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OPTIMAL DESIGN OF LATTICE STRUCTURE FOR COLUMN-LIKE MESO-ELEMENTS

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Summary. Wire-and-Arc Additive Manufacturing (WAAM) allows for the fabrication of lattice structures made of branches having fixed area, with a certain freedom in terms of the overall form. In this contribution, the design of spatial networks for WAAM is addressed, adopting a combined approach of optimization and funicular analysis that is especially conceived for column-like meso-elements. The equilibrium of funicular networks is handled through the force density method, i.e. writing the problem in terms of the ratio force to length in each branch of the network. Independent sets of branches arise when enforcing sector symmetry of the column-like meso-element. An optimization problem is stated in terms of any independent subset of the force densities aiming at retrieving solutions of minimal weight. Local enforcements are formulated to control the location of a selected set of nodes, the range of variation of the length of the branches, and the radius of the column-like structural component. The optimization problem is handled by methods of sequential convex programming, exploiting suitable approximations of the objective function and the constraints. Peculiar features of the achieved layouts are pointed out, as well as of the presented approach.

1 Introduction

Promising applications of metal 3D printing in construction pertain to the adoption of Wire-and-Arc Additive Manufacturing (WAAM) [1]. Indeed, this technique allows for fabricating optimal layouts both exploiting automated production and reduced material use. Among the various applications, WAAM can be adopted to build lattice structures at various scales. This includes the fabrication of meso-elements, as the lattice column whose optimal design is herein investigated [2]. In this application, a "dot-by-dot" deposition technique is considered, which can build networks of thin bars whose cross-section is fixed. This mainly depends on the adopted printing devices and parameters.

Lattice shells have double curvature, while consisting of branches that mainly undergo axial forces [3]. Optimal forms for this kind of structures can be conveniently investigated by using equilibrium-based methods, such as funicular analysis, see e.g. [4, 5]. Reticulated shells can be modelled as networks consisting of branches (either struts or ties) with given connectivity. Boundary supports are prescribed at the restrained nodes of the network, whereas unrestrained ones are such that the forces arising in the branches are in equilibrium with the applied point loads. The equations governing the equilibrium of the unrestrained nodes are not linear in the coordinates of the nodes. However, by introducing the concept of force densities, i.e. the ratio force to length in the branches of the network [6], these equations become linear and uncoupled in the three spatial directions. The force densities may be conveniently adopted as unknowns for any form-finding process, especially when coupled with an optimization strategy, see in particular [7, 8, 9].

In this contribution, a special version of the approach presented in [7] and [8] is explored, by embedding the Force Density Method (FDM)[6] within a multi-constrained minimization problem and taking benefit of the symmetry conditions that are requested in the investigated structural application. Due to its peculiar form, the arising optimization problem can be efficiently solved through techniques of sequential convex programming [10] that were originally conceived to handle large-scale multi-constrained formulations of size optimization for elastic structures, see [11] among the others.

The research of the optimal shape of lattice columns is made by adopting as minimization unknowns a reduced set of independent force densities that descends from the required rotational symmetry. The adopted objective function is the weight. Constraints are of geometric type, being related to the minimum and maximum length of the members, the minimum radius of the column-like meso-element, and the minimum value of the vertical coordinate of the top nodes of the spatial network.

In the next sections, a brief overview of the force density method is given, and the multi-constrained problem is presented. A numerical example is shown to assess the method, explore some anti-funicular lattice layouts, and draw some preliminary conclusions.

2 Force density method

The force density method [6] is used as “state equation” of the optimization problem to handle the equilibrium of spatial lattices. A funicular network is made of $n_s = n + n_f$ nodes and m branches, which can withstand axial forces only. The axes of the Cartesian reference system with origin O are denoted by x, y, and z. According to the original notation, $\mathbf{x}_s$, $\mathbf{y}_s$, $\mathbf{z}_s$ are vectors that gather the coordinates of the n_s nodes: $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$ refer to the n unrestrained nodes, i.e. the nodes where external forces are exerted; $\mathbf{x}_f$, $\mathbf{y}_f$, $\mathbf{z}_f$ collect the n_f restrained nodes, i.e. those where reactions arise. The connectivity matrix that fully describes the shape of the grid making the lattice is labelled as $\mathbf{C}_s$, having subset $\mathbf{C}$ for the unrestrained nodes and $\mathbf{C}_f$ for the restrained ones. The vectors that collect the coordinate difference of the nodes along the axis x, y, z are denoted by $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$, respectively:

$$\begin{aligned} \mathbf{u} &= \mathbf{C}_s\mathbf{x}_s, \\ \mathbf{v} &= \mathbf{C}_s\mathbf{y}_s, \\ \mathbf{w} &= \mathbf{C}_s\mathbf{z}_s. \end{aligned} \tag{1}$$

The force densities, i.e. the ratios force to length for each branch of the network, are stored in the vector $\mathbf{q} = \mathbf{L}^{-1}\mathbf{s}$, being $\mathbf{s}$ the vector that collects the forces in the m branches. The length of the i-th branch reads $l_i = \sqrt{u_i^2 + v_i^2 + w_i^2}$, being gathered in the square matrix $\mathbf{L} = \mathrm{diag}(\mathbf{l})$. Only vertical loads are prescribed in this study, through vector $\mathbf{p}_z$. Due to the introduction of the vector $\mathbf{q}$, the equilibrium of the unrestrained nodes is given by a set of linear equations that are uncoupled in the three axes, i.e.:

$$\begin{aligned}
&\mathbf{C}^T\mathbf{Q}\mathbf{C}\mathbf{x} + \mathbf{C}^T\mathbf{Q}\mathbf{C}_f\mathbf{x}_f = \mathbf{0},\\
&\mathbf{C}^T\mathbf{Q}\mathbf{C}\mathbf{y} + \mathbf{C}^T\mathbf{Q}\mathbf{C}_f\mathbf{y}_f = \mathbf{0},\\
&\mathbf{C}^T\mathbf{Q}\mathbf{C}\mathbf{z} + \mathbf{C}^T\mathbf{Q}\mathbf{C}_f\mathbf{z}_f = \mathbf{p}_z,
\end{aligned} \tag{2}$$

where $\mathbf{Q} = \mathrm{diag}(\mathbf{q})$.

In this contribution, column-like meso-elements for which rotational symmetry of the lattice is required are investigated. This implies that only a reduced set of force densities, i.e. the independent force densities stored in the vector $\bar{\mathbf{q}}$, govern the form finding process. One has:

$$\mathbf{q} = \mathbf{B}\bar{\mathbf{q}}, \tag{3}$$

where B_{ij} is equal to one either if $i = j$ or $q_j = \bar{q}_i$ (due to the symmetry requirement), otherwise $B_{ij} = 0$.

3 Multi-constrained optimization problem

A multi-constrained minimization problem is stated in terms of any sub-set of independent force densities $\bar{\mathbf{q}}$, with the aim of designing lightweight lattices. It reads:

$$\begin{cases}
\min\limits_{\bar{q}_i} \ f & \text{(4a)}\\
\text{s.t.} \quad \mathbf{C}_x^T\mathbf{Q}\mathbf{C}_x\mathbf{x} + \mathbf{C}_x^T\mathbf{Q}\mathbf{C}_{fx}\mathbf{x}_f = \mathbf{0}, & \\
\qquad \mathbf{C}_y^T\mathbf{Q}\mathbf{C}_y\mathbf{y} + \mathbf{C}_y^T\mathbf{Q}\mathbf{C}_{fy}\mathbf{y}_f = \mathbf{0}, & \text{(4b)}\\
\qquad \mathbf{C}_z^T\mathbf{Q}\mathbf{C}_z\mathbf{z} + \mathbf{C}_z^T\mathbf{Q}\mathbf{C}_{fz}\mathbf{z}_f = \mathbf{p}_z, & \\
\qquad \mathbf{q} = \mathbf{B}\bar{\mathbf{q}}, & \text{(4c)}\\
\qquad \left(\dfrac{l_i}{l_{min}}\right)^2 \geq 1 \quad \text{for } i = 1...m, & \text{(4d)}\\
\qquad \left(\dfrac{l_i}{l_{max}}\right)^2 \leq 1 \quad \text{for } i = 1...m, & \text{(4e)}\\
\qquad \dfrac{x_i^2 + y_i^2}{r_{min}^2} \geq 1 \quad \text{for } j = 1...n, & \text{(4f)}\\
\qquad z_{j_t} \geq z_t^{min} \quad \text{for } j = 1...n_t, & \text{(4g)}
\end{cases}$$

In the above discrete problem, the objective function is the total length of the network, which can be computed as:

$$f = \sum_{i=1}^{m} l_i. \tag{5}$$

The "dot-by-dot" WAAM technology is such that all printed members have the same cross-section, meaning that minimizing the total length of the bars in the network provides the same result as minimizing the overall weight of the column-like meso-element.

The system of Eqns. (4b) states the equilibrium of the unrestrained nodes in the three spatial directions, which allows for computing the coordinates $\mathbf{z}$ from the whole set of force densities in $\mathbf{q}$. The relation linking $\mathbf{q}$ to the minimization unknowns $\bar{\mathbf{q}}$ is given in Eqn. (4c).

Eqns. (4d) and (4e) are set of local constraints that are used to prescribe the minimum (l_{min}) and maximum (l_{max}) value of the length of each branch in the optimal network. The coordinate difference of the connected points given in Eqn. (1) are used to enforce these geometric constraints in a straightforward way. They also play a key role in regularizing the optimization problem. Controlling l_{min} prevents the arising of dense regions which may be difficult to print, whereas limiting l_{max} means enforcing a maximum slenderness to prevent member buckling.

Eqn. (4f) is used to enforce a minimum radius (r_{min}) for the column-like meso-element, aiming at preserving a minimum value of the second moment of area, against column buckling.

The inequalities in Eqn. (4g) are used to prescribe a minimum elevation for a subset of the nodal coordinates $\mathbf{z}$, i.e. the top nodes. Indeed, z_t^{min} is the minimum height of the column.

The multi-constrained minimization problem is solved by means of the Method of Moving Asymptotes [10], see the discussion in Section 1. Being MMA a first order approach, the sensitivity of the objective function and constraints with respect to the force densities $\mathbf{q}$ is needed, see e.g. [8] and [12]. The chain rule may be used to compute the derivatives with respect to the minimization variables $\bar{\mathbf{q}}$, see Eqn. (4c). It must be remarked that a Nested Analysis and Design (NAND) approach is herein adopted. This means that, at each iteration, Eqns. (4b) are solved for the current set of $\bar{\mathbf{q}}$ (and $\mathbf{q}$). Also, due to the limited number of minimization unknowns, the enforcement of the local constraints in Eqns. (4d)–(4g) does not call for the implementation of ad hoc techniques to preserve numerical efficiency, see in particular [13].

4 Numerical example

A lattice column is addressed, inspired by the investigations presented in [2, 14] in the field of Wire-and-Arc Additive Manufacturing (WAAM), see also [1, 15]. Using the "dot-by-dot" deposition technique with a 304L stainless steel wire, bars with a diameter of 6 mm can be fabricated [8]. A column-like meso-element with bottom and top fixed radius equal to $r = 0.9/\pi$ m is herein investigated, considering three types of lattice, whose connectivity is represented in Figure 1. The lattice type A is diamond-like, whereas type B and type C are diagrid-like lattices.

The vertical load is applied at the top 12/24 nodes, with a resultant of 10 kN. These top nodes are restrained along the x and y axes, whereas the 12/24 bottom nodes are restrained in the three Cartesian axes. The order of rotational symmetry enforced by Eqn. (4c) is twelve.

The minimization of the total length of the members making the network is addressed by implementing the formulation in Eqn. (4) with the following parameters: $l_{min} = 0.05$ m, $l_{max} = 0.15$ m, $r_{min} = 0.075$ m, and $z_t^{min} = 2.0$ m. It is remarked that, according to the investigation reported in [8], a single bar can withstand a maximum of approximately: i) 7 kN in tension, and ii) 2 kN in compression for a slenderness equal to 100, which corresponds to the herein enforced $l_{max} = 0.15$ m. If not differently specified, the optimization is initialized by using a homogeneous distribution of compressive force densities. For all the optimal lattices, the constraints in Eqn. (4g) are active, meaning that the height of the column-like meso-element is the same, i.e. z_t^{min}. The same applies for a subset of the constraints in Eqn. (4e), meaning that in each solution there exist some branches whose length equals l_{min}. It is finally reported that all the achieved solutions fulfill the entire set of enforced constraints.

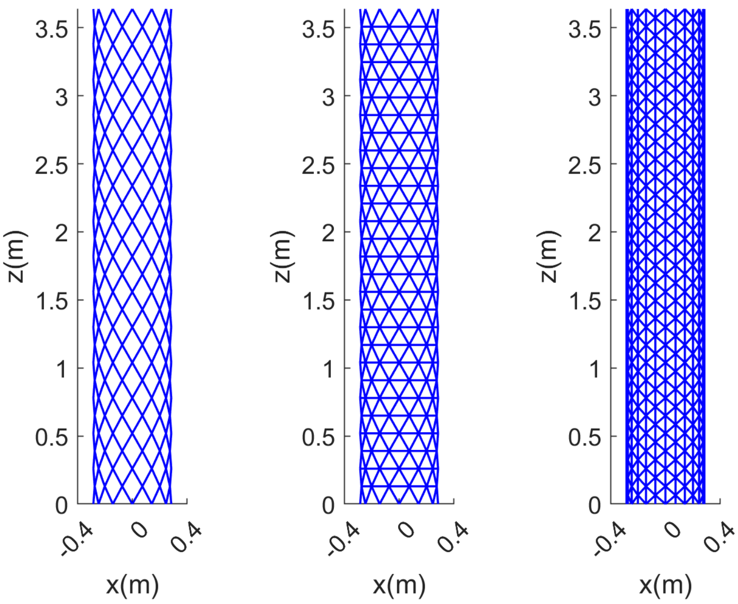

Figure 1: Connectivity of the members for a column-like meso-element: lattice type A (left), lattice type B (center), lattice type C (right).

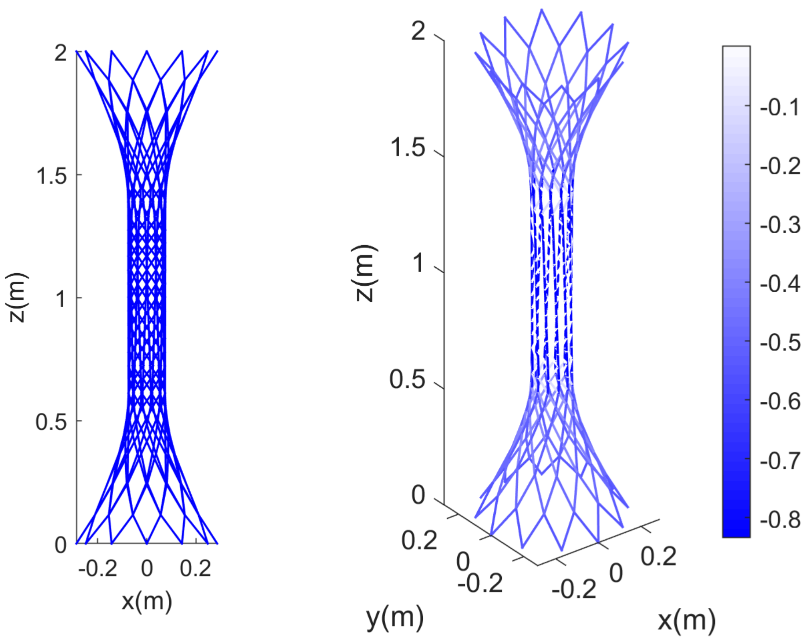

Figure 2: Optimal design (left) and map of the element forces (right, in kN) for lattice type A.

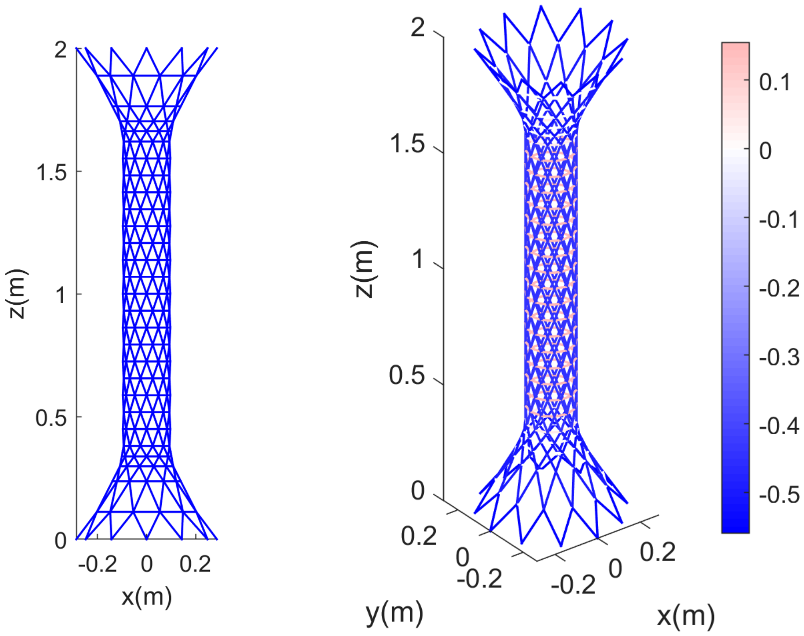

Figure 3: Optimal design (left) and map of the element forces (right, in kN) for lattice type B.

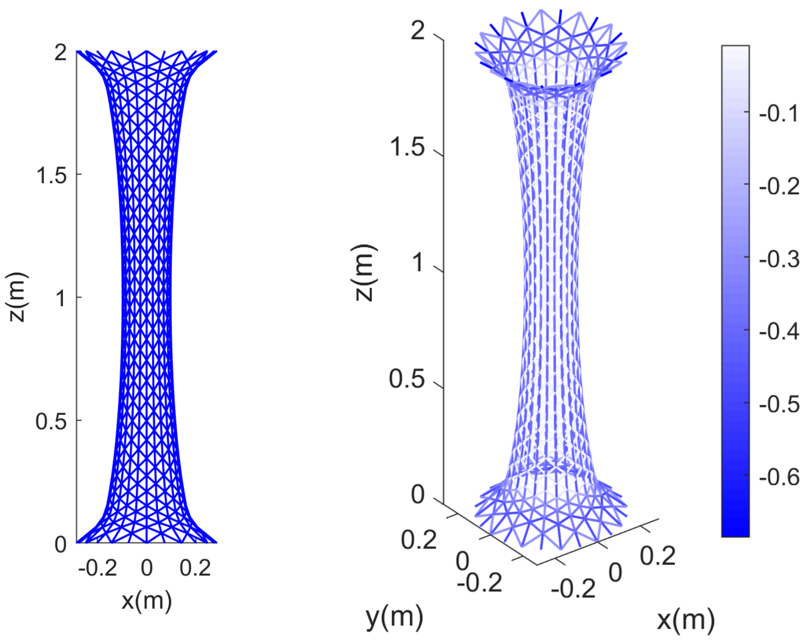

Figure 4: Optimal design (left) and map of the element forces (right, in kN) for lattice type C.

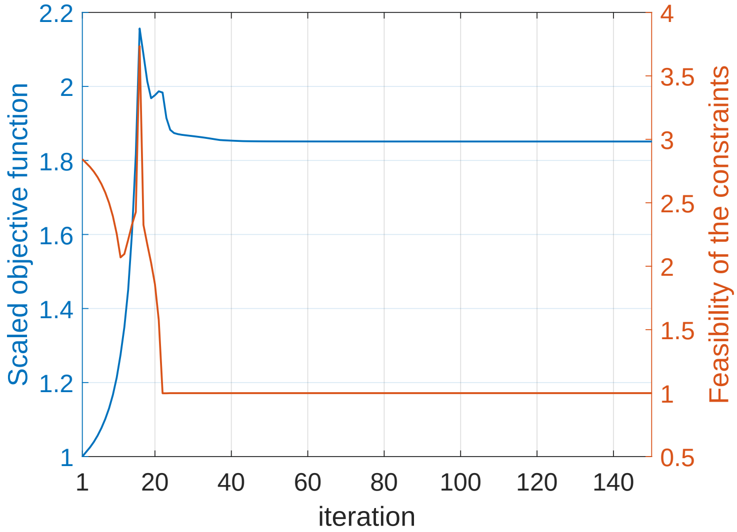

Figure 5: Convergence plots for the form-finding using lattice type C.

The optimal layout found for the lattice type A is represented in Figure 2, along with a map of the forces $\mathbf{s} = \mathbf{Lq}$ computed at convergence. The set $\mathbf{s}$ is in equilibrium with the prescribed vertical load, as per Eqns. (4b). The total length, i.e. the value of the objective function at convergence, equals 54.4 m. Some constraints on l_{max} and r_{min} are active at convergence. Overall, the diamond-like patterns is preserved at the extreme, whereas in the central part of the column the bearing structure mainly consists of vertical elements, see in particular the branches with no force in Figure 2(right).

The optimal result for the lattice type B and the relevant map of forces are given in Figure 3. In this case an initialization with tensile force densities has been adopted for the elements in the hoops. At the ends of the column the form-finding retrieves a diamond-like layouts that resembles that of Figure 2. However, in the inner region the original diagrid-like pattern with straight vertical elements and horizontal tensile-stressed hoops is maintained. The overall length is 72.9 m, whereas the minimum radius of the column is larger than r_{min}. Some constraints on l_{max} are active at convergence.

The result obtained when using lattice type C and the relevant map of forces are depicted in Figure 4. The overall length is 121.5 m. As for the lattice type B, the minimum radius of the column is larger than r_{min}, and some of the constraints on l_{max} are active at convergence. The optimal design is characterized by a central region made of bars with a smooth curvature, which mainly transfer the vertical forces to the ground restraints. The diagrid scheme is entirely active at the ends.

It is remarked that neither stress constraints nor overhang constraints have been implemented in this preliminary numerical investigation, see [8]. Indeed, the achieved force maps were found to be compatible with the strength of the bars. Concerning overhang, while optimal results for the lattices type A and type C are marginally affected by such an issue, the printing strategies discussed in [1] and [14] are referred to when coping with the hoops of the optimal solution found for the lattice type B.

Finally, in Figure 5, two convergence curves reported for the optimization of the lattice type

C are given. They refer to the history plot of the objective function (scaled to its initial value) and of the feasibility of the constraints (equal to one when all the enforcements are fulfilled). A few iterations are needed to reach a feasible region of the design domain, whereas smooth convergence is reported in the subsequent steps.

5 Concluding remarks

In this contribution, a numerical tool has been implemented to address the design of column-like meso-elements through funicular analysis. As investigated in the recent literature, the force density method can be conveniently implemented to cope with the equilibrium of spatial networks of bars, especially when coupling the form-finding tool with optimization routines. Dealing with “dot-by-dot” WAAM, the overall length of the bars in the lattice has been adopted as objective function. Independent sets of branches have been used as minimization unknowns, exploiting sector symmetry. Local enforcements have been formulated to control the location of a selected set of nodes, the range of variation of the length of the branches and the radius of the column-like structural component. The arising multi-constrained problem has been handled by techniques of sequential convex programming.

A preliminary numerical investigation has been performed, considering a diamond-like lattice and two diagrid-like ones. Peculiar optimal solutions have been found, reporting smooth convergence and full feasibility of the achieved layouts with respect to the enforced constraints. These results must be intended as possible shapes to be further investigated and enhanced by tools of structural analysis, especially dealing with second order effects.

The ongoing research is mainly devoted to the exploration of energy-based objective functions and to the investigation of efficient strategies to control global buckling in the design stage.

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