, named after its inventors Bhatnagar-Gross-Krook [24]:

$$\Omega_i^{\mathrm{BGK}} = -\omega_i(f_i - f^{\mathrm{eq}}), \tag{2}$$

with f^{eq} being the equilibrium PDF and ω_i the relaxation frequency. The latter one is coupled to the kinematic lattice viscosity via

$$\nu = c_{\mathrm{s}}\Delta\mathrm{t}(\frac{1}{\omega_i} - \frac{1}{2}). \tag{3}$$

Depending on the number of relaxation frequencies, ranging from one up to 27 for all stencil directions in *3D*, different collision schemes are available. In this work, we mainly consider the three collision schemes MRT [25], HRR-BGK [5] and parCUM [8] (parameterized Cumulant). The collision schemes themselves reproduce the Navier-Stokes equations, but miss the energy dissipation that would happen in turbulent flows on the subgrid scale. To overcome this and enable the simulation of high Re flows with lower resolution, a turbulence modeling like *LES* needs to be introduced. Such a model locally modifies the viscosity of the flow (ν_{tot}) by adding an eddy-viscosity (ν_{turb}) to the physical one of the fluid (ν_{phy}):

$$\nu_{\mathrm{turb}} = \left(C_{\mathrm{x}}\Delta\mathrm{x}^2\right)\overline{OP} \quad (4) \qquad \nu_{\mathrm{tot}} = \nu_{\mathrm{phy}} + \nu_{\mathrm{turb}} \quad (5)$$

with $\Delta\mathrm{x}$ being the spatial resolution, C_{x} a model constant and $\overline{OP}$ the model operator. Both, C_{x} and $\overline{OP}$ are dependent on the chosen *LES* model. In addition to the aforementioned LES models, a wall function can be applied to model boundary layers at the wall instead of resolving them.

3 Review of Investigations

This section is dedicated to recent investigations [12–15] of state-of-the-art key components for high Re that were indentified and implemented in *Musubi*. In general, MRT, HRR-BGK, PRR-BGK and parCUM were used as collision schemes along with the D3Q27 stencil. As MRT was found to be unstable with D3Q27 [12], this collision scheme was exceptionally used with the D3Q19 stencil. Except for the parCUM collision scheme, which includes an implicit *LES* (*ILES*), they were combined with one of the three *LES* models: the Smagorinsky [26], Vreman [27] or Wall-Adapting Local Eddy-viscosity (WALE) model [28]. The wall was either modeled by the implicit Musker function [29], the implicit Reichardt function [30] or the explicit combination of the Werner and Wengle [31] with the Schmitt [32] profile, called Power-Law profile.

3.1 *LES* and Collision Scheme Investigations

Spinelli et al. [12] investigated different combinations of collision schemes and LES models for the flow around a cylinder aiming to find the best results in terms of accuracy and performance. They used the following components that are available in *Musubi*:

- Collision schemes: MRT, HRR-BGK, PRR-BGK and parCUM
- LES models: Smagorinsky with Van-Driest damping, Vreman and WALE

For their investigations, they used the quasi *3D* cylinder test case with a diameter based Reynolds number of 3900. The quality of the LES solution according to the LES quality index (*LESQI*) of Celik et al. [33] was assessed. Details on the setup with local grid refinement (multi-level) as well as the *LESQI* can be found in [12]. We briefly summarize and discuss their three comparisons.

Comparison 1 – Influence of Collision Schemes In their first comparison, they compared typical flow quantities against experimental data for the different collision schemes with Vreman as fixed *LES* model and parCUM with its *ILES*. The best results compared to the *DNS* reference data were obtained with parCUM followed by the combinations of HRR-BGK-Vreman and MRT-Vreman. PRR-BGK-Vreman failed to reproduce the Reynolds stresses of the experiment and yielded unsatisfactory results. Therefore, it was excluded from the two following comparisons.

Comparison 2 – Influence of *LES* Models In their second comparison, they investigated the impact of the different *LES* models on the solution and performance. For that, they used HRR-BGK and MRT in conjunction with Smagorinsky, Vreman or WALE. HRR-BGK-Vreman and MRT-WALE were shown to give good results compared to the reference data. However, the results with HRR-BGK-Smagorinsky and HRR-BGK-WALE were not good. The authors have identified the double usage of the velocity gradient as a reason for this. It is used in the collision scheme itself as well as in the *LES* model. Further, they showed that MRT lacks dissipation. As the WALE model is more dissipative than the Vreman, MRT-WALE is a better combination to overcome this issue.

Comparison 3 – Quantitative Comparison of Characteristic Quantities In their third comparison, they assessed the suitability of the best combinations of the first two comparisons – HRR-BGK-Vreman and MRT-WALE – as well as of the parCUM (*ILES*) to accurately predict the characteristic quantities of the test case. Again, the best results were obtained with parCUM.

Performance Investigation Finally, they evaluated the performance of the different collision schemes combined with the Vreman *LES* model, except for the parCUM (*ILES*). In this investigation a uniform mesh along with the following combinations were used: HRR-BGK-Vreman and PRR-BGK-Vreman both with D3Q19 and D3Q27, MRT-Vreman with D3Q19 only and parCUM with D3Q27 only. The parCUM collision scheme with its *ILES* was shown to be both, the best in terms of accuracy and compuational cost, followed by the combination of HRR-BGK-Vreman and the one of MRT-WALE. MRT with D3Q19 was slower than the HRR-BGK counterpart. The data locality of parCUM makes it fast. Different to that, HRR-BGK, PRR-BGK as well as all the *LES* models make use of the velocity gradient. For the calculation of it, they need to access non-local data, namely of the neighbors, which is more costly.

3.2 *WMLES* and Collision Scheme Investigations

The components previously discussed in Subsection 3.1, namely collision schemes and *LES* models, were now extended by using wall functions. In the corresponding publication, Spinelli et al. [13] aimed for a parametric investigation of the most common collision schemes, *LES* models and wall functions using the bi-periodic turbulent channel flow (*TCF*). The *TCF* is a well-known and well-documented canonical test case for wall-bounded flows [34, 35]. In this case, the flow was induced by an external force [36, 37]. The results were compared against *DNS* reference data of [35], amongst other by determining the relative L2-norm. The setup for the test case is described in detail in [13]. For the sake of completeness, the applied components are listed as:

- Collision schemes: MRT, HRR-BGK and parCUM
- LES models: Smagorinsky with Van-Driest damping, Vreman and WALE
- Wall functions: Musker, Power-Law and Reichardt

The same components as in [12] were used except for the PRR-BGK collision scheme (unsatisfactory results) and extended by the wall functions. As before, the parCUM was used with its *ILES* only. In general, the D3Q27 stencil was used, except for MRT (see Section 3). In a total of four comparisons, Spinelli et al. iteratively examined the suitability of each component by changing one at a time.

Comparison 1 – Influence of Domain Size In the first comparison, they empirically analyzed the effects of the domain size on the normalized velocity profiles as well as the normalized Reynolds stresses for three different resolutions and a fixed combination of MRT-Vreman-Musker. For the smallest domain size and the coarsest resolution, they observed fluctuations in the normalized Reynolds stress. But these got mitigated by increasing the resolution. Thus, the smallest domain size was found to be a compromise between accuracy and computational costs. It was then used for further comparisons.

Comparison 2 – Influence of Wall Functions In the second comparison, they investigated the impact of the different wall functions on performance and accuracy for a fixed combination of MRT-Vreman. All wall functions were shown to have a linear convergence order [13, Figure 10 on p. 15]. In terms of accuracy, Musker yielded good results for all three resolutions. In contrast to that, Power-Law was shown to be unsuitable if the resolution lead to the first cell being located in the buffer layer. Although the Power-Law function was the only explicit one, the implicit Reichardt function was cheaper. The computational cost of the Musker function was only slightly higher than the Power-Law function, making the latter one the most expensive ones. Nevertheless, Musker was identified to be a good compromise between computational cost and accuracy for all resolutions. Thus, it was used for the following investigation of different *LES* models and collision schemes.

Comparison 3 – Influence of *LES* Models In the third comparison, Spinelli et al. investigated the effect of the *LES* models for a fixed combination of MRT-Musker. The accuracy obtained with WALE was slightly better at the wall such that it resulted in the following order in terms of accuracy: WALE, Vreman and Smagorinsky. In terms of computational cost, the order looked alike: Vreman, Smagorinsky, WALE, with Vreman being the cheapest. Based on these two rankings, the authors concluded to prefer Vreman. They also stated [13, p. 20]'[...] on average the Vreman model is 10% computationally cheaper than the WALE model.'

Comparison 4 – Influence of Collision Schemes In the fourth comparison, they investigated the influence of the collision scheme on computational cost and accuracy for a fixed combination of Vreman-Musker (except for parCUM due to its *ILES*). The best results in terms of accuracy were obtained by parCUM, except for the middle resolution. For that one, MRT-Vreman-Musker yielded better results. HRR-BGK was observed to severely under-predict the peaks of the profiles as well as being unable to match the *DNS* data for the two coarser resolutions. In terms of computational cost, HRR-BGK was again the worst. MRT was the cheapest. Although it was used with Vreman-Musker it was cheaper than parCUM with that combination. Overall, in terms of both, accuracy and performance, the parCUM-Musker combination (*ILES*) outperformed the other combinations. The HRR-BGK collision scheme failed to reproduce the peaks as well as the trends of the *DNS* reference data in most cases.

3.3 Influence of Blending Parameter σ of HRR-BGK on *WMLES*

While HRR-BGK delivered suboptimal results independent of the combination it was used to run the *TCF* test case (see Subsection 3.2), the ones with HRR-BGK-Vreman for the flow around a cylinder (see Subsection 3.1) were accurate. In order to assess the reasons for that, the *TCF* test case with $\mathrm{Re}_\tau = 1000$ as in the investigation of Subsection 3.2 was used. Based on the previous findings of HRR-BGK-Vreman in Subsection 3.1 and of Vreman-Musker in Subsection 3.2 being good combinations plus the fact, that Vreman is the computationally cheapest *LES* model, the combination of HRR-BGK-Vreman-Musker was used for the next investigation. In that investigation, Spinelli and Gericke [14] varied the blending parameter σ between $[0.9, 1.0]$. In all the previous investigations it was fixed to 0.98 as recommended in literature [5]. HRR-BGK was further enhanced by utilizing a correction term as proposed by Feng [38] and employing the D3Q19 as well as the D3Q27 stencil. For a better overview, the components are listed in the following:

- Collision scheme: HRR-BGK and HRR-BGK-Correction
- LES model: Vreman
- Wall function: Musker
- Blending parameter σ: [0.9, ..., 1.0]

They showed that HRR-BGK was still under-predicting the peaks independent of the applied stencil. Furthermore, they showed that the HRR-BGK-Vreman-Musker combination with D3Q19 was more accurate for the two lowest resolutions. For these two resolutions, the first cell (y_1^+) at the wall was located in the buffer layer ($y_1^+ = 25$) or in the logarithmic layer $y_1^+ = 50$. On the other hand, the combination with D3Q27 gave more accurate results for the highest resolution. In this case, the first cell was located in the viscous sub-layer ($y_1^+ = 12.5$). The authors also stated, that [14, p. 3] '[...] the correction term improve[d] the accuracy of the results on average by 0.1%. For few cases [..., it] worsened.'. Regarding the choice of σ, they found that it was dependent on the chosen stencil: for D3Q19-HRR-BGK-Vreman-Musker, the best results were obtained with $\sigma = 0.998$. For D3Q27-HRR-BGK-Vreman-Musker, the best results were obtained with $\sigma = 1.0$. For the latter one, the HRR-BGK scheme corresponds to the Recursive Regularized BGK (RR-BGK) scheme proposed by Malaspinas [37].

3.4 Influence of Blending Parameter σ of HRR-BGK on Selection of LES and WM

Gericke et al. [15] extended the previous research by analyzing the influence of σ of HRR-BGK with and without correction term on the selection of LES model and the wall function along with its performance impact. They used the most promising ones from previous studies [12,13] discussed in Subsection 3.1 and Subsection 3.2. To account for the entire range of σ-values, they extended the previous range of [14] by 0.0 and 0.5. Again, the used components are listed for a better overview:

- Collision scheme: HRR-BGK and HRR-BGK-Correction
- LES models: Vreman and WALE
- Wall functions: Musker and Power-Law
- Blending parameter σ: [0.0, 0.5, 0.9, 0.92, 0.94, 0.96, 0.98, 1.0]

For $\sigma = 0.0$, the collision scheme corresponds to the PRR-BGK one discussed in [12], for $\sigma = 1.0$ to the RR-BGK one of Malaspinas [37]. In their investigation, Gericke et al. compared their results to those of Vreman-Musker of [14] as well as the *DNS* reference data of Lee and Moser [35]. They stated that the choice of σ depends on the wall function. For Vreman-Musker the best results were obtained with $\sigma = 0.9$, for Vreman-Power-Law with $\sigma = 0.94$. As in the previous investigation (see Subsection 3.3), they observed no benefit, but higher computational costs, if the correction term was used.

3.5 Performance Investigations and Operational Parameters

In the context of *Musubi*, also other performance investigations were done: they were not only helpful to identify and improve performance bottlenecks like the communication patterns [10,11], but also to determine the node-level performance of the kernels (without *LES* and *WM*) [9]. In this investigation it was shown that even with the more expensive D3Q27 stencil, RR-BGK is cheaper than the cheapest HRR-BGK (obtained with D3Q19). Besides, the best relation of *OpenMP* threads to *MPI* processes was determined by means of intra- and inter-node performance measurements [21]. All these information were then combined to determine the operational parameters for the airfoil target case. This, as well as the setup of it were extensively discussed in [22].

4 Insights from the Meta-Perspective: Conclusion and Next Steps

After discussing the details of several investigations in Section 3, we can now step back and look at those findings from a higher-level. This helps us to recognize similarities and differences, but also patterns and underlying principles. Therefore, this section is dedicated to the meta-perspective. We first summarize the four main investigations, before the best combinations are collected in an overview table.

The investigations for combinations of *LES* and collision schemes in Subsection 3.1 revealed that the combination HRR-BGK-Vreman gave accurate results but not HRR-BGK-WALE. MRT-WALE in turn was better than MRT-Vreman. The reason for this was found to be a slight difference in the dissipative behaviour of the two *LES* models and the the lack of it in MRT (see [12, Figure 7, p. 32]). This emphasizes the necessity to investigate the interaction of different components and their characteristics to consider their individual needs when combining them.

The investigations for combinations of *WMLES* and collision schemes in Subsection 3.2 showed that overall, in terms of both, accuracy and computational cost, the parCUM-Musker combination (*ILES*) outperformed the other combinations. If a *LES* model is needed, the best compromise in terms of accuracy and computational cost was found to be Vreman-Musker. The HRR-BGK collision schemes failed to reproduce the peaks as well as the trends of the *DNS* reference data in most cases. As the authors used the suggested blending parameter of $\sigma = 0.98$ as in [5], they attributed the reason for this failure to the recursive regularization. In that, Hermite coefficients of an order higher than two are neglected. While that procedure was stated to be negligible for high Re in [39], the authors suspected that it was not negligible for this test case.

The investigations of the influence of σ on the results obtained with Vreman-Musker in Subsection 3.3 revealed that adjusting σ dependent on the stencil as well as the resolution improved the accuracy. With respect to performance and keeping in mind that for application runs, we do not want to employ the highest resolution they used, there is no benefit by using the more expensive D3Q27 stencil or the correction term for this test case. Besides, this investigation revealed the cause of the previous problem: the failure to reproduce the peaks and general trends was due to the choice of σ, which influences the dissipation of the HRR-BGK scheme. As these investigations were conducted for a fixed *WMLES* combination, further comparisons were made.

The investigations of the influence of σ on the selection of *LES* and *WM* in Subsection 3.4 showed that the choice of σ also depends on the wall function. The best combination was the D3Q19-Vreman-Power-Law-HRR-BGK combination with a σ value of 0.9 and without the correction term independent of the resolution. It was shown that there is no large difference in terms of accuracy if Vreman or WALE were used, but in terms of computional cost. The last observation is different to the one of the first investigation. There, the results with HRR-BGK-WALE were unsatisfactory, emphasizing the dependence of the components on the test case.

All the presented findings are summarized in Table 1 and Table 2. They are presented in such a way that the differences quickly become apparent.

Table 1: Overview of reviewed publications, test cases and systems used for that.

ID	Publication	Test Case	System	Details
#1	Spinelli et al. [12]	Cylinder Re = 3900	CARA	Subsec. 3.1
#2	Spinelli et al. [13]	Turbulent channel flow $Re_\tau = 1000$	CARA	Subsec. 3.2
#3	Spinelli & Gericke [14]	Turbulent channel flow $Re_\tau = 1000$	CARO	Subsec. 3.3
#4	Gericke et al. [15]	Turbulent channel flow $Re_\tau = 1000$	CARO	Subsec. 3.4

In Table 2 a combination is obtained via all components with the same sub- or superscript. For investigation #2, MRT is part of the second and third best combination in terms of accu-

Table 2: Overview of investigations as listed in Table 1 along with the used components. (If a component is used, it is marked via x, otherwise via $-$. The stencil is indicated via $\circ$ (D3Q19) or Δ (D3Q27). The best σ value is given on the left side of the stencil marker (e.g. ${}^{0.98}\Delta$). For investigation #1, PRR-BGK was removed from the table due to the unsatisfactory results. For investigation #3 and #4, RR is employed via HRR $\sigma = 1.0$. #4 is splitted into two rows for better readability. The best combinations are highlighted by the same number in the sub- or superscript per ID. Thereby, the number indicates the rank, with subscripts for a ranking in terms of compuational cost and superscripts in terms of accuracy, e.g. for #1 the second best combination in terms of accuracy (superscript 2) is D3Q19-HRR-BGK-Vreman with $\sigma = 0.98$.
Details on the combinations and abbreviations – Sma (Smagorinsky), Vre (Vreman), WAL (WALE), Mus (Musker), PwL (Power-Law) and Rei (Reichardt) – can be found in the sections mentioned inTable 1.

ID	Collision Scheme				LES			Wall Function		
	RR	HRR	MRT	parCUM	Sma	Vre	WAL	Mus	PwL	Rei
#1	–	${}^{0.98}\Delta^{2}_{2}$	$\circ^{3}_{3}$	Δ^{1}_{1}	x	x^{2}_{23}	x^{3}	–	–	–
#2	–	${}^{0.98}\Delta$	$\circ^{23}_{23}$	Δ^{1}_{1}	x	x^{3}_{2}	x^{2}_{3}	x^{123}_{123}	x	x
#3	$\sigma = 1.0$	${}^{0.998}\circ, {}^{1.0}\Delta$	–	–	–	x	–	–	–	–
#4	$\sigma = 1.0$	${}^{0.9}\circ, \Delta$	–	–	–	x	x	x	–	–
		${}^{0.94}\circ, \Delta$	–	–	–	x	x	–	x	–

racy (superscript 2 & 3). The full combination for the second best in terms of computational cost according to the table is MRT-Vreman-Musker (#2, subscript 2). Like that, we can easily determine the best combinations in terms of accuracy and computational cost for each test case and investigation discussed in Section 3. From the meta-perspective, looking at all these investigations again, it can be seen that parCUM with its *ILES* outperforms the other combinations in terms of accuracy and computational cost. Further, it can be seen that Smagorinsky even in its improved version with Van-Driest damping never ends up in the top three combinations (no sub- or superscript). For #3, there are two different sigma values indicating their depends on the stencil. The best result for D3Q19-Vreman-Musker was obtained with $\sigma = 0.998$, while for D3Q27-Vreman-Musker it was $\sigma = 1.0$. As discussed in Subsection 3.3, for $\sigma = 1.0$ HRR-BGK corresponds to the RR-BGK scheme. From the node-level measurements of Wendler et al. [9], we also know that the RR-BGK performs better and faster than the HRR-BGK collision scheme. With this information in mind, we can choose the combination more wisely. What we can also see from the table is the fact that the choice of σ for investigation #4 depends on the wall function only. And finally, the table nicely highlights the familar dilemma: cost or accuracy. For investigation #2 in terms of computational cost for example, the combination MRT-Vreman-Musker is better (subscript 2) than the MRT-WALE-Musker (subscript 3). But in terms of accuracy (superscripts), it is exactly the other way round. The table provides a fairly comprehensive and clear presentation of four different studies at a glance. It also highlights the added value of looking at several studies from a meta-perspective.

Conclusion and Next Steps

The analysis of several investigations from a meta-perspective underlines the importance of knowing and taking the characteristics of components into account. As the analysis reveals it is important to utilize the correct combination of components to achieve optimal results. parCUM stands out with both good results and good performance. One reason for the latter is its *ILES* so that the usage of a LES model is unnecessary. Accordingly, computational resources can be saved. However, the assumption for the target test case is that with a high Re of 1.2 million, a *LES* model will also be necessary for parCUM. For this reason, parCUM will be combined with different *LES* models in future studies and evaluated in terms of computational costs and accuracy. Up to now, the investigations with HRR-BGK and its blending parameter were conducted for flat walls. To enable a more robust validation of the findings in Subsection 3.4, future work should include additional test cases: e.g. the flow around a cylinder as in Subsection 3.1 or our target case of the flow around an airfoil. The results can then be used to supplement the compact overview of all the studies introduced throuhgout this work in Table 1 and Table 2.

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